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Gap Analysis and Review of the Utah Core Standards for Mathematics (UCS-M)

Final Report

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Executive Summary

In May 2026, WestEd contracted with the Utah State Board of Education (USBE) to conduct an independent, third-party review of the proposed draft of the Utah Core Standards for Mathematics (UCS-M). This review was intended to provide a thorough review of the UCS-M for its rigor, clarity, measurability, instructional focus, developmental appropriateness, and the integration of mathematical practices, along with a Benchmark comparison of the UCS-M with high-performing national and international frameworks and standards. This report integrates the findings across two phases of WestEd's reviews and includes recommendations and a summary of the strengths and opportunities for refinement of the draft UCS-M.

This review was completed in two complementary phases. The first phase compared the UCS-M with three benchmark systems: the Massachusetts Mathematics Curriculum Framework, the Minnesota Academic Standards in Mathematics, and the Singapore Mathematics Syllabus. These comparisons included standards-level crosswalks, domain-level analyses of content coverage, and comparisons of the Utah Mathematical Practices with the Standards for Mathematical Practices (SMPs) used by most U.S. state mathematics standards. The second phase consisted of a standards-level review in which each Utah standard was evaluated using rubrics (see Appendix-A for the rubrics used) that addressed the measurability, instructional focus, clarity and accessibility, mathematical rigor, Depth of Knowledge (DOK), developmental appropriateness, and the integration and appropriateness of the Utah Mathematical Practices. Together, these analyses provide a comprehensive evaluation of the quality of the draft UCS-M.

Overall Findings

Overall, the review found that the UCS-M provide a strong and coherent framework for mathematics teaching and learning across the PK–12 continuum. The standards demonstrate substantial alignment with nationally recognized mathematics standards while incorporating several distinctive features that reflect Utah's educational priorities. Across both phases of the review, the evidence suggests that the standards are generally well organized, instructionally focused, developmentally appropriate, and mathematically rigorous. The review did not identify evidence suggesting a need for major structural revisions to the standards. Rather, the findings point primarily toward opportunities to improve the clarity, measurability, and sequencing of selected standards.

Major Strengths

Several strengths emerged consistently across the benchmark comparisons and standards review.

- **Strong alignment with nationally recognized mathematics standards.** The UCS-M were found to be highly comparable to the Massachusetts Mathematics Curriculum Framework. While broader differences were found in comparison to the Minnesota and Singapore mathematics standards, these mostly reflected alternative approaches to sequencing and curricular emphasis rather than fundamental differences in mathematical expectations.
- **High-quality, well-balanced mathematics standards.** The standards demonstrate a strong balance of conceptual understanding, procedural skill and fluency, and application, while maintaining instructional focus consistent with high-quality mathematics standards used throughout the United States.
- **Coherent PK–12 progression.** The review found that the standards are developmentally appropriate and establish a logical PK–12 progression of mathematical learning. Cognitive demand also progresses appropriately from foundational knowledge in the early grades to increasingly sophisticated mathematical reasoning in the upper grades and high school.
- **Distinctive emphasis.** The standards include a comprehensive PK–12 Data Science progression and explicit integration of the Utah Mathematical Practices throughout the content standards, distinguishing the UCS-M from many existing state mathematics standards.

Opportunities for Refinement

While the review identified many strengths, several recurring opportunities for refinement emerged across the benchmark comparisons and standards review.

- **Clarity and measurability of selected standards.** Reviewers identified opportunities to improve the clarity, measurability, and specificity of performance expectations within a limited number of standards. Most observations related to the wording of standards rather than the underlying mathematical content.
- **Integration of the Utah Mathematical Practices.** The review found that the Utah Mathematical Practices were generally well integrated into the standards, while identifying opportunities to further clarify how they are expressed within selected content standards.
- **Developmental progression of selected standards.** Reviewers identified a small number of standards that may warrant further consideration regarding their placement or expected level of reasoning, particularly in the early grades.
- **Localized benchmark differences.** The benchmark analyses identified selected differences

in content emphasis and sequencing between the UCS-M and the benchmark standards that USBE could consider during revisions. These differences were particularly found in Geometry and Measurement, some advanced Secondary topics, and Utah's PK–12 Data Science progression.

Recommendations

Based on the evidence collected during this review, WestEd offers the following recommendations for USBE's consideration:

- **Preserve the overall structure and organization of the UCS-M.** The review found substantial alignment with nationally recognized mathematics standards and no evidence supporting major structural revisions.
- **Prioritize targeted revisions.** Targeted revisions could improve the clarity, measurability, and accessibility of selected standards while preserving their underlying mathematical intent. Review the placement of the relatively small number of standards identified through the developmental appropriateness review to determine whether adjustments to sequencing or expected reasoning would strengthen the overall progression.
- **Continue refining the integration of the Utah Mathematical Practices.** The review found opportunities to clarify the Utah Mathematical Practices while maintaining their explicit integration throughout the standards.
- **Use the attached analyst rating worksheets to inform standard-specific revisions.** These resources provide detailed reviewer comments, suggested revisions, and benchmark comparisons that complement the findings presented in this report.

Concluding Remarks

Taken together, the benchmark analyses and standards review indicate that the draft of the UCS-M provides a high-quality foundation for mathematics instruction across the PK–12 continuum. The standards are broadly aligned with nationally recognized mathematics frameworks while incorporating several distinctive features that reflect Utah's educational priorities. The opportunities for refinement identified through this review are concentrated primarily in the wording, implementation, and sequencing of selected standards rather than the overall design of the standards themselves. Consequently, this report should be viewed as supporting the continued refinement of an already strong standards framework, with the accompanying analyst worksheets and benchmark crosswalks providing detailed guidance for future revisions.

Introduction

In May 2026, WestEd was contracted by the Utah State Board of Education (USBE) to conduct an independent, third-party review of the proposed draft of the Utah Core Standards for Mathematics (UCS-M). The purpose of the review was to provide an objective evaluation of the standards using multiple sources of evidence, including comparisons with nationally and internationally recognized benchmark systems and a detailed standards-level review of the quality and characteristics of the Utah standards themselves.

High-quality mathematics standards serve as the foundation for curriculum, instruction, assessment, and professional learning. Consequently, the review was designed to examine not only whether the Utah standards address the breadth and depth of mathematical content reflected in high-performing benchmark systems but also whether the standards are written in a manner that supports clear interpretation, high-quality instruction, coherent learning progressions, and valid assessment. Together, these complementary analyses provide a comprehensive evaluation of the UCS-M and identify areas of alignment, distinction, and opportunities for refinement.

This report presents the results of WestEd's independent review of the UCS-M and synthesizes findings from the two phases of the review integrating evidence across all analyses. Unlike the interim memos (i.e., the Gap-Analysis Memo and the Standards-Quality-Review Memo) prepared during the course of the project, which separately summarized individual components of the review, this report presents an integrated evaluation of the Utah standards, highlighting overarching patterns, recurring themes, and evidence-based considerations to support USBE's review of the draft standards.

Purpose

The purpose of this report is to provide a comprehensive evaluation of the proposed UCS-M. Specifically, the report examines

a) the extent to which the Utah standards align with nationally and internationally recognized mathematics standards;
b) the overall quality of the standards with respect to measurability, clarity, instructional focus, rigor, developmental appropriateness, and the integration of the Utah Mathematical Practices; and
c) the overarching strengths, notable differences, and opportunities for refinement that emerge when findings are synthesized across all analyses.

Findings from these complementary analyses are integrated to provide evidence-based observations and considerations to support USBE's review of the proposed UCS-M.

Scope of Work

The review consisted of two complementary phases. First, WestEd conducted standards-level benchmarking analyses comparing the UCS-M with three nationally and internationally recognized benchmark systems: the Massachusetts Mathematics Curriculum Framework, the Minnesota Academic Standards in Mathematics, and the Singapore Mathematics Syllabus. These analyses examined standards-level alignment, content coverage, domain emphasis, grade-level placement, and the progression of major mathematical ideas across the PK–12 continuum.

Second, WestEd conducted a comprehensive standards-quality review of all Utah mathematics standards. Using structured review rubrics developed in collaboration with USBE (see Appendix-A), reviewers evaluated each standard for measurability, instructional focus, clarity and accessibility, DOK, mathematical rigor, developmental appropriateness, and the integration and appropriateness of the Utah Mathematical Practices. Independent reviews, calibration activities, double-coding, and quality-control procedures were used to promote consistency in the application of the review criteria.

The analyses presented in this report synthesize findings across these complementary lines of evidence to provide a comprehensive evaluation of the UCS-M. Throughout the report, findings are considered collectively rather than in isolation to identify recurring themes, cross-cutting observations, and areas for consideration.

The Utah Core Standards for Mathematics (UCS-M)

The draft UCS-M present a PK–12 progression designed to develop conceptual understanding, procedural fluency, and mathematical reasoning. The standards are organized around major mathematical domains in PK–8 and at the high school level. For the purposes of comparisons with Benchmark standards, the domains within the UCS-M in PK–8 are summarized into the domains of Number and Operations, Ratios and Proportional Reasoning, Algebra, Functions, Geometry and Measurement, and Statistics and Data Science; at the high school level, the UCS-M are summarized by the domains of Algebra, Functions, Number Systems, Geometry, Data Science, and Matrices and Vectors. In addition to the content standards, the UCS-M explicitly integrate eight Utah Mathematical Practices throughout the standards and identify Essential Competencies at each grade level to highlight foundational learning priorities. The standards also include expectations related to mathematical modeling and data science across the PK–12 progression.

Methodology

WestEd completed the review process in two phases: (a) a benchmarking analysis and (b) a standards-quality review. Together, these complementary analyses provided multiple sources of evidence regarding the content, organization, quality, and instructional utility of the proposed UCS-M.

The benchmarking analysis included standards-level crosswalks comparing each Utah standard with corresponding standards in the Massachusetts Mathematics Curriculum Framework, the Minnesota Academic Standards in Mathematics, and the Singapore Mathematics Syllabus. These analyses examined standards-level alignment, domain-level content coverage, differences in grade-level placement, and the relative emphasis placed on major mathematical domains. Reviewers also compared the Utah Mathematical Practices with the Standards for Mathematical Practice reflected in the Massachusetts and Minnesota frameworks.

The standards-quality review employed a structured set of review rubrics developed in collaboration with USBE (see Appendix-A). Four mathematics content experts independently evaluated each Utah standard for measurability, instructional focus, clarity and accessibility, DOK, mathematical rigor, developmental appropriateness, and the integration and appropriateness of the Utah Mathematical Practices. Reviewers completed calibration exercises prior to coding, approximately 20 percent of standards were independently double-coded to evaluate reviewer consistency, and standards requiring additional discussion were adjudicated through a structured quality-control process led by a senior mathematics content expert.

This section provides a high-level overview of the review methodology. Detailed descriptions of the benchmark selection rationale, standards crosswalk methodology, review rubrics, reviewer training and calibration procedures, quality-control processes, inter-rater reliability analyses, and supporting technical documentation are provided in the Benchmarking Gap-Analysis Memo and the Standards-Quality Review Memo, which accompany this report as attachments.

Benchmark Comparisons

A central component of this review was the comparison of the UCS-M with nationally and internationally recognized benchmark systems: the Massachusetts Mathematics Curriculum Framework, the Minnesota Academic Standards in Mathematics, and the Singapore Mathematics Syllabus. Together, these benchmarks provide multiple perspectives on high-quality mathematics standards, representing nationally recognized expectations, high-performing state standards, and an internationally respected mathematics curriculum.

Standards-level crosswalks were developed to compare the Utah standards with each benchmark system and were subsequently synthesized into analyses of content coverage, domain-level emphasis, grade-level placement, and overall progression. Rather than evaluating whether the standards are identical, these comparisons identify areas of broad alignment; distinguish meaningful differences in emphasis and organization; and provide context for understanding the unique characteristics of the Utah standards. The following sections summarize the major findings from these benchmark comparisons and highlight the most significant similarities and differences observed across the benchmark systems.

Overall Comparisons Across Benchmark Systems

Overall, the benchmarking analyses indicate that the UCS-M are highly comparable to the benchmark systems reviewed. The strongest correspondence was observed with the Massachusetts mathematics standards, where the Utah standards demonstrated substantial alignment across nearly all domains and grade levels. In both comparisons, the vast majority of Utah standards exhibited strong or partial grade-level alignment, indicating that the mathematical content, sequencing, and overall organization of the standards closely reflect nationally recognized expectations.

Comparisons with the Minnesota mathematics standards also demonstrated broad alignment, although several differences in emphasis and organization were identified. These differences were generally concentrated in the treatment of advanced secondary mathematics; the organization of selected concepts across grade levels; and the relative emphasis placed on Statistics and Data Science, where Utah provides a more explicit and comprehensive progression. Despite these differences, the overall mathematical progression and content coverage remained highly comparable across the two systems.

The comparison with the Singapore Mathematics Syllabus revealed the greatest differences among the benchmark systems. However, these differences were driven primarily by variations in the sequencing and organization of mathematical concepts rather than by substantial differences in overall content coverage. Singapore generally introduces many mathematical ideas earlier in the elementary grades and places greater emphasis on advanced mathematics, including calculus, during the pre-university years. Conversely, Utah places considerably greater emphasis on Data Science and statistical reasoning throughout the PK–12 progression. As a result, many apparent differences reflected alternative approaches to curriculum design rather than omissions of mathematical content.

Taken together, the benchmarking analyses suggest that the Utah standards reflect a comprehensive and rigorous mathematics curriculum that is broadly aligned with nationally recognized expectations while also incorporating several distinctive features. The most notable characteristics of the Utah standards include a sustained emphasis on Data Science; an explicit integration of Mathematical Practices throughout the content standards; and organizational decisions that distinguish the Utah standards from the benchmark systems without substantially altering the overall breadth of mathematical content.

Benchmark Spotlights

Massachusetts

Massachusetts was selected due to its longstanding reputation for rigorous, coherent expectations, and strong emphasis on foundational skills and fluency. Massachusetts is frequently used as a reference point in external reviews and cross-state comparisons, making its standards a credible high bar for evaluating depth, grade-to-grade progression, and readiness for advanced coursework.

The UCS-M demonstrated a high degree of alignment with the Massachusetts mathematics standards. Strong or partial alignment was observed across nearly every domain, indicating that the Utah standards reflect mathematical expectations comparable to one of the nation's most rigorous state mathematics frameworks.

Across grades K–8, the two systems demonstrated particularly strong correspondence in most domains including Number and Operations, Ratios and Proportional Reasoning, Algebra, and Functions. While Geometry and Measurement also exhibited substantial overlap, Massachusetts seemed to place a slightly greater emphasis on measurement-based problem-solving, including multistep measurement conversions and generating and representing measurement data using line plots. At the high school level, Algebra, Functions, Geometry, Data Science, and Matrices and Vectors all demonstrated nearly complete alignment.

The primary distinctions between the two systems were relatively modest. Massachusetts places somewhat greater emphasis on selected Geometry and Measurement concepts in K–8, whereas Utah provides a substantially more extensive progression in Statistics and Data Science. Overall, however, the analyses indicate that the Utah standards closely reflect the breadth, depth, and organization of mathematical content represented in the Massachusetts framework.

Minnesota

Minnesota was selected because their mathematics standards are widely viewed as clear, coherent, and practical for instruction, with a strong focus on grade-by-grade progression and readiness for algebra. The Minnesota standards provide a strong comparison point for usability and coherence alongside rigor, and the state's recent revisions further emphasize reasoning, problem-solving, and access for all students. Because Minnesota adopted a substantially revised mathematics framework in 2022[1], the benchmark comparison was conducted using these most recently adopted standards.

Unlike the Massachusetts mathematics standards, the 2022 Minnesota Academic Standards in Mathematics are organized using a different conceptual framework. Rather than traditional mathematics domains, the standards are organized around three broad content strands: Patterns and Relationships, Spatial Reasoning, and Data and Probability. These strands are supported by seven K–12 Anchor Standards that articulate learning progressions across grade levels. The standards also incorporate the Standards for Mathematical Practice (SMPs) from the Common Core framework, together with an additional cross-cutting dimension, Contribution and Context, which emphasizes the application of mathematics in culturally and locally relevant contexts. These structural differences provide important context for interpreting the benchmark comparison, as mathematical ideas are grouped, sequenced, and integrated differently than in the Utah standards.

Consistent with these structural differences, the comparison of the UCS-M with the Minnesota mathematics standards revealed more noticeable differences in organization and instructional emphasis than were observed for the Massachusetts Mathematics Curriculum Framework.

Across the elementary and middle grades, the Utah and Minnesota standards demonstrated stronger overall alignment in the domains of Ratios and Proportional Reasoning, Algebra, and Functions. Alignment was also substantial in Number and Operations, although Minnesota places greater emphasis on selected elementary concepts related to whole-number place value, operations, and number relationships. The greatest differences occurred within Geometry and Measurement, where approximately 69 percent of Utah standards exhibited strong or partial alignment. The crosswalk identified differences in the treatment of geometric constructions, classification of geometric figures, transformations, and selected measurement concepts, reflecting both differences in emphasis and the sequencing of these topics across grade levels.

The K–8 comparison also highlighted important differences in the treatment of Statistics and Data Science. While nearly all Utah standards demonstrated strong or partial alignment, the two systems organize statistical content differently. Utah establishes a comprehensive PK–12 progression in Data Science that explicitly includes data investigations, sampling, bias, study

[1]https://education.mn.gov/mdeprod/idcplg?IdcService=GET_FILE&dDocName=PROD083852&RevisionSelectionMethod=latestReleased&Rendition=primary

design, interpretation of evidence, and statistical reasoning. Minnesota incorporates many of these ideas within broader statistics standards and places comparatively less emphasis on Data Science as a distinct strand spanning the PK–12 continuum.

At the secondary level, alignment was strongest in Data Science, while somewhat greater differences emerged in Algebra, Functions, Geometry, Number Systems, and particularly in Matrices and Vectors. The crosswalk indicates that these differences were concentrated within specific topic areas rather than reflecting broad differences in mathematical expectations. For example, differences were observed in the treatment of complex numbers, geometric transformations and circle geometry, and matrix and vector operations, the latter reflecting a substantially greater emphasis within the Utah standards. Overall, the analyses indicate that the Utah and Minnesota standards are broadly comparable in their mathematical content and progression, with the primary distinctions arising from differences in the organization and emphasis of selected concepts rather than the overall scope of the curriculum.

Singapore

Singapore was selected as an international benchmark because of its sustained record of high performance on international mathematics assessments—including the Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA)—and its internationally recognized, coherent mathematics curriculum.

Unlike the UCS-M and other U.S. state standards, the Singapore Mathematics Syllabus[2] is not organized as a comprehensive set of grade-level content standards. Rather, it is organized separately for the primary and secondary levels and structured around a curriculum framework centered on mathematical problem-solving, with content organized into broad topic areas that are revisited and extended over time. The syllabus places considerable emphasis on mastery of a relatively small number of concepts at each grade level before progression. The Singapore syllabus introduces many foundational concepts earlier than is typical in U.S. curricula; and culminates in differentiated upper-secondary pathways that include advanced mathematics topics beyond those typically expected for all U.S. high school students. These structural differences provide important context for interpreting the benchmark comparison.

The comparison between the UCS-M and the Singapore Mathematics Syllabus revealed the greatest differences among the benchmark systems reviewed. Nevertheless, the two systems demonstrated substantial alignment across many core mathematical concepts, with most differences reflecting variations in sequencing, organization, and curricular emphasis rather than omissions of mathematical content.

[2] https://www.moe.gov.sg/media/files/primary/92bff26d-b2b4-4535-b868-b8415c744b91.pdf
https://www.moe.gov.sg/media/files/secondary/fsbb/syllabus/3118d4e6-1722-475e-a21a-7117aba21277.pdf
https://www.moe.gov.sg/media/files/secondary/fsbb/syllabus/d415c25d-cf29-4b05-83da-9713f38edd14.pdf
https://www.moe.gov.sg/media/files/secondary/fsbb/syllabus/2155cce5-f6b4-4532-897c-c5a8fa1852c6.pdf

Across the elementary and middle grades, the strongest alignment was observed in Number and Operations and Ratios and Proportional Reasoning, where approximately 85 percent and 100 percent of Utah standards, respectively, exhibited strong or partial alignment with corresponding Singapore expectations. Alignment was also notable in Geometry and Measurement (76%) and Algebra (72%). Overall, Singapore generally introduces multiplication and division, fractions, decimals, percentage, and many measurement concepts earlier than Utah in elementary grades, while Utah introduces concepts related to coordinate geometry, integer operations, and algebraic reasoning earlier in the middle grades. The crosswalk also identified differences in the treatment of functions, reflecting Singapore's later introduction of formal function concepts.

The most substantial differences emerged in Statistics and Data Science. Utah establishes an explicit PK–12 progression in Data Science that includes statistical investigations, sampling, bias, study design, interpretation of evidence, and statistical reasoning beginning in the elementary grades. Singapore addresses data analysis and probability throughout the curriculum but places considerably less emphasis on Data Science as a distinct strand, resulting in substantially lower alignment within this domain. These differences reflect contrasting curricular priorities rather than gaps in mathematical rigor.

At the secondary level, differences became more pronounced. Strongest overall alignment was observed in Algebra (82%), while moderate alignment was observed in Functions (52%). The greatest differences occurred in Number Systems, Matrices and Vectors, Geometry, and Data Science. The crosswalk indicates that many Utah standards related to complex numbers, vectors, matrix operations, and data science have no direct counterparts within the Singapore syllabus. Conversely, Singapore places greater emphasis on advanced algebraic manipulation, mathematical modeling, and topics associated with pre-university preparation, including content that extends beyond the scope of Utah's required secondary mathematics standards. Overall, the analyses indicate that Utah and Singapore provide comparable coverage of core mathematics while differing substantially in how mathematical ideas are organized, sequenced, and emphasized across the PK–8 continuum, but somewhat more significant differences in emphasis at the high-school level.

Standards Quality Review

In addition to benchmarking the UCS-M against nationally and internationally recognized mathematics frameworks, WestEd conducted a comprehensive review of the quality of the standards themselves. This review examined the extent to which the standards are measurable, instructionally focused, clear and accessible, appropriately rigorous, and developmentally appropriate. This review employed a structured set of review rubrics developed in collaboration with USBE (see Appendix-A). The review then evaluated the integration and appropriateness of the Utah Mathematical Practices within the content standards. The following sections summarize the findings for each review criterion, highlighting patterns observed across grade levels and identifying opportunities for refinement where applicable.

Measurability

Measurability refers to the extent to which a standard clearly specifies an observable and assessable student performance. Measurable standards focus on the intended outcomes of teaching and learning by describing what students should know and be able to do using observable, low-inference performance expectations. They employ precise language, avoid vague or non-observable verbs that require subjective interpretation; provide sufficient specificity to communicate the expected level of performance without being overly prescriptive; and describe learning expectations that can reasonably be demonstrated through a single, assessable performance. Each standard was evaluated using a three-level rubric (see Appendix-A) and classified as “Measurable and Well Specified”; “Partially Measurable and/or Well Specified”; or “Not Measurable or Poorly Specified”.

Overall, the UCS-M demonstrated strong performance on this criterion. Of the 418 standards reviewed, 55 percent were rated as Measurable and Well Specified; 42 percent were rated as Partially Measurable and/or Well Specified; and only 3 percent were rated as Not Measurable or Poorly Specified. These findings indicate that the large majority of standards clearly communicate assessable learning expectations, while a relatively small proportion would benefit from revisions to improve measurability.

Across grade levels, the most common measurability concerns reflected recurring writing patterns rather than shortcomings in the mathematical content itself. The most frequently identified issues included multiple performance expectations embedded within a single standard; the use of non-observable verbs (e.g., “understand,” “recognize,” or “attend to”); vague or subjective language (e.g., “flexibly,” “efficiently,” or “reasonable”); and expectations that were insufficiently specified to support consistent interpretation by educators and assessment developers. In many cases, reviewers concluded that these concerns could be addressed through relatively minor revisions, such as replacing non-observable verbs with observable student behaviors, clarifying vague terminology, or separating compound standards into distinct performance expectations.

Representative examples illustrate these patterns. For example, P3.CC.2 asks students to "begin to recognize" that written numbers represent a quantity and to "use a variety of tools" to model numbers, requiring substantial interpretation because the expected student behaviors are not explicitly defined. Similarly, 4.MG.4 asks students to "attend to precision and reasonableness" when measuring and sketching angles without specifying an observable performance that would demonstrate proficiency. Other standards, such as 5.MG.3, combine several distinct expectation—including constructing and using coordinate plane models, interpreting coordinate pairs, graphing points, and interpreting coordinates in context—making it difficult to assess proficiency through a single performance. Additional representative examples, together with reviewers' explanations and suggested revisions, are provided in Appendix B (Table B1).

Overall, the findings suggest that the Utah mathematics standards are generally written in a manner that supports classroom instruction and assessment development. The relatively small number of measurability concerns identified by reviewers primarily reflect opportunities to improve precision and assessability through clearer wording, more observable performance expectations, and the separation of compound standards into discrete, measurable learning expectations.

Clarity

Clarity refers to the extent to which a standard communicates its intended mathematical expectations in language that is understandable, precise, and accessible to its intended users, including educators, curriculum developers, parents, policymakers, and other stakeholders. Clear standards describe mathematical content and practices using accurate and unambiguous language; provide sufficient guidance to communicate the intended learning expectations; avoid unnecessary jargon; and are free from grammatical or mathematical inaccuracies that could impede interpretation. Each standard was evaluated using a three-level rubric (see Appendix-A) and classified as "Clear and Accessible," "Partially Clear and/or Accessible," or "Not Clear or Accessible."

Overall, the UCS-M performed well on this criterion. Of the 418 standards reviewed, 60 percent were rated as Clear and Accessible, and another 39 percent were rated as Partially Clear and/or Accessible; only 1 percent of the standards were rated as Not Clear or Accessible. These findings indicate that most standards effectively communicate the intended mathematical expectations, while a subset would benefit from revisions to improve clarity and accessibility.

Across grade levels, the most common clarity concerns reflected recurring communication issues rather than deficiencies in the mathematical content. Reviewers frequently identified vague or ambiguous language; insufficient guidance to communicate the intended performance expectation; overly long or compound standards; and technical terminology that could be made more accessible to a broader audience. Importantly, reviewers distinguished between terminology that is appropriately technical for the mathematics being taught—particularly in middle and high school—and terminology or phrasing that unnecessarily obscures the intended learning expectation. In several instances, clarity concerns arose from the incorporation of language associated with the Utah Mathematical Practices (e.g., “build and use models,” “attend to structure,” “attend to precision”), where the practice language sometimes made it less clear what specific mathematical knowledge or skill students were expected to demonstrate.

In many instances, clarity concerns overlapped with measurability concerns, as standards that were difficult to interpret were also more difficult to assess consistently. Reviewers generally concluded that these issues could be addressed through relatively modest revisions, such as simplifying language; replacing abstract or ambiguous phrases with more mathematically precise terminology; providing additional examples or context where appropriate; and separating compound standards into more focused learning expectations.

Representative examples illustrate these patterns. For example, K.CC.8 uses the term “subitize,” which is mathematically appropriate but may not be familiar to all intended audiences without additional explanation or an illustrative example. Similarly, standards such as 6.NS.7 use phrases such as “build and use models” without clearly specifying the type of mathematical model or representation expected, while other standards incorporate Mathematical Practice language (e.g., “attend to structure” or “attend to precision”) in ways that may obscure the primary mathematical expectation. Finally, standards such as 5.NBT.6 combine multiple expectations into lengthy statements that reduce readability without adding instructional clarity. Additional representative examples, together with reviewers’ explanations and suggested revisions, are provided in Appendix B (Table B2).

Overall, the findings suggest that the Utah mathematics standards are generally written in language that is understandable and mathematically sound. The clarity concerns identified by reviewers primarily reflect opportunities to improve accessibility and interpretability through more precise wording, clearer descriptions of expected mathematical behaviors, and a more judicious integration of Mathematical Practice language within the content standards. Addressing these issues would improve readability for educators, families, and other stakeholders while preserving the mathematical intent of the standards.

Instructional Focus

Instructional focus refers to the extent to which the standards emphasize the most important mathematics for each grade level or course while avoiding unnecessary or extraneous content. For grades K–8, each standard was classified according to whether it addressed the major work, supporting work, additional work, or extraneous work of the grade, as reflected in the mathematics standards adopted by most U.S. states. For high school, standards were classified as addressing widely applicable prerequisites, appropriate supporting work, or extraneous or inappropriate work for the course.

Overall, the instructional focus of the UCS-M was found to be highly consistent with the instructional priorities reflected in other contemporary mathematics standards used by various U.S. states. Across the PK–8 standards, 60 percent of the standards were classified as addressing the major work of the grade; 30 percent as supporting work; 10 percent as additional work; and only one standard (in Grade 1) was identified as addressing extraneous work. At the high school level, 63 percent of the standards addressed widely applicable prerequisites for college and career readiness, while the remaining 37 percent of the standards addressed appropriate supporting content. No high school standards were identified as extraneous or inappropriate for the intended course sequence.

These findings indicate that the Utah standards maintain a strong emphasis on the mathematical content that is widely recognized as most important for developing mathematical proficiency across the PK–12 continuum. The distribution of standards across major, supporting, and additional work closely mirrors the instructional priorities reflected in the mathematics standards adopted by most other U.S. states. Reviewers found little evidence that instructional attention would be diverted toward content that falls outside the expected scope of grade-level mathematics.

The single PK–8 standard identified as extraneous work represented an isolated instance rather than a broader pattern, and reviewers did not identify evidence of systematic overemphasis on peripheral or inappropriate content. Instead, the findings suggest that the Utah standards generally prioritize foundational mathematical ideas while including supporting and additional content in a manner that is consistent with nationally recognized expectations for coherent mathematics instruction.

Overall, the review indicates that the UCS-M exhibit a strong instructional focus that is well aligned with the grade-level priorities reflected in high-quality U.S. mathematics standards. This alignment supports coherent curriculum development, instructional planning, and assessment while maintaining an appropriate emphasis on the mathematical knowledge and skills that prepare students for increasingly sophisticated mathematics.

Depth of Knowledge

Depth of Knowledge (DOK) refers to the level of cognitive complexity required for a student to successfully demonstrate proficiency on a standard. In assigning DOK ratings, reviewers considered the minimum depth of mathematical knowledge a student would need to successfully complete the performance expectation described in each standard. Rather than focusing on the difficulty of the mathematics itself, the ratings reflect the complexity of the thinking required to demonstrate proficiency. Standards were classified using Webb's four-level

DOK framework: DOK 1 (Recall and Reproduction); DOK 2 (Skills and Concepts); DOK 3 (Strategic Reasoning); and DOK 4 (Extended Investigation). In some cases, a standard could reasonably support tasks at multiple DOK levels depending on how the task was implemented. In these instances, reviewers assigned the lower DOK level to represent the minimum DOK the performance expectation was likely to elicit.

Overall, the cognitive demand reflected in the UCS-M was well balanced and consistent with expectations for high-quality mathematics standards. Of the 418 standards reviewed, 7 percent were classified as DOK 1; 61 percent as DOK 2; 31 percent as DOK 3; and 1 percent as DOK 4. The predominance of DOK 2 and DOK 3 standards indicates that the Utah standards generally require students to apply conceptual understanding, make mathematical decisions, justify reasoning, and engage in multistep problem-solving rather than relying primarily on memorization or routine procedures. Exemplar UCS-M standards that are representative of each DOK level are presented in Table 1 below.

Table 1. UCS-M That Are Illustrative of Specific DOK Levels

DOK Level	Descriptors/Criteria	Examples
DOK 1	• Well-defined, routine tasks involving recall or recognition of facts, definitions, or terms; carrying out step-by-step procedures and algorithms; or rote application of formulas • Can be completed with little or no conceptual understanding of the topic • DOK revealed is rote memorization and reproduction of procedures	**P4.CC.1**—With support, *accurately* recite number names and the counting sequence to 20. (MP 6) **K.CC.2—***Accurately* count forward from a given number between 1 and 100. Attend to the *counting structure*. (MP 6, 7) **1.NBT.1—**Count to 120 *accurately*, starting at any number. Represent a number of objects up to and including 120 with a written numeral. (MP 6) **2.NBT.3—**Read three-digit whole numbers and *represent the place value structure in* those numbers using numerals, base 10 word form, and expanded form. (MP 7) **3.MG.3—***Add or remove context* to solve one-step story problems involving length, liquid volume, mass, and time. (MP 6)

DOK 2	• Requires some conceptual understanding of the underlying mathematics, but the task is still routine in nature; there is a straightforward process to follow once the student identifies the problem type • Requires some amount of reasoning or decision-making (e.g., which common denominator, which method to solve the system, which property to check) • Often involves multiple steps	**4.NF.8**—Compare two decimal numbers to the hundredths place by reasoning about their size. Justify the comparison. *Represent the relationship* between two numbers using comparison symbols <, =, and >. (MP 3, 7) **7.D.3**—Create *precise and reasonable* visual representations, including histograms (with technology), dot plots, and box plots. Analyze and interpret visual representations and numerical summaries for measures of center (mean and/or median) and measures of variability (mean absolute deviation and/or inter-quartile range). (MP 6) **S1.G.6—***Construct, justify and communicate clear arguments* that show how the criteria for triangle congruence (ASA, SAS, SSS) can be established using the definition of congruence in terms of rigid motions. (MP 3) **S3.A.2**—Create equations and inequalities in one variable to *model a context* and use them to solve problems. Include equations arising from linear, quadratic, simple cubic (e.g., $f(x)=2x^3$), simple exponential, square root and cube root functions. (MP 3)
DOK 3	• Nonroutine task; no one straightforward process that can be followed, so strategic thinking is required. A need for experimentation and tinkering is common • In addition to applying conceptual understanding of the underlying math, the task requires justification, comparison, evaluation, reasoning within a single problem • While verbs are not 100% reliable for determining DOK, DOK 3 tasks often include verbs like explain, justify, construct an argument, decide, evaluate, justify, and analyze • May require making connections between different mathematical ideas or topics	**2.NBT.**6—*Build and use concrete and pictorial models* to determine the sums and differences of three-digit numbers with sums and minuends up to and including 999. Use the models to *represent and describe the structure of* combining hundreds with hundreds, tens with tens, ones with ones, and composing and decomposing tens and hundreds. *Ask questions to compare* concrete and pictorial models with efficient numerical strategies. (MP 1, 4, 7) **5.NF.3—***Build and use models*, including area models, to multiply fractions, whole numbers, and mixed numbers. Connect the properties of multiplication to multiplying fractions. Solve problems involving multiplication of fractions and mixed numbers *using real-world contexts and equations* (fraction by a fraction, fraction by a mixed number, mixed number by mixed number). (MP 2, 4) **8.G.1**—Use rigid transformations to establish a definition of congruent figures. *Justify* that two shapes are congruent through the use of rigid transformations. *Make and evaluate conjectures* about attributes of, rotations, reflections, and translations with and without coordinates. (MP 3, 8) **S2.D.4**—Use the statistical evidence to evaluate results from analyses to answer questions about associations between two quantitative variables. Identify ways to refine investigative questions, data collection, visualization, or analysis. Represent the relationship with a curve of best fit and use the model of the equation to solve problems. *Justify* statistical reasoning and results to others in a variety of formats including verbal, written, and visual. (MP 3, 8)

DOK 4	• Requires investigation, complex reasoning, planning, and developing ideas over an extended period of time • Nonroutine task with a wide range of possible solutions, approaches, and ways of engaging • Often requires time to consider and process task constraints • Often involves drawing on mathematical knowledge and understanding to deal with ambiguity or "messiness"	**4.D.1**—Determine a topic question and *ask investigative questions* that will lead to collecting, analyzing, and interpreting data with up to two variables. *Make conjectures* about the impact of the sampling process and sample size on results. (MP 1, 8) **5.D.1**—Determine a topic question and *ask investigative questions* that will lead to collecting, analyzing, and interpreting data with up to four variables. *Ask questions* to identify potential sources of bias in data collection, representation, and interpretation. (MP 1) **5.D.2**—*Select and use tools strategically* to collect and organize data. Build data visualizations. *Ask questions* to consider a given data set with up to four variables. *Make conjectures* about potential sampling approaches and how those approaches impact results. (MP 1, 5, 8) **6.D.2**—*Select and use tools appropriately and strategically* to generate, collect, and organize data. Ask questions about how the data were collected and whether the data are useful to answer the statistical question of interest. (MP 1, 5) **S2.D.2**—*Select the appropriate statistical tools* to create a data collection plan. Collect and organize primary data to investigate the association between two variables and make conjectures. For secondary data, determine if data collected are useful to answer the statistical question of interest. (MP 5)

This distribution of the UCS-M predominantly representing DOK 2 and DOK 3 is also broadly consistent with the cognitive expectations reflected in contemporary U.S. mathematics standards adopted by many other states, where the majority of standards are expected to elicit DOK 2 and DOK 3 thinking. As expected, DOK 1 standards occurred primarily in the early grades where foundational knowledge and fluency are essential precursors to more complex mathematical reasoning. Likewise, relatively few standards were classified as DOK 4, reflecting the fact that state mathematics standards are generally intended to support learning and assessment through classroom instruction and on-demand statewide assessments rather than extended investigations conducted over prolonged periods of time.

The five standards classified as DOK 4 require students to engage in extended investigation, planning, and decision-making over time—for example, designing statistical investigations, selecting appropriate data collection methods, and evaluating the suitability of data for answering statistical questions. While these expectations reflect authentic mathematical practice, they may present challenges for traditional on-demand statewide assessments. As Utah continues refining the standards and associated assessment system, it may be worthwhile to consider whether these standards are best represented as DOK 4 expectations or whether they could be reframed to preserve the intended mathematical learning while supporting valid measurement through on-demand assessment formats.

Overall, the review indicates that the UCS-M reflect an appropriate progression of cognitive demand across the PK–12 continuum. The standards place substantial emphasis on conceptual reasoning and mathematical decision-making while maintaining an appropriate role for foundational knowledge and procedural fluency in the early grades.

Mathematical Rigor

Mathematical rigor refers to the complementary development of procedural skill, conceptual understanding, and application. Reviewers identified each dimension reflected in a standard's performance expectations; a standard could receive more than one classification when it required students to integrate one or more of concepts, procedures, and/or applications. Procedural classifications indicated that students were expected to demonstrate a mathematical skill or procedure; conceptual classifications required mathematical understanding beyond the execution of a procedure; and application classifications required students to use mathematics in a real-world situation or context.

It should be noted that some early-grade standards (particularly in PK and K) were classified as procedural skill because they emphasize the recall of foundational mathematical knowledge (e.g., counting sequences, number names, or basic facts). Although these expectations are more appropriately characterized as foundational recall than procedural fluency, they were coded as procedural to maintain consistency within the procedural-conceptual-application framework used in this review.

Overall, the UCS-M place the greatest emphasis on conceptual understanding while maintaining substantial attention to procedural skill and application. Across all 418 unique standards summarized in the standards review memo, 68 percent emphasized conceptual understanding; 48 percent addressed procedural skill or fluency; and 38 percent incorporated application. Every standard addressed at least one dimension of rigor. However, since the categories were not mutually exclusive, the percentages across the standards need not sum to 100 percent.

The grade-level results show that conceptual understanding remains prominent throughout the PK–12 progression, accounting for most of the standards in every grade or course. The percentage emphasizing conceptual understanding ranged varied across grades. Particularly strong concentrations of conceptual expectations occurred in Grade 5—where students extend their understanding of fractions, decimals, volume, and data—and in Secondary I, where the standards emphasize relationships among algebraic, functional, geometric, and statistical representations.

The emphasis on procedural skill or fluency varied more noticeably across grades. Procedural expectations were relatively prominent in kindergarten (73%) and reflect emphasis on topics such as foundational counting, number, and early operations skills. They generally accounted for a smaller share of standards during the upper elementary and middle grades, reaching their lowest level in Grade 8 (29%). At the high school level, procedural emphasis increased, where

45 percent of standards addressed procedural skill or fluency in Secondary I, 63 percent in Secondary II and 53 percent in Secondary III. Importantly, these high school courses also remained conceptually oriented with 88 percent of the standards in Secondary I, 70 percent of the standards in Secondary II, and 68 percent of the standards in Secondary III emphasizing conceptual understanding; the greater procedural emphasis therefore did not replace conceptual expectations.

Application also varied by grade and course. It was most prominent in Grade 5 (62%), followed by Secondary III—Data Science and Algebra II—Data Science (60%), Grade 7 and Secondary I (50% each), and Grade 6 (46%). These concentrations correspond to grades and courses in which students are frequently expected to solve contextual problems, interpret quantities and representations, model relationships, or conduct statistical investigations. Beyond the early elementary grades, application was least prevalent in Secondary II (26%) and Algebra I (7%). Secondary II features a significant amount of abstract algebra content as well as geometry-focused content where connections to real-world applications are not explicitly included in the standards. (In Geometry, this is balanced by the large number of data science standards, which are heavily applications-focused.) Algebra I likewise includes a significant amount of abstract content that is not explicitly connected to applications in the standards, and also includes none of the data science standards that are included in Secondary I.

Aside from patterns related to specific high school courses observed above, the mathematical rigor patterns we observed in the UCS-M are broadly compatible with the content emphasis of the Massachusetts standards, which were found through the crosswalk analyses to be highly aligned with Utah. However, the Massachusetts standards were not independently coded using the same mathematical-rigor rubric; consequently, these comparisons should be interpreted as evidence of broad consistency in content and performance expectations rather than as direct comparisons of rigor percentages.

Overall, the findings indicate that the UCS-M maintain a sustained emphasis on conceptual understanding while varying the relative attention to procedures and applications in ways that generally correspond to the mathematical content of particular grades and courses.

Vertical Alignment and Developmental Appropriateness

Evaluation Approach

The Utah State Board requested an evaluation of the extent to which the draft UCS-M establish a logical progression of mathematical knowledge and skills from preschool through grade 12, ensuring that prerequisite knowledge is established before more advanced concepts are introduced. To address this question, WestEd synthesized evidence from both phases of the review. Specifically, the analysis draws upon (a) the standards-quality review of developmental appropriateness, including reviewer comments and developmental issue flags; (b) benchmark comparisons examining the placement and sequencing of mathematical content relative to nationally and internationally recognized mathematics standards; and (c) supporting evidence from the instructional focus, mathematical rigor, and DOK analyses. Collectively, these analyses provide evidence regarding the overall progression of the standards, while recognizing that the review did not constitute a comprehensive concept-by-concept learning progression analysis.

Overall Findings

Overall, the findings indicate that the UCS-M establish a developmentally appropriate PK–12 progression that is broadly aligned with nationally recognized mathematics standards. Of the 418 standards reviewed, 98 percent were rated Developmentally Appropriate; and no standards were rated Not Developmentally Appropriate. The strongest ratings were observed across grade 2, grades 4–8, and Secondary III, where all standards were judged developmentally appropriate for the grade and/or placed appropriately within the course sequence. A number of standards in Secondary II and III were denoted as extended standards; while not developmentally inappropriate, these standards typically appeared one to two years earlier than in most other states.

The benchmark comparisons similarly indicate that Utah's overall mathematics progression is highly comparable to the Massachusetts mathematics standards. Across both K–8 and high school, the vast majority of Utah standards demonstrated strong or partial grade-level alignment with these benchmark systems, and very few standards were rated as appearing in a different grade-level in the benchmark systems. This suggests that the overall sequencing of mathematical content closely reflects nationally recognized expectations. Comparisons with Minnesota and Singapore revealed somewhat greater differences in sequencing and organization, although many of these differences reflected alternative curricular structures and instructional pathways rather than fundamentally different mathematical expectations.

Taken together, these findings suggest that the Utah standards establish a logical progression of mathematical ideas across the PK–12 continuum while remaining broadly consistent with the sequencing reflected in other high-quality mathematics standards.

Patterns Identified Across the Review

Although the overall progression was found to be developmentally appropriate, reviewers identified several recurring patterns that may warrant further consideration.

In the early grades, the mathematical content itself was generally judged to be appropriate for the intended grade levels. The majority of developmental concerns instead related to the level of reasoning or abstraction expected within selected standards. In several instances, reviewers noted that students were expected to engage in activities such as constructing arguments, making or evaluating conjectures, or reasoning about mathematical structures in ways that may exceed typical developmental expectations for the grade. In many cases, these concerns were closely related to the incorporation of Mathematical Practice language within the content standards rather than to the mathematical content itself.

In the high-school pathways, reviewers noted that content that was denoted as extended topics standards in year 1 (matrices and matrix algebra) and 2(vector and vector algebra, some complex number topics, and conic sections) courses, while not developmentally inappropriate, appeared one to two years earlier than in most other states. For students who intend to go on to Calculus in year 4 of high school, these topics are an important part of the necessary preparation for that course and are typically taught in honors versions of year 1 and 2 courses. Importantly, these observations do not suggest that the content itself is inappropriate for high school mathematics, but rather that teachers should take special care regarding developing prerequisite knowledge and ensuring that students are prepared for these topics and appropriately supported when the topics are introduced.

The benchmark analyses provide additional context for these observations. Relative to the Massachusetts standards, only limited differences in content placement were identified, suggesting broad agreement regarding the progression of most mathematical concepts. Comparisons with Singapore revealed more substantial differences in sequencing, with Singapore introducing several foundational topics—such as multiplication and division, fractions, percents, and some measurement and geometry ideas—earlier than Utah. Conversely, Utah introduces and develops Data Science substantially earlier and more extensively than the benchmark systems, reflecting an intentional curricular emphasis rather than a developmental concern.

Table 2 presents the various sources of evidence used for this synthesis.

Table 2. Evidence Base for the Evaluation of Vertical Alignment and Developmental Appropriateness

Source of Evidence	Summary of Findings
Developmental Appropriateness Review	Overall, the developmental appropriateness review found that the UCS-M establish a developmentally appropriate progression of mathematical content across the PK–12 continuum. Out of the 418 standards reviewed, 97% of standards were rated as Developmentally Appropriate. The relatively small number of partial ratings were concentrated in selected early-grade standards, where reviewers questioned the level of reasoning or abstraction expected.
Benchmark Comparisons to Minnesota	**Broadly comparable progression with localized differences in sequencing and emphasis.** The comparison with the Minnesota mathematics standards indicated that the overall progression of the Utah standards is broadly comparable, although more differences in sequencing, organization, and content emphasis were identified than in the comparisons with the Massachusetts standards. Most mathematical concepts demonstrated strong or partial alignment, with differences concentrated in Number and Operations, Geometry and Measurement, and high school domains, where Minnesota organizes and sequences some concepts differently. These differences largely reflect alternative approaches to structuring mathematical content rather than fundamentally different expectations for student learning.
Benchmark Comparisons to Singapore	**Different curricular pathway with comparable overall mathematical scope.** The comparison with the Singapore Mathematics Syllabus revealed the greatest differences in the progression of mathematical content among the benchmark systems. These differences are driven primarily by the organization and sequencing of the curriculum rather than by the overall scope of mathematical content. Singapore generally introduces several foundational concepts, including multiplication and division, fractions, percentages, measurement, and selected geometry topics, earlier than Utah and other U.S. states. On the other hand, Utah places substantially greater emphasis on Data Science throughout the PK–12 progression.

Overall, the evidence from both phases of the review supports the conclusion that the UCS-M establish a coherent and developmentally appropriate progression of mathematical learning across the PK–12 continuum. The standards are broadly aligned with the progression reflected in nationally recognized benchmark systems and demonstrate few instances of content that reviewers judged to be developmentally inappropriate. The relatively small number of standards receiving Partial Developmental Appropriateness ratings primarily reflect opportunities to refine the level of expected reasoning in selected early-grade standards.

Secondary I–III versus AGA Pathways

The Utah draft standards include two different three-year high school mathematics pathways: (1) an integrated pathway (Secondary I, II, and III) in which conceptual categories like algebra, geometry, probability, statistics, data science, and so on are woven together to highlight authentic mathematical connections between these categories, as is done in grades K through 8, and (2) an Algebra I–Geometry–Algebra II (AGA) pathway in which topics related to the conceptual category of Geometry and Data Science are separated out from other mathematics topics. While the full set of standards appears in both pathways, the standards are divided differently across Years 1, 2, and 3 of high school.[3]

Nearly every other national system except the United States, including the four highest-achieving PISA nations (Singapore, Japan, South Korea, and Estonia), teaches each year of secondary math using an integrated approach, with each year including a blend of algebra, geometry, trigonometry, statistics, and functions (Learning Policy Institute, 2025). The more siloed AGA approach can be traced back to the 1892 National Education Association's Committee of Ten in an effort to standardize a chaotic high school curriculum and inconsistent college entrance requirements across the states. The Committee's report set the disciplines and their rough ordering, solidifying the rigid one-subject-per-year siloing over the following decades through mechanisms like the Carnegie unit; the dominance of textbook publishers who standardized around separate Algebra and Geometry volumes; and college admission exams organized by subject. The AGA sequence became the near-universal national norm in the mid-20th century after the space race when the drive to funnel students toward calculus reinforced a linear, tracked, algebra-heavy high school mathematics progression (Burdman, 2015).

Scholarly research does not definitively support the clear superiority of either approach in terms of student achievement, with contextual factors (such as access to skilled and experienced teachers; instructional practices and pedagogy; balancing conceptual understanding with procedural fluency; teacher beliefs; positive learning environments; and students' perception of teacher support and strong reciprocal relationships) all having a greater impact on student success (Cardichon et al., 2020; Corkin & Ekmekci, 2019; Hoge, 2016; Osborne, 2021; Allensworth et al., 2021; Hiebert & Grouws, 2007; Yu & Singh, 2018; Gutshall, 2016; Yeager et al., 2022; Kroeper & Murphy, 2020; Priniski & Thoman, 2020).

An important feature of high-quality secondary mathematics courses is that they are organized around mathematical "big ideas" and the connections between them rather than a checklist of procedures, facts, and formulas for students to memorize and reproduce by rote. Charles and Carmel (2005) described a mathematical "big idea" as "a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" (p. 10). They and other experts in the field contend that "big ideas" are

[3] Editorial notes: A1.CE.2 is tagged "(MF 3, 4)" rather than MP; A1.CE.4 contains a stray "(Sk 2)"; and G.GMD.4's cross reference is truncated to "S3.G." where it should read S3.G.3.

important because they enable us to see mathematics as a coherent set of ideas that encourage a deep understanding of mathematics, enhance transfer, promote memory, and reduce the quantity of information that must be memorized. A focus on big ideas also enables teachers to understand how mathematical topics and ideas are connected across years and grade levels, rather than viewing them as separate entities (Charles & Carmel, 2005; Fosnot & Jacob, 2010; Ma, 2010).

For standards documents that specifically call out topics to be taught in each year of high school, this means that teachers and designers of instructional materials need to be able to view the standards for each year and clearly understand (1) what the coherent "mathematical story" is for each grade level in terms of big mathematical ideas and (2) how those big ideas are intentionally and methodically developed over multiple years to create even larger-scale mathematical storylines so that students have opportunities to develop the type of deep understanding that enables transfer and authentic problem-solving. This should be the case regardless of whether students are following an integrated pathway or an AGA pathway.

A senior reviewer with extensive expertise in secondary mathematics reviewed both pathways that appear in the Utah standards (Secondary I-III and AGA), paying special attention to big ideas and coherent mathematical storylines; findings are discussed below.

Integrated Pathway (Secondary I–III)

Secondary I includes five major areas of work:

1. Solving linear and exponential equations
2. Understanding, comparing, and representing linear and exponential functions
3. Describing characteristics of linear and exponential functions
4. Understanding and applying congruence in terms of geometric transformations
5. Analyzing data to compare distributions or explore associations between two quantitative variables

These areas of work can be thought of as different aspects of two "big ideas":

1. **Comparing situations with a constant *rate* of change versus situations with a constant *ratio* of change**
2. **Congruence and what does and does not change under rigid transformations**

Figure 1. Comparing arithmetic sequences and linear functions with geometric sequences and exponential functions

Arithmetic Sequence

n	1	2	3	4	5
a_n	2	6	10	14	18

+4 +4 +4 +4

6 – 2 = 4 → constant **rate** of change

$$a_n = 2 + 4(n - 1)$$

Geometric Sequence

n	1	2	3	4	5
a_n	2	6	18	54	162

· 3 · 3 · 3 · 3

6 ÷ 2 = 3 → constant **ratio** of change

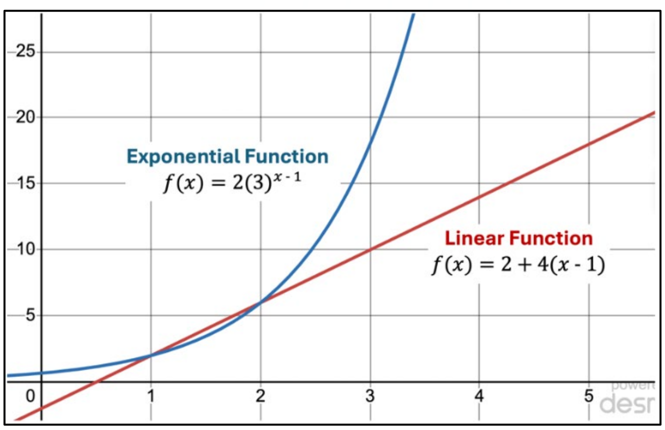
$$a_n = 2(3)^{n-1}$$

Linear Function

$$f(x) = 2 + 4(x - 1)$$

Exponential Function

$$f(x) = 2(3)^{x-1}$$



Arithmetic and geometric sequences (S1.F.4; S1.F.9) serve the entry point to the first idea (see Figure 1), which are then connected to linear and exponential functions (their continuous counterparts), with arithmetic sequences and linear functions changing at a constant *rate,* while geometric sequences and exponential functions change with a constant *ratio*. The Functions strand of the course culminates in S1.F.11, in which students are expected to understand how and why exponential growth eventually exceeds linear growth.

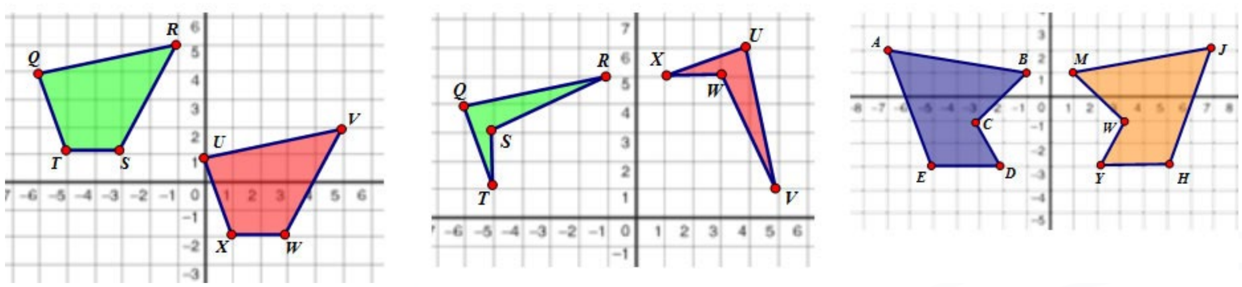
The Geometry strand of the course connects to the Functions strand via the idea of rigid transformations of geometric figures, with S1.G.2 asking students to define rigid transformations **as functions** that take points as inputs and give points as outputs (see Figure 2). This is where we see the second "big idea" addressed, with congruence defined as the existence of such a function (S1.G.3) and with triangle congruence criteria derived from that definition (S1.G.6). While "functions" are often presented as an algebra topic, this Secondary I course treats the idea of function as a key structural idea for the year that marries important ideas from both Algebra and Geometry.

Figure 2. Rigid transformations as functions that map points in the plane to other (or the same) points in the plane

A translation (x,y) --- > (x + 6,y - 3) maps these two quadrilaterals, so Quad QRST ≅ Quad UVWX

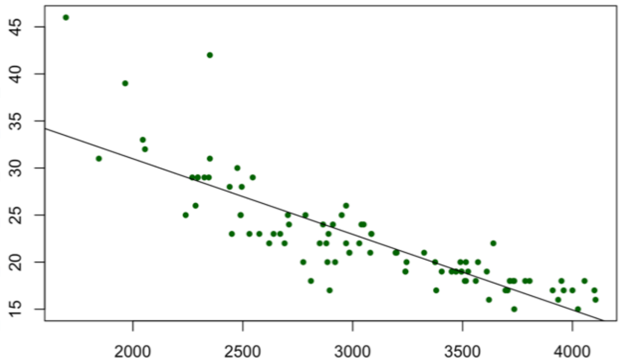
A rotation of 270° about the origin maps these two quadrilaterals, so Quad QRST ≅ Quad UVWX

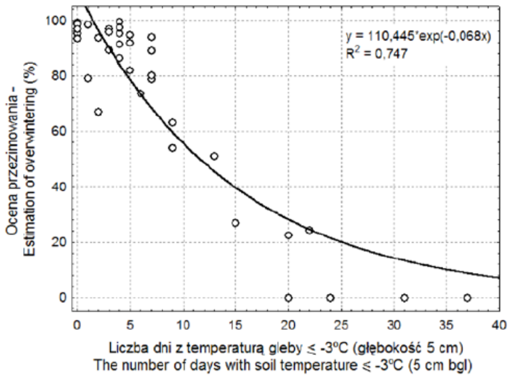
A rotation over the y axis maps these two pentagons, so ABCDE ≅ JMWYH



In the Data Science strand, we again see a connection to functions as well as to the first "big idea" of constant rate versus constant ratio as students explore bivariate relationships, determine whether they are linear or nonlinear, and consider how best to model linear relationships using line of best fit (linear functions).

Figure 3. Modeling a relationship between two variables with a linear function (left) versus with an exponential function (right)





While matrix algebra topics are not typically taught in a first-year high school course due to their more abstract nature, matrices are strongly related to linear equations, functions, and geometric transformations. As long as instructional materials address the content skillfully, the matrix algebra standards that appear under "extended topics" can be naturally interwoven with standards related to linearity, linear equations and functions, systems of linear equations, and transformations.

Secondary II includes five major areas of work that build on those from Secondary I:

1. Solving linear, exponential, and **quadratic** equations
2. Understanding, comparing, and representing functions from these three families
3. Describing characteristics of linear, exponential, and **quadratic** equations
4. Proving congruence **and similarity** via transformations
5. Analyzing and characterizing bivariate associations

Secondary II's central organizing "big ideas" pick up where Secondary I's two "big ideas" left off:

1. **What expands or changes when a relationship is nonlinear**
2. **What does and does not change when we include dilations**

The first idea arises in the Number Systems and Algebra threads as students extend their understanding of integer exponents to rational exponents and begin to solve quadratic equations with no real solutions. Imaginary and complex numbers are motivated by the

Fundamental Theorem of Algebra (S2.NS.1–NS.4), arriving as the answer to a natural question: How do we understand the solution(s) to an equation such as $x^2 + 1 = 0$?

$$x^2 + 1 = 0 \rightarrow x^2 = -1 \rightarrow \ x = \sqrt{-1}$$

There is no real number that can be squared to give us -1, and so in Secondary II, the symbol *i*, an imaginary number, is introduced to represent $\sqrt{-1}$. This allows students to find algebraic solutions to a class of equations that do not have real solutions:

$$x^2 + 1 = 0 \rightarrow x^2 = -1 \rightarrow \ x = \pm i$$

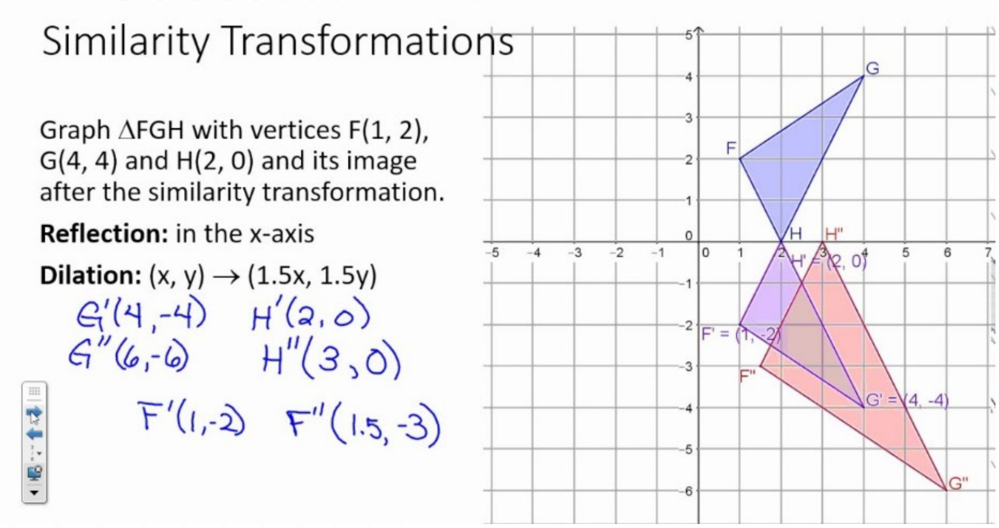
The first big idea above appears again in the Functions strand as students begin to explore quadratic functions in addition to linear and exponential ones. This new function family motivates two new analytic tools: (1) using equivalent forms of functions to reveal different properties (S2.A.1, S2.F.5—vertex vs. factored vs. standard) and (2) finding the average rate of change over an interval (S2.F.2). In grade 8 and Secondary I, students explore what happens when functions are transformed via reflections and vertical translations; in Secondary II, the "transformation toolkit" is completed as students explore function dilation (i.e., $kf(x)$) and horizontal translation (i.e., $f(x+k)$) in S2.F.7, which is supported by access to equivalent forms (e.g., vertex form, standard form).

The second big idea appears in the Geometry strand as students extend their understanding of congruence and rigid transformations to make sense of similarity, dilation, and scaling. Just as in Secondary I students defined congruence as the existence of a function composed of rigid transformations that map one figure to another, they now define similarity as the existence of a congruence transformation composed with a dilation (or contraction) function. They then explore which features of shapes similarity transformations preserve (angle relationships, ratios of corresponding sides) and which they do not necessarily preserve (e.g., side lengths).

Defining and exploring similarity lays the groundwork for trigonometry, as the equivalence of ratios of corresponding sides in similar triangles is what makes trigonometry possible. S2.G.4 defines the trigonometry ratios by observing that side ratios are invariant across similar right triangles; then in S2.G.7, students investigate the similarity of circle, leading to radian measure as a ratio of arc length to radius (S2.G.9). S2.G.11 then builds on the idea of scaling distances and line segments by a scale factor k, leading to the idea of area and volume scaling by k^2 and k^3.

High quality instructional materials will highlight a key connection between these two big ideas, namely how the scale factor k in the Geometry strand echoes the parameter k in the Function transformations. The Data Science strand again connections to Functions thread as students are asked in S2.D.3 to investigate bivariate relationships and decide whether a linear, exponential, or quadratic model best fits the data, with "curve of best fit" replacing "line of best fit."

Figure 4. A similarity transformation that maps a figure onto a similar figure in the plane



While the vector algebra standards under "Extended Topics" are most closely tied to the study of linear functions and/or equations and transformations rather than to functions, in general, they can be naturally interwoven with topics related to linearity and transformations of linear functions.

All versions of **Secondary III** include four areas of major work:

- Solving polynomial, logarithmic, and radical equations
- Understanding, comparing, and representing polynomial, exponential, logarithmic, and trigonometric (sine and cosine) functions
- Describing characteristics of functions from these families
- Drawing and justifying conclusions from surveys, experiments, and observational studies

The hallmark of a coherent mathematical progression is the "coming together" of mathematical ideas over time, rather than simply adding more and more skills and ideas where the connections are unclear. We see this coherence emerge in Secondary III in two key areas:

- **Functions and Geometry.** While the study of functions expands to include logarithmic functions (the inverse of exponential functions) and radical functions (the inverse of power functions), the characteristics of functions that students describe also expands to include end behavior (i.e., how a function behaves as the independent variable gets very far from zero). Additionally, a number of "big ideas" from the Functions category are now consolidated under the umbrella of "function families as taxonomy." This "coming together" culminates in S3.F.7, which asks students to choose which function family makes the most sense for modeling a particular situation. Ideas from the Geometry category are also largely subsumed by Functions in Secondary III, where sine and cosine model periodic phenomena and the domain is extended past 0°–90° (S3.F.2). Ratios of side lengths of right triangles also evolve from features of similar right triangles to become functions in their own right ($f(x) = \sin(x)$, $f(x) = \cos(x)$, etc.).

- **Statistics and Data Science.** After two years of describing samples, students in Secondary III now synthesize a wide range of knowledge and skills related to probability, conditional probability, the normal distribution, deliberate study design (survey vs. observational study vs. experiment), simulation, sampling distributions, *p*-values, and statistical significance (S3.D.1–D.8) in order to reason from sample to population under chance variation.

The three separate Secondary III options (Calculus pathway, Intermediate Algebra pathway, Data Science pathway) then build on these two pillars in different ways. In the **Calculus** and **Intermediate Algebra pathways**[4], students continue expanding their knowledge of different function families as they explore rational functions, spend more time with inverse function relationships, and use algebraic arithmetic techniques to build new functions. They also deepen their understanding of trigonometric functions and identities via the unit circle, and extend their work with the complex number system through complex number arithmetic. Students who opt for the **Calculus pathway** encounter a few additional topics designed to prepare them for that course—specifically arithmetic and geometric series; sigma notation; parametric equations; and additional work with trigonometric identities and complex numbers. In the **Data Science pathway**, students go deeper into topics related to statistics, probability, and modeling rather than more advanced algebra topics, content that is likely to prepare students for an advanced statistics or quantitative reasoning course and for careers that involve collecting, analyzing, and interpreting real-world data, modeling real world phenomena with mathematics, and/or critically consuming data and data summaries collected and presented by others.

Coherence Across Secondary I–III

Six clear "mathematical storylines" carry across the three integrated courses, creating a coherent mathematical narrative:

1. **Function families gradually expand on a deliberate schedule, via a stable analytic template.** Students' exploration of function families proceeds from linear and

[4] The Intermediate Algebra pathway codes composition as CS3.F.16 and inverse-by-composition as CS3.F.17, while the Calculus pathway codes those same two standards as CS3.F.15 and CS3.F.16, meaning the code CS3.F.17 denotes different content in the two courses, which conflicts with the stated convention that shared CS3 codes carry across. Also, Intermediate Algebra's compound inequality standard is typed "CSS3.A.13" rather than CS3.A.13, which we assume to be a typo. Finally, while the Calculus pathway's fifth major area of work is billed as parametric **and polar** functions, we observed a parametric standard (CS3.F.14) with no corresponding polar standard.

exponential; to quadratic (plus absolute value and piecewise); to polynomial, radical, and logarithmic; and then to trigonometric (plus rational functions in the Calculus Pathway). Exploration of each new family also proceeds via the same five topics: key features, multiple representations, transformations, modeling, and comparison to previously explored families. (We can see this in the parallel wording of S1.F.5, S2.F.1, and S3.F.1.)

2. **Transformations of both graphs and geometric figures are gradually expanded side-by-side, allowing for specific connection.** Students first make sense of translation and reflection of graphs and figures, which helps to connect Geometry and Algebra topics. They then move into exploring dilation of figures and similarity, which can be connected to dilation of functions and the effect on their graphical representations. The "big idea" behind this thread is that when a function acts on a figure or a graph, some properties are preserved while others change in predictable ways. By Secondary III, students are transforming trigonometric graphs using reasoning they first constructed through work with triangles in Secondary I.
3. **The idea of rate of change is methodically developed as students move toward calculus.** Students explore and compare constant difference (slope) and constant ratio in Secondary I, then make sense of average rate of change over a closed interval in nonlinear functions in Secondary II, then examine end behavior specifically as well as comparative growth of different functions and function families in Secondary III, creating a natural runway toward the idea of the derivative for students who eventually plan to take Calculus.
4. **Solving is framed as mathematical reasoning; the number system expands through the grades to keep solutions available.** In S1.A.4, students are asked to use mathematical reasoning to solve linear equations and inequalities, beginning from the assumption that a solution exists and justifying each algebraic manipulation. Rational exponents and complex numbers arrive in Secondary II as the logical consequence of solving new equation types, as do logarithmic, radical, and rational functions in Secondary III.
5. **Geometry gradually evolves into trigonometry as the study of functions and geometric transformations meld.** In Secondary I, students explore rigid motion and connect it to congruence of figures; in Secondary II, these topics lead naturally into the study of dilation, similarity, right triangle side ratios, and radians. Finally, in Secondary III, these threads merge in students' study of trigonometric functions and identities and the unit circle.
6. **The statistical investigative cycle is revisited annually, with deepening scope each year.** While S1.D.1–D.6, S2.D.1–D.4, and S3.D.5–D.8 are structurally identical (ask a statistical question; plan and collect data (or evaluate secondary data); analyze, interpret and refine), the scope and complexity of what students are asked to do gradually increases over time, from univariate comparison and linear bivariate comparison in Secondary I, to nonlinear bivariate models in Secondary II, to categorical data, probability, and formal

inference in Secondary III. Data Science is also where each year's algebra is applied, with the strand always including opportunities for mathematical modeling with students' increasing lexicon of function families.

While the "Extended Topics" sections (matrices in Secondary I; vectors, the complex plane, and conics in Secondary II) sit slightly outside this narrative, they provide interesting connections between systems of equations and plane transformations.

Algebra I–Geometry–Algebra II Pathway

Algebra I includes three major areas of work:

1. Solving linear, exponential, and quadratic equations
2. Understanding, comparing, and representing linear, exponential, and quadratic functions
3. Describing characteristics of functions from these families

While including fewer Algebra standards and more Geometry standards (as is done in Secondary I) provides opportunities to develop rich connections across seemingly different topics, focusing tightly on one conceptual category in Year 1 has the advantage of providing a clearer sense of mathematical coherence at a level that may be more immediately obvious to students. They have the opportunity to compare and contrast three different types of equations and/or function families while investigating how algebraic structure manifests in a variety of representations—symbolic, graphic, tabular, real-world situations, and so on—in ways that are legible and recognizable as "linear-ness," "exponential-ness," or "quadratic-ness." A focus on the algebraic structure of different equation and function families also helps to motivate basic transformations (e.g., vertical translation and reflection).

Limiting Year 1 to the study of algebraic topics also enables a tight focus on solving equations of different types as a process of logical reasoning, which leads naturally to the extension of the number system when familiar numbers are not sufficient (as we saw in the example above under Secondary II in which imaginary and complex numbers arise from attempting to algebraically solve equations such as $x^2 - 1 = 0$.)

Geometry likewise includes three major areas of work:

1. Understanding and applying congruence via geometric transformations
2. Proving congruence and similarity via geometric transformations
3. Analyzing and interpreting data related to comparing distributions or exploring associations

The "mathematical storyline" of this course begins with students exploring rigid motions defined as functions on the plane (G.CO.2), then defining congruence as the existence of such a function (G.CO.3) and deriving triangle congruence criteria (G.CO.6), as in Secondary I. Students then have opportunities to justify these criteria using coordinate geometry via the Pythagorean Theorem, distance formula, and slope (G.CO.7), which is eventually developed into formal proof in multiple formats (G.CO.8). From congruence, students investigate dilation and its connection to similarity (G.SRT.1), then prove the Pythagorean Theorem by similarity (G.SRT.2). This leads naturally into the study of trigonometric ratios as similarity invariants (G.SRT.3) and basic trigonometric identities (G.SRT.4). An understanding of triangle similarity can then be leveraged to prove formally that all circles are similar (G.C.1) and can define radian as arc length over radius (G.C.3). Students are then prepared to investigate area and volume scaling by k^2 and k^3 (G.GMD.1) and apply this understanding to geometric modeling and density (G.GMD.2–4). In the integrated sequence, this same learning progression is spread across three courses and tied more explicitly into Algebra topics.

It is worth noting that G.D.1 through G.D.10 is the **complete** Secondary I and Secondary II data program, accounting for one of the course's three billed major areas of work (roughly a third of the course). These standards have essentially no connection to the geometric content and are collocated, more than integrated with geometry content. That said, the inclusion of the Data Science standards in the Geometry course could be seen as quietly providing opportunities for students to maintain algebra skills, keeping the Algebra I function families in active use during a year that would otherwise contain little symbolic algebra. This partially mitigates a well-documented weakness of AGA sequences.

Algebra II includes four major areas of work, identical to those of Secondary III:

1. Solving polynomial, logarithmic, and radical equations
2. Understanding, comparing, and representing polynomial, exponential, logarithmic, and trig functions
3. Describing characteristics of these types of functions
4. Drawing and justifying conclusions from surveys, experiments, and observational studies

As in Secondary III, we see a number of mathematical "big ideas" beginning to crystallize in two key areas: **Functions and Geometry**—where students' study of a variety of function families over the past three years is consolidated under the umbrella of "function families as taxonomy," with many geometry-focused ideas such as similarity and right triangles subsumed by the Functions category—and **Statistics and Data Science**, where students synthesize a wide range of knowledge and skills related to probability, study design, sampling, and statistical significance in order to reason from sample to population under chance variation. The key difference between Algebra II and Secondary III is the absence of the three geometric modeling standards which, in the AGA sequence, appear in the Geometry course instead.

Coherence Across the AGA Sequence versus the Integrated Sequence

Because the individual standards remain the same across both pathways, the same six "mathematical storylines" we saw in the integrated pathway can be traced through the AGA pathway. However, it is important to note that the forced separation of the Algebra and Geometry categories arguably increases **within-strand coherence** at the expense of some of the **cross-grade coherence** that these six storylines provide so well through the integrated sequence:

1. **Function families expand in a "choppy" rather than steady rhythm.** Students explore linear, exponential, and quadratic function families all in Year 1, see no new function families in Year 2, then encounter polynomial, radical, logarithmic, and trigonometric functions all in Year 3. The integrated sequence provides a steady drip of new function families, whereas the AGA front-loads and back-loads work with function families, leaving a gap in the middle.
2. **Transformation is no longer a clear unifying idea across Algebra and Geometry.** In the integrated sequence, rigid motions and transformations of graphs can be presented as the same concept explored in parallel within the same course; students can transform triangles and graphs within the same semester and can be shown that they are applying the same mathematical idea in two different contexts. In the AGA sequence, all function transformations (A1.BF.4; A1.BF.5) are presented in Algebra I while all geometric transformations appear in Geometry, a year apart. This means the connection between these two ideas must be expressly reconstructed by a teacher who notices it (if the connection is not explicitly addressed by instructional materials). This also creates a small sequencing issue in that A1.M.5 has students use 2 × 2 matrices to model transformations of the plane a full year before students encounter transformations of the plane in Geometry.
3. **Exploration of rate of change is compressed, then paused.** Students explore both constant difference versus constant ratio **and** average rate of change in Algebra I. They do not encounter rate of change at all in Geometry the following year, then pick up the idea again in Algebra II in the context of end behavior and comparative growth. While the "runway to calculus" and the derivative is still present, two-thirds of it appears in Year 1, limiting its utility for students who will eventually take Calculus.
4. **Solving and number system closure is addressed entirely in Years 1 and 3.** In Algebra I, students solve quadratic equations, leading to the Fundamental Theorem of Algebra and closure of the complex numbers. Students then encounter no new ideas around solving equations in Geometry (apart from applying basic solving techniques in geometric contexts), then solve polynomial, logarithmic, radical, and trigonometric equations in Algebra II. As mentioned above, the year-long algebraic gap that this creates is a well-documented weakness of an AGA sequence. While the inclusion of the Data Science standards in the Geometry course somewhat mitigates this gap in the draft Utah

standards, teachers should still expect that students will need to revisit foundational ideas from Algebra I and earlier grades related to functions and solving in order to be successful in Algebra II.

5. **The geometry-to-trigonometry arc is accomplished in two years (Geometry and Algebra II) rather than three.** The concept of similarity is developed in geometry via ratios of corresponding side lengths, leading natural into trigonometric ratios and identities and then radians. These ideas are then picked up in Algebra II in the context of periodic functions. While this progression may feel cleaner in structure, it is also likely more fragile in practice, since the handoff depends entirely on Algebra II teachers reactivating an idea that students have not seen since the year before (versus the Integrated progression, where the three-year development of this arc is more continually present).
6. **The statistical investigative cycle runs only twice instead of three times.** In the integrated sequence, students execute the full investigative cycle (ask; plan and collect; analyze; and interpret and refine) once a year for three years, incorporating topics from algebra and geometry each year with increasing depth and complexity. In AGA, the cycle likely runs twice during the Geometry year (G.D.1–6, then G.D.7–10 with nonlinear models) and once in Algebra II with inference. While two iterations back-to-back within a single year may offer certain affordances in terms of continuity, this structure forfeits the opportunity for annual reinforcement, and Year 1 students end up doing no statistics at all. It is also worth noting that if a third of the Geometry course is to be spent on statistics and data science, teachers must be conscientious about ensuring that appropriate time is spent on those topics rather than allowing them to be "squeezed out" should geometry-focused units run long. If this happens, students can arrive at Algebra II inference standards having never completed a descriptive statistics cycle, since there is no opportunity in Algebra I.

Development of Computational Fluency

The Utah draft standards define procedural fluency as applying procedures efficiently, flexibly, and accurately, transferring them across contexts, building procedures from other procedures, and recognizing when one strategy or procedure is more appropriate than another, all of which are well supported by research literature. The document then lays out an explicit theory of action by which computational fluency is to be developed (conceptual understanding → reasoning strategies → automatic basic fact retrieval → computational fluency), thus laying the groundwork for more complex procedural fluency through the grade levels.

The phrase "flexibly, accurately, and efficiently" functions as a marker of standards explicitly focused on fluency, while earlier standards laying conceptual groundwork for fluency can often be identified by the phrase "build and use models." The standards also employ a three-tier vocabulary—basic fact fluency, computational fluency, and procedural fluency—to label the

Essential Competencies related to fluency at each grade. The table below shows how fluency standards create a coherent progression through each tier from pre-K through grade 8.

Table 3. Progression of Fluency-Related Expectations Through the Grade Levels

Grade Level	Fluency-Related Standards		
	Computation with Conceptual Strategies	Expected Fact Fluency	Expected Computational Fluency
PK–3	• P3.CC.4—Counting and cardinality within 5 • P3.CC.7—Subitize within 3	N/A	N/A
PK–4	• P4.CC.3, 4—Counting and cardinality within 10 • P4.CC.7—Subitize within 5	N/A	N/A
Kindergarten	K.NBT.1—Counting, cardinality, and addition or subtraction within 10; decompose 11-19 into ten + ones	K.OA.4—Addition and subtraction within 5	N/A
Grade 1	• 1.OA.6—Counting, cardinality, and addition or subtraction within 20 • 1.NBT.4—Two-digit addition with concrete or pictorial models	1.OA.7—Addition and subtraction within 10	N/A
Grade 2	2.NBT.6—Sums or differences up to and including 999; decompose three-digit numbers into hundreds, tens, and ones	2.OA.2—Addition and subtraction within 20	2.NBT.5—Addition of up to four two-digit numbers; differences of two two-digit numbers
Grade 3	• 3.OA.3—Multiplication of division within to 100 • 3.NBT.4—Multiplication of multiples of 10	3.OA.6—One-digit multiplication and division facts within 100	3.NBT.3—Addition and subtraction of three-digit numbers

Grade 4	• 4.NBT.6—Multiplication of two four-digit numbers by one-digit numbers and two-digit numbers by two-digit numbers • 4.NBT.7—Division of three-digit numbers by one-digit	N/A	4.NBT.5--Multi-digit addition and subtraction using "efficient algorithms"
Grade 5	• 5.NBT.7—Division of four-digit numbers by one-digit • 5.NBT.8–10—All decimal operations • 5.NF—All fraction operations	N/A	5.NBT.6—Multiplication of four-digit numbers by one-digit numbers and two-digit numbers by two-digit numbers
Grade 6	6.NS.1—Dividing fractions by fractions	N/A	6.NS.2—Multi-digit decimal division via an algorithm 6.NS.3—All four operations with multi-digit whole numbers and decimals
Grade 7	N/A	N/A	7.NS.4—All four operations with rational numbers, including signed decimals and fractions

Comparisons of Fluency Progressions Across Benchmark Standards

The Utah draft standards and all benchmark standards reviewed make a foundational commitment to developing conceptual understanding of number and operations first, with procedural fluency growing out of that conceptual understanding. All five sets of standards also share the same ordering of number types (whole numbers and place value, then fractions, then decimals, then signed numbers), and all lean on visual models as the bridge from conceptual understanding to procedural fluency (area models, number lines, arrays, and Singapore's bar model tradition).

As expected, the U.S. standards are the most closely related with only minor differences in how fluency is treated and discussed, while Singapore's standards have a few notable differences.

- The word "fluency" appears only once in Singapore's primary syllabus; rather than stating a fluency **policy**, Singapore simply schedules fluency as **content**. For example, "mental calculation involving addition and subtraction within 20 / of a 2-digit number and ones without renaming / of a 2-digit number and tens" is a P1 (grade 1) learning objective with the same status as counting or using place value.

- Algorithms are listed generally rather than specifically; for example, “multiplication and division algorithms, up to 3 digits by 1 digit,” whereas U.S. standards tend to include modifiers that add specificity (e.g., “efficient algorithms,” “the standard algorithm”).
- Singapore’s are the only benchmark documents that include an explicit calculator policy. From P5 on, individual objectives are tagged “without calculator,” indicating that calculators are otherwise available
- Singapore’s are the only standards that differentiate by track, splitting into “Standard” and “Foundation” at P5, with the “Foundation” track deliberately reteaching P1–P4 material in P5.

The table below illustrates where fluency milestones fall in different standards documents.

Table 4. Comparison of Fluency-Related Milestones Across Benchmarks

Milestone	Utah	Massachusetts	Minnesota	Singapore
Addition and/or subtraction facts from memory	End of grade 2 (3.OA-style “know from memory” in 2.OA.2)	End of grade 2	Never explicitly stated; fluency within 20 via mental strategies required by end of grade 2	P1–P2 (grades 1–2)—as mental calculation objectives
Multiplication and/or division facts	End of grade 3, all one-digit products	End of grade 3, within 100	**Grade 4**, and framed as 0–12; grade 3 covers only 2s, 5s, 10s, squares	P2 (grade 2) –with 2, 3, 4, 5, and 10 P3 (grade 3) –with 6, 7, 8, and 9
Multi-digit addition and subtraction	Grade 4, “efficient algorithms”	Grade 4, **standard algorithm**	No fluency benchmark; grade 4 asks for *estimation*	P2 (grade 2) –with three-digit numbers P3 (grade 3) –with four-digit numbers
Multi-digit multiplication	Grade 5, “efficient numerical strategies”	Grade 5, **standard algorithm**	Grade 5, “efficient strategy”—decomposition or area model	P3–P4 (grades 3 and 4)

Milestone	Utah	Massachusetts	Minnesota	Singapore
Multi-digit division	Grade 5, efficient strategies; no algorithm named	Grade 6, **standard algorithm**	Grade 5, procedures that *may* include partial quotients and standard algorithms	P3–P4 (grades 3 and 4)
Decimals (all four operations)	Grade 5 with models; grade 6 applies an algorithm	Grade 6, **standard algorithm** for each	Grade 6, "solve situations requiring arithmetic"	P4–P5 (grades 4 and 5)
Fraction addition and subtraction	Grade 4 with like denominators, using models Grade 5 with unlike denominators	Grade 4 with like denominators Grade 5 with unlike denominators	Grade 4 with like denominators, using models Grade 5 with unlike denominators	P2—like denominators within one whole P3—related fractions within one whole P4—no more than two different denominators P5—fractions and mixed numbers
Fraction multiplication	Grade 4, fractions by whole numbers Grade 5, fractions by fractions with models	Grade 4, fractions by whole numbers Grade 5, fractions by fractions	Grade 6, with generalizable procedures	P5
Fraction division	Grade 5, unit fractions by whole numbers and whole numbers by unit fractions with models Grade 6, fractions by fractions with models	Grade 5, unit fractions by whole numbers and whole numbers by unit fractions Grade 6, fractions by fractions	Grade 6, with generalizable procedures	P6
Signed rational numbers	Grade 7	Grade 7	Grade 7	Secondary 1 (grade 7)

Utah Mathematical Practices

One of the distinguishing features of the UCS-M is the explicit integration of the Utah Mathematical Practices throughout the content standards. Unlike many mathematics standards documents, which present mathematical practices as a separate companion framework, the Utah standards intentionally embed one or more mathematical practices within every content standard to reinforce the development of mathematical reasoning alongside mathematical content. This approach reflects the view that mathematical proficiency is developed not only through mastery of mathematical concepts and procedures but also through engaging students in productive mathematical habits of mind, including reasoning, communication, modeling, strategic use of tools, and mathematical inquiry.

As part of this review, WestEd evaluated the Utah mathematical practices from two complementary perspectives. First, reviewers examined the extent to which the Utah Mathematical Practices identified within individual standards were meaningfully integrated into the standards' performance expectations and appropriately aligned with the mathematical content. Second, the Utah Mathematical Practices were compared with the Standards for Mathematical Practice (SMPs) that serve as the mathematical practice framework adopted or adapted by most U.S. state mathematics standards, including the Massachusetts and Minnesota mathematics standards used as benchmarks in this review. Together, these analyses provide evidence regarding both the implementation of the Utah Mathematical Practices within the standards and the extent to which they reflect nationally recognized conceptions of mathematical proficiency.

Integration of the Utah Mathematical Practices Within Standards

The standards review found that, in general, the Utah Mathematical Practices are consistently and meaningfully integrated throughout the UCS-M. Reviewers evaluated whether the Utah Mathematical Practice(s) identified within each standard were reflected in the mathematical performance students were expected to demonstrate rather than serving as isolated or disconnected additions to the content standard.

Across the 418 standards reviewed, 90 percent were rated as having all identified Utah Mathematical Practices meaningfully integrated into the performance expectations, while 5 percent were rated as having some identified practices integrated and 5 percent were rated as having no meaningful integration. These findings indicate that the UCS-M are generally successful in connecting mathematical practices with mathematical content in ways that support coherent instruction instead of treating the practices as separate instructional goals.

Strong integration was observed throughout the elementary grades, where nearly all standards received the highest integration rating. Similar patterns were observed across middle school, with only isolated instances in which reviewers identified opportunities to strengthen the relationship between the selected Utah Mathematical Practice and the mathematical expectation. At the high school level, integration ratings remained generally strong, although reviewers identified somewhat more standards in which one or more practices were only partially integrated or not fully reflected in the stated performance expectation. Even within the high school standards, however, approximately four out of every five standards demonstrated complete integration of the identified Mathematical Practices.

Across grade levels, reviewers consistently observed that the UCS-M intentionally embed mathematical reasoning within content learning. Rather than treating the Utah Mathematical Practices as instructional recommendations or separate process standards, the UCS-M incorporate them directly into the student performance expectations. This design encourages students to develop conceptual understanding, procedural skill, and mathematical reasoning simultaneously and reflects an integrated view of mathematical proficiency.

The relatively small number of standards receiving partial integration ratings did not generally reflect concerns with the Utah Mathematical Practices themselves. Instead, reviewers most often concluded that the selected practice was only partially represented within the wording of the standard or that additional performance expectations would be needed for the practice to be fully realized. In some cases, reviewers noted that the identified Utah Mathematical Practice reflected only one aspect of the mathematical task, while another practice would also have been appropriate given the complete performance expected of students.

Overall, the findings suggest that the explicit integration of the Utah Mathematical Practices represents a significant strength of the UCS-M. The standards consistently reinforce mathematical reasoning alongside content learning and demonstrate a high degree of coherence between mathematical practices and mathematical expectations.

Appropriateness of the Utah Mathematical Practices

In addition to evaluating integration, reviewers examined whether the Utah Mathematical Practice(s) identified within each standard were appropriate for the mathematical content and performance expectations described by the standard. Whereas the integration review focused on whether the practice was meaningfully reflected in the standard, the appropriateness review considered whether the selected practice represented the most suitable mathematical habit of mind for the content being taught.

Overall, reviewers found a very high degree of alignment between the Utah Mathematical Practices and the mathematical content of the standards. Across all 418 standards, 95 percent were rated as having all identified Utah Mathematical Practices appropriate, while only 2 percent were rated as having some identified practices appropriate and 2 percent were rated as having no identified practices appropriate. These findings indicate that the Utah Mathematical Practices selected for individual standards are generally well matched to the mathematical knowledge, reasoning, and problem-solving behaviors students are expected to demonstrate.

The strongest results were observed throughout the elementary and middle grades, where nearly all standards were judged to have appropriately selected Utah Mathematical Practices. Similarly high levels of agreement were observed across the secondary courses, although reviewers identified a small number of standards where alternative or additional Utah Mathematical Practices could have strengthened the connection between the mathematical activity and the intended learning outcome, or where a typo was suspected. These instances were relatively uncommon and did not suggest any systematic pattern of inappropriate practice assignments.

Where reviewers identified opportunities for improvement, they most often concluded that multiple Utah Mathematical Practices could reasonably apply to the same mathematical task. In these cases, the identified practice was not necessarily incorrect; rather, reviewers observed that an additional practice—or, less frequently, a different practice—might more completely capture the mathematical thinking required by the standard. These observations reflect the interconnected nature of mathematical proficiency, where students frequently engage in reasoning, modeling, communication, strategic tool use, and use of structure simultaneously while solving mathematical problems.

Taken together, the integration and appropriateness findings suggest that the Utah Mathematical Practices have been thoughtfully incorporated throughout the standards and generally align well with the mathematical content they accompany. The relatively small number of partial ratings reflects opportunities to further strengthen the connection between individual standards and the practices they are intended to promote rather than concerns with the overall design or intent of the Utah Mathematical Practices themselves.

Comparison With the Standards for Mathematical Practice Used by Most U.S. States

As part of the benchmark review, WestEd compared the Utah Mathematical Practices with the Standards for Mathematical Practice (SMPs) that serve as the mathematical practice framework adopted or adapted by most U.S. state mathematics standards, including the Massachusetts and the Minnesota mathematics standards. The purpose of this comparison was not to determine whether the two frameworks are identical but rather to examine the extent to which they reflect similar expectations for mathematical reasoning, problem-solving, communication, modeling, precision, and strategic use of tools.

Overall, the comparison indicates substantial overlap between the Utah Mathematical Practices and the SMP framework. Both frameworks emphasize mathematical reasoning, justification, communication, strategic problem-solving, identification of patterns and structure, precision, and purposeful use of tools. The principal differences lie less in the underlying competencies themselves than in how those competencies are organized, framed, and emphasized. In several instances, the Utah framework broadens or reorganizes ideas that are distributed across multiple SMPs, while in other cases the SMP framework provides a more explicit emphasis on particular mathematical habits of mind. These differences represent alternative approaches to organizing mathematical proficiency rather than fundamentally different expectations for student learning.

Table 5 summarizes the alignment between Utah Mathematical Practices and the Standards for Mathematical Practice (SMPs) used by three of the benchmark systems. Because Utah reorganizes and reframes some mathematical practices differently than the benchmark systems, some Utah practices align with multiple SMPs and vice versa. Detailed comparisons of each mathematical practice follow this table.

Table 5. Utah Draft Mathematical Practices Alignment to Standards for Mathematical Practice

Strongly aligned **Partially aligned** **Not aligned**

Utah Mathematical Practices (MP)	Standards for Mathematical Practice (MA and MN)							
	SMP 1	SMP 2	SMP 3	SMP 4	SMP 5	SMP 6	SMP 7	SMP 8
MP 1	Not aligned							
MP 2		Partially aligned						
MP 3			Strongly aligned					
MP 4				Partially aligned	Partially aligned			
MP 5					Strongly aligned			
MP 6	Partially aligned					Strongly aligned		
MP 7							Partially aligned	
MP 8	Partially aligned							Not aligned

Practice 1: Sense-Making versus Inquiry

The first Practice Standard (SMP) that occurs in nearly every state's standards ("Make sense of problems and persevere in solving them") is built around a problem-solving narrative: Students explain the meaning of a problem to themselves; look for entry points; plan a solution pathway; monitor and evaluate progress; change course if necessary; persevere when stuck; and check answers using a different method. (This practice is famously the "productive struggle" practice.)

In contrast, Utah's MP 1 ("Ask questions to explore mathematical ideas") centers around student-generated questioning. This is a substantial shift in intent, not merely a change in language. Most notably, the language found in other states' version of SMP 1–including "perseverance," "planning," "monitoring," and "checking"—is not present in Utah's MP 1 description. Some of the ideas found in SMP 1 appear in the general introduction to the Utah standards or are embedded in other Utah MPs—such as MP 6's reasonableness and MP 8's testing of conjectures—but the headline emphasis on planning, metacognition, perseverance, and persistence is absent in MP 1.

Practice 2: Contextualizing and Decontextualizing—Keeping the Literal Meaning but Lacking the Bigger Picture

The second SMP in most other states ("Reason abstractly and quantitatively") is specifically focused on the process of solving contextualized problems, often referred to as "word problems" or "story problems." This process involves two complementary mathematical skills: 1) decontextualizing—or abstracting a concrete situation by representing it symbolically and manipulating the symbols to arrive at a numerical or symbolic solution—and 2) recontextualizing, or pausing to probe the referents in the original problem during the manipulation and interpreting the numerical or symbolic answer in context. This requires both quantitative reasoning—or attending to what in a problem can be countered or measured and how those quantities are related—and abstract reasoning, or manipulating pure symbols without attending to their referents.

Utah's MP 2 ("Add or remove context to make sense of mathematics") keeps the literal meaning of decontextualizing and recontextualizing but not the reason behind why these two skills are important. It mentions the need for students to "flexibly move between concrete, pictorial, and abstract representations" and "move back and forth between a problem's context and its representation to use the form that best fits the situation," but the emphasis on the process of solving contextualized problems is notably absent. Instead, the standard says that "working in and out of context throughout a learning progression empowers students to make sense of mathematics and persevere as they examine abstract and real-world problems," which is not necessarily untrue but is vague and does not address with specificity why a student might need to work in and out of context when solving contextualized problems. Versions of MP 2 in different grade bands also reference assigning a context when working with noncontextualized problems or ideas to better understand them, which does not appear in the version of SMP 2 that nearly all other states use.

Practice 3: Constructing Arguments—Close, With One Change in Emphasis

The version of SMP 3 that appears in most other states' standards ("Construct viable arguments and critique the reasoning of others") has two main pillars: 1) building one's own mathematical and logical arguments and 2) engaging critically with those of others. The "critique" element is explicit and prominent.

Utah's MP 3 ("Construct, justify, and communicate clear and reasonable arguments") keeps "construct" and also adds "justify" and "communicate," with grade-band descriptions emphasizing respectful agreement and disagreement. For example, in preschool, it states, "They develop strategies to agree and disagree respectfully. They begin to justify their arguments with evidence," and at the high school level, it reads, "construct arguments to distinguish between situations that can be modeled by various function types." While the overlap is strong, the explicit language around critiquing others' mathematical arguments is notably softer in the Utah Mathematical Practices. While the version that most states use of SMP 3 names "critique the reasoning of others" as a headline expectation, Utah folds the listening-and-disagreeing piece into grade-band descriptions. The substance is similar, but the visibility and emphasis differs.

Practice 4: Modeling—Utah Broadens the Meaning

The version of SMP 4 that appears in most other states' mathematics standards ("Model with mathematics") is unambiguously focused on applied modeling—using mathematical models to solve problems in everyday life, society, and the workplace; identifying important quantities; analyzing relationships; drawing conclusions; and revising the model if necessary. The "model" in this version of practice refers specifically to mathematical models, particular types of representations, usually symbolic, that can be used to make predictions about the situation they model. This definition is in line with the widely referenced definition from the Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME), which describes modeling as "a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena" (Garfunkel & Montgomery, 2019, p. 8). This process is distinctive for its cyclic nature, where, starting from a real-world situation, students or modelers formulate a mathematical representation, work through it, and then return to the context to assess the model's validity and applicability.

In contrast, Utah's MP 4 ("Build and use models") broadens the definition of "model" to include not just **mathematical** models but also "verbal, contextual, visual … and physical models." The description includes mathematical models and applied modeling but also explicitly broadens the practice to include a second meaning: "Students build and use models in two primary ways. One of these is to use models to connect mathematical concepts to one another. Another way that students use models in high school is to model real-world phenomena." The elementary versions of the practice indicate that "models" can also include manipulatives, area models, number lines, and base-ten blocks (objects that most other state standards would classify as "tools").

This is the most consequential framing difference, since the verb phrase "Build and use models to …" appears in hundreds of Utah content standards. This usage would not meet the definition of "modeling with mathematics" that is used in nearly all other states' mathematical practices and in GAIMME; instead, "modeling with mathematics" would be considered "using appropriate tools strategically" to support representations and make meaning of mathematical ideas and objects. The risk is that the concept of "modeling" gets diluted; when so many standards begin with "Build and use models," the disciplinary practice of applied modeling and using mathematical models, rather than physical or visual ones, can be neglected.

Practice 5: Use of Tools — Again, a Broadened Meaning

SMP 5, as it appears in most state standards ("Use appropriate tools strategically"), considers "tools" to be things like pencil and paper, concrete models, rulers, protractors, calculators, spreadsheets, statistical package, dynamic geometry software, and computer algebra systems. It also includes representations that students might use to help them solve a problem, such as number lines, tape diagrams, and graph sketches. The focus of the practice is on a student's ability to consider the problem, make decisions about which tools are best for the job, and then use them strategically (rather than arbitrarily).

The spirit of Utah's MP 5 is the same ("Select and use tools appropriately and strategically"), though the practice explicitly broadens the definition of "tool" to include not only physical, concrete, and representational tools but also mathematical techniques (e.g., completing the square, isolating x, the Zero Product Property) and bodies of mathematical knowledge (e.g., ratio reasoning, multiplicative thinking).

As with the verb phrase "Build and use models to…," phrases involving the use of "tools" that refer to something other than physical tools or commonly used representations appear in hundreds of Utah content standards. As with the concept of "modeling," there is a risk that by using the word "tool" so broadly, it could refer to virtually any mathematical idea, concept, or technique. As a result, its meaning is diluted to the point that it is no longer useful to educators, curriculum writers, and assessment designers.

Practice 6: Precision—Utah Adds "Reasonableness"

The version of SMP 6 that appears in most states' mathematics standards ("Attend to precision") focuses on precise language and precise definitions, along with careful use of symbols such as the equal sign, units of measure, and labeling axes. Additionally, it includes calculating accurately and efficiently while expressing numerical answers with precision appropriate to the context.

Utah's MP 6 ("Attend to precision and reasonableness") broadens this description by including everything addressed by the more common version of SMP 6 and explicitly adding the idea of considering reasonableness: "They regularly evaluate whether actions, ideas, processes, and solutions make sense and are reasonable … The continuous evaluation of reasonableness ensures that students improve their ability to identify and learn from mathematical errors." In the version of SMP 6 that appears in most states' standards, the idea of reasonableness is treated under SMP 1 as part of the problem-solving process in which students go back to the original problem and consider whether or not their solution makes sense given the context before accepting it and SMP 2, which contextualizes a solution during and after the problem-solving process to determine its actual meaning in the original problem.

Practice 7: Structure—Similar Concept; Utah Lacks Key Specificity

The version of SMP 7 that appears in most states' mathematics standards ("Look for and make use of structure") focuses on noticing the patterns that arise from mathematical structure and leveraging that structure to efficiently characterize mathematical objects and solve problems. For example, a student who understands the base-ten structure of numbers can mentally leverage that structure to efficiently solve a problem like 7 x 13 like so:

$$7 \times 13 = 7 \times (10 + 3) = 7 \times 10 + 7 \times 3 = 70 + 21$$

$$70 + 21 = 70 + (20 + 1) = (70 + 20) + 1 = 90 + 1 = 91$$

Looking for and making use of structure also means seeing mathematical objects as being constructed from other mathematical objects and considering the implications of that construction holistically to draw conclusions. For example, seeing $5 - 3(x - y)^2$ not just as a mathematical object in its own right but understanding it as 5 – 3* [some positive number] and thus 5 – [some positive number], and then concluding that the value of the entire expression can never be greater than 5.

Utah's MP 7 addresses a similar idea in many ways ("Describe and represent structures, patterns, and relationships") but lacks the specificity of the more common version of SMP 7 that makes it so powerful: "Students describe and represent structures, patterns, and relationships to make sense of mathematics … They identify patterns and make connections across mathematical ideas." This description is quite vague and, as compared to the more common version, lacks a clear description of specifically how students can leverage mathematical structure to draw mathematical conclusions and solve problems more efficiently. Grade-level practice descriptions are similarly vague.

Practice 8: Regularity versus Conjecture, Test, and Evaluate

The version of SMP 8 that appears in most states' standards ("Look for and express regularity in repeated reasoning") invites students to notice when calculations are repeated; to look for general methods and shortcuts; and to abstract patterns into algebraic rules—all examples of mathematical regularity (which often arises from mathematical structure). A student might notice while carrying out long division that the calculations have become repeatable and conclude that the quotient is a repeating decimal. Or a student may say, "Every time, I am doubling and then adding five," and represent that regularity with a function like $y = 2x + 5$. At its heart, this version of SMP 8 is about abstracting from regularity to a general formula or method.

Utah's MP 8 ("Make conjectures and evaluate the results") has a different focus that is quite different from this more common version. This practice is reminiscent of the scientific method, inviting students to engage in mathematical investigations, "[using] their understanding of structures, patterns, and relationships to speculate about the nature of a relationship and then test their conjecture to determine how to proceed." Once again, in the more common version of SMP 8, this habit of mind is addressed by SMP 1 as part of the problem-solving process. The net effect is that the Utah Mathematical Practice trades the more common version's emphasis on algebraic generalization for an emphasis on conjecture-and-test investigation. While this aligns with Utah's data-science orientation, the cost is that the specifically algebraic habit of spotting regularity and leveraging it to generalize mathematically is no longer an emphasis.

Overall Comparison

Taken together, the comparison indicates that the Utah Mathematical Practices and the version of the SMPs that appear in most other states' standards reflect a mostly common vision of mathematical proficiency while organizing that vision in different ways. Both frameworks emphasize reasoning, communication, precision, strategic use of tools, modeling, and recognition of mathematical structure as essential components of mathematics learning.

The Utah version places comparatively greater emphasis on inquiry, conjecture, reasonableness, and broad use of representations, while the version that most other states use more explicitly emphasizes perseverance in problem-solving; abstract quantitative reasoning; mathematical modeling in authentic contexts; and recognizing regularity through repeated reasoning. These differences primarily reflect alternative organizational and instructional emphases rather than fundamentally different expectations for student learning. Overall, the comparison suggests that the Utah Mathematical Practices remain broadly consistent with nationally recognized conceptions of mathematical proficiency while reflecting several design decisions that distinguish the Utah practices from those adopted by most other U.S. states.

Conclusions

This report integrates findings from two complementary phases of WestEd's independent review of the draft UCS-M. The first phase examined the standards relative to nationally and internationally recognized benchmark systems through standards-level crosswalks and domain-level comparisons. The second phase evaluated the standards themselves using a structured review process focused on measurability, clarity, instructional focus, mathematical rigor, DOK, developmental appropriateness, and the integration and appropriateness of the Utah Mathematical Practices. Together, these analyses provide a comprehensive evidence base for evaluating the overall quality of the draft standards.

Overall, the findings indicate that the UCS-M provide a strong and coherent framework for mathematics teaching and learning across the PK–12 continuum. The standards demonstrate substantial alignment with nationally recognized mathematics standards, place a strong emphasis on conceptual understanding and mathematical reasoning, and incorporate several distinctive features, including an expanded Data Science progression and explicit integration of the Utah Mathematical Practices, that distinguish them from many existing state mathematics standards. At the same time, the review identified recurring opportunities to improve the precision, clarity, developmental progression, and implementation of selected standards. These observations are intended to support continued refinement of the standards rather than fundamental redesign.

Overall Strengths of the UCS-M

Across both phases of the review, several consistent strengths emerged.

Strong Alignment With Nationally Recognized Mathematics Standards

The benchmark analyses found that the UCS-M are highly comparable to the Massachusetts Mathematics Curriculum Framework. More than 90 percent of K–8 standards and nearly all high school standards demonstrated strong or partial alignment with these benchmark systems, indicating broad agreement in the scope, progression, and organization of mathematical content. Although somewhat greater differences were observed relative to Minnesota and Singapore, these differences generally reflected alternative approaches to sequencing and emphasis of mathematical ideas rather than substantial differences in mathematical expectations.

Well-Balanced Mathematical Content

The standards reflect an appropriate balance of conceptual understanding, procedural skill, and mathematical application while maintaining instructional focus consistent with nationally recognized mathematics standards. The review found that the standards appropriately prioritize the major work of each grade and emphasize conceptual understanding throughout the PK–12 progression. The distribution of DOK also reflects an appropriate progression from foundational knowledge in the early grades toward increasingly sophisticated reasoning in later grades.

Developmentally Appropriate Progression

The standards establish a generally coherent PK–12 progression of mathematical learning. A majority of the standards were rated as developmentally appropriate, with reviewers identifying relatively few concerns regarding the placement of mathematical content across grade levels. The benchmark comparisons similarly suggest that the overall progression closely reflects those found in nationally recognized mathematics standards with concepts and topics being introduced at about the same time, especially when compared to the Massachusetts standards.

Distinctive Emphasis on Data Science and Mathematical Practices

The review identified two particularly distinctive features of the Utah standards. First, the standards establish a comprehensive PK–12 progression in Data Science that extends beyond the traditional treatment of statistics found in many benchmark systems. Second, the explicit integration of the Utah Mathematical Practices throughout the content standards represents an innovative approach to embedding mathematical reasoning within content learning. Reviewers found that the Utah Mathematical Practices were generally well integrated into the standards and appropriately aligned with the mathematical expectations they accompany.

Opportunities for Refinement

Although the review identified many strengths, several recurring themes emerged across the standards review that present opportunities for further refinement.

Improve Clarity and Measurability of Selected Standards

The most consistent observations across reviewers were related to the clarity and measurability of some standards rather than any underlying issues with the mathematical content. Across grade levels, reviewers identified opportunities to improve measurability and clarity by replacing non-observable verbs with observable performance expectations; reducing ambiguity; simplifying unnecessarily complex or compound standards; and providing more precise descriptions of expected student performance. These issues were concentrated in a relatively small subset of standards and generally could be addressed through targeted wording revisions rather than substantive changes to mathematical content.

Continue Refining the Integration of Mathematical Practices

Reviewers found that the Utah Mathematical Practices were generally well integrated and appropriately aligned with the mathematical expectations they accompany. However, the review also found that embedded Mathematical Practice language frequently reduced the clarity or measurability of individual standards by making it less obvious what specific mathematical knowledge or skill students were expected to demonstrate. Future revisions may therefore benefit from considering how Mathematical Practice language can continue to reinforce mathematical reasoning while preserving clear, observable performance expectations.

Review Selected Developmental Progressions

The standards were generally found to be developmentally appropriate; however, reviewers identified a limited number of standards whose placement within the PK–12 progression may warrant additional consideration, in particular a subset of early-grade standards require relatively sophisticated reasoning. These observations represent targeted opportunities for refinement rather than broad concerns regarding the overall progression of the standards.

Consider Localized Benchmark Differences

Although the benchmark analyses demonstrated strong overall comparability with nationally recognized mathematics standards, they also identified localized differences in emphasis and placement. These include selected Geometry and Measurement topics; the sequencing of certain advanced secondary concepts; and Utah's expanded treatment of Data Science. These findings do not suggest deficiencies in the UCS-M; rather, they provide additional context for policy decisions regarding the intended scope and sequencing of mathematics content.

Recommendations for USBE

Based on the evidence collected during this review, WestEd offers the following recommendations for USBE's consideration.

1. **Preserve the overall structure and organization of the UCS-M.** The review found substantial alignment with nationally recognized mathematics standards and identified no evidence suggesting the need for major structural revisions to the standards.
2. **Prioritize targeted revisions to standards language.** Future revisions should focus primarily on improving the clarity, measurability, and accessibility of individual standards while preserving the underlying mathematical intent.
3. **Review selected standards identified through the developmental appropriateness analysis.** Particular attention may be given to the placement of a small number of early-grade standards where the expected level of reasoning may warrant reconsideration.

4. **Continue refining the implementation of the Utah Mathematical Practices.** The review supports maintaining the explicit integration of Mathematical Practices throughout the standards while considering opportunities to clarify how those practices are incorporated into individual content standards.
5. **Use the detailed analyst review worksheets as the primary resource for standards revision.** This report intentionally synthesizes findings across the review. Detailed, standard-specific observations—including reviewer comments, flagged issues, suggested revisions, recommended Mathematical Practices, and benchmark crosswalks—are documented in the accompanying analyst rating worksheets and benchmark crosswalk attachments. These materials provide the most appropriate foundation for any future standards revision process.

Final Remarks

Overall, the findings from this independent review suggest that the Utah Core Standards for Mathematics provide a high-quality foundation for mathematics instruction across the PK–12 continuum. The standards are broadly aligned with nationally recognized mathematics frameworks while incorporating several distinctive features that reflect Utah's educational priorities, particularly in the areas of Data Science and the explicit integration of Utah Mathematical Practices. The opportunities for refinement identified throughout this review are concentrated primarily in the wording, sequencing, and implementation of selected standards rather than the overall design of the standards themselves. Consequently, the analyst review worksheets and benchmark crosswalks accompanying this report should be viewed as complementary resources that provide detailed, standard-level guidance for any future revisions undertaken by USBE.

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Appendices

Appendix A. Rubrics Used for the Utah Standards Quality Review

Table A1. Rubric Used for Measurability and Specificity

<table>
<tr><th>Criteria & Description</th><th>Score</th><th>Indicators</th></tr>
<tr><td rowspan="4"><u>Measurability & Specificity</u>: Assess whether the standards are written in a way that allows for the development of clear assessment items</td><td>2—Measurable & well specified</td><td>The standard…
• Clearly indicates what is expected of students.
• Includes specific, low-inference performance expectations.
• Includes an appropriate level of detail
• Is not too broad or overly prescriptive.</td></tr>
<tr><td>1—Partially measurable / well specified</td><td>• Aspects of the standard clearly indicate what is expected of students and include specific, low-inference performance expectations, while other aspects do not.
• Aspects of the standard may be too broad or lacking in sufficient detail, or may be overly prescriptive.</td></tr>
<tr><td>0—Not measurable or poorly specified</td><td>• It is not clear from the standard what is expected of students.
• Performance expectations are not included or may be high inference.
• The standard may be too broad or overly prescriptive.</td></tr>
<tr><td colspan="2">If a score of 1 or 0 is assigned, then the primary reason(s) measurability is reduced is flagged. One or more of the following flags will be assigned: Too vague; Too broad; Multiple expectations; Non-observable verb; Lacks performance expectation; Overly prescriptive; Poorly Specified; Other.</td></tr>
</table>

Table A2. Rubric Used for Instructional Focus

Criteria & Description	Score	Indicators
Instructional Focus (K-8): Identify whether the standard addresses the "major work" of the grade; supporting work; additional work; or extraneous content.	**Major work**	The standard addresses the **major work** of the grade as defined by most US states.
	Supporting work	The standard addresses the **supporting work** of the grade as defined by most US states.
	Additional work	The standard addresses **additional work** of the grade as defined by most US states.
	Extraneous work	The standard **does not address** major, supporting, or additional work as defined by most US states.
Instructional Focus (High School): Identify whether the standard addresses 'widely applicable prerequisite; appropriate supporting work; or extraneous work for the grade level/course.	**Widely applicable prerequisite**	Standards that fall under the clusters, domains & conceptual categories considered **widely applicable prerequisites** for high school as defined by most US states.
	Appropriate supporting work	The standard addresses something that is not tagged as widely applicable prerequisites as defined by most US states, but seems appropriate for the course / grade level.
	Extraneous or inappropriate work	The standard either **does not appear** for most US states' high school standards in some form or seems inappropriate for the year/course.

Table A3. Rubric Used for Clarity and Accessibility

Criteria & Description	Score	Indicators
Clarity & Accessibility: Ensure the terminology is simple, concise, mathematically precise, and still accessible to educators and parents.	**2—Clear and accessible**	The standard… • Is understandable to key users such as teachers and curriculum directors • Is free of mathematical errors and imprecisions • Includes sufficient guidance to understand the PE • Uses language that most people can understand and is, as much as possible without sacrificing precision or rigor, free from jargon; any technical terms are appropriate for the grade level/course.
	1—Partially clear and accessible	• Some aspects of the standard are understandable to users, while other aspects could be confusing or ambiguous. • Mathematical errors or imprecisions may make aspects of the standard confusing. • Some aspects may need additional guidance to understand the PE. • Some aspects of the standard may include unnecessary jargon or technical language that is not appropriate for the course/grade level.
	0—Not clear and accessible	• Is likely to be confusing to key users like teachers and curriculum directors • Mathematical errors or imprecisions may make the standard confusing. • May lack sufficient guidance regarding the relevant content knowledge and math practices • May include unnecessary jargon that makes it difficult for the general public to understand.
	If a score of 1 or 0 is assigned, then the primary reason(s) clarity is reduced is flagged. One or more of the following flags will be assigned: Vague/ambiguous language; Jargon; Mathematical error/imprecise usage of mathematical term(s); Unclear grammar/typo(s); Insufficient guidance to understand PE; Other	

Table A4. Rubric Used for Depth of Knowledge

Criteria & Description	Score	Indicators
<u>**Depth of Knowledge**</u>: Evaluate the cognitive complexity required for students to successfully demonstrate the knowledge and skills described in a standard.	**DOK 1** (Recall, reproduction, rote procedures)	The performance expectation in the standard requires only **recall of information** (such as a fact, definition, term, or simple procedure), **performing a simple algorithm**, or **applying a formula**.
	DOK 2 (Skills/Concepts)	The performance expectation in the standard requires the student to **exercise judgment** as to how to appropriately apply conceptual understanding. The task is still **routine** in nature, with a clear solution path.
	DOK 3 (Strategic Reasoning/ Evidence/ Justification)	The performance expectation in the standard is **nonroutine**, without a clear solution path; the student must engage in **strategic thinking** in order to explain, justify, evaluate, analyze, and so on.
	DOK 4 (Extended investigation/ Synthesis over time)	The performance expectation in the standard requires drawing on mathematical knowledge and understanding to engage in **investigation, longer-term planning**, and **dealing with ambiguity,** often over an extended period of time.
	Not Applicable	The standard does not include a clear performance expectation, so it is not possible to give a DOK rating.

Table A5. Rubric Used for Mathematical Rigor

Criteria & Description	Score	Indicators
Mathematical Rigor: Identify whether each standard addresses mathematical skills and procedures; conceptual understanding of a mathematical idea; and/or applications of a mathematical skill/procedure.	**Procedural Skill / Fluency**	The standard requires demonstrating a **mathematical skill or procedure**; may or may not also require conceptual understanding or application**.**
	Conceptual Understanding	The standard requires **demonstrating mathematical understanding** that **goes beyond demonstrating a skill or procedure**; while verbs are not 100% reliable for determining whether conceptual understanding is involved, such standards often include verbs like explain, justify, evaluate.
	Applications	The standard requires **applying** conceptual understanding and/or procedural skills **to a real-world situation or context**.
	None	The standard doesn't address procedural skills, conceptual understanding, or applications.

Table A6. Rubric Used for Developmental Appropriateness

<table>
<tr><th>Criteria & Description</th><th>Score</th><th>Indicators</th></tr>
<tr><td rowspan="4"><u>Developmental Appropriateness:</u> Ensure that each standard is developmentally appropriate for the grade level/course.</td><td>2—Developmentally appropriate</td><td>• The mathematical skill, idea, or application addressed by the standard is developmentally appropriate for the grade level/course.
• The level of depth and complexity in which the performance expectations require students to engage with the content is developmentally appropriate for the grade level/course.</td></tr>
<tr><td>1—Partially developmentally appropriate</td><td>• Aspects of the mathematical skill, idea, or application addressed by the standard is developmentally appropriate for the grade level/course, while other aspects are not.
• The level of depth and complexity in which parts of the performance expectations require students to engage with the content is developmentally appropriate for the grade level/course, while others are not.</td></tr>
<tr><td>0—Not developmentally appropriate</td><td>• The mathematical skill, idea, or application addressed by the standard is not developmentally appropriate for the grade level/course.
• The level of depth and complexity in which the performance expectations require students to engage with the content is not developmentally appropriate for the grade level/course.</td></tr>
<tr><td colspan="2">If a score of 1 or 0 is assigned, then the primary reason(s) developmental appropriateness is reduced is flagged, and then reviewed later for progression review. One or more of the following flags will be assigned: Prerequisite missing; Too early; Too late; Repeats without growth; Abrupt jump; Other.</td></tr>
</table>

Table A7. Rubric Used for Integration and Appropriateness of Mathematical Practices

Criteria & Description	Score	Indicators
Math Practice Integration: Determine whether or not the MP(s) named in each standard are meaningfully integrated into the standard's performance expectation(s).	**2—All named practices integrated**	**All** practices explicitly named in the standard are **meaningfully integrated** into the standard and aligned with its performance expectations.
	1—Any named practices integrated	**At least one** of the practices explicitly named in the standard is meaningfully integrated into the standard and aligned with its performance expectations, but **at least one is not**.
	0—No practices integrated	**None** of the practices explicitly named in the standard are meaningfully integrated into the standard and aligned with its performance expectations.
	For scores of 1, the practice that is not meaningfully integrated will be identified.	
Math Practice Appropriateness/ Alignment: Determine whether or not the MP(s) named in each standard are appropriate to the mathematical content of the standard.	**2—All named practices appropriate**	**All practices** explicitly named in the standard are **appropriate to and aligned to** the mathematical content of the standard.
	1—Any named practices appropriate	**At least one** of the practices explicitly named in the standard is **appropriate to and aligned to** the mathematical content of the standard, but **at least one is not**.
	0—No practices appropriate	**None** of the practices explicitly named in the standard are appropriate to and aligned to the mathematical content of the standard.
Additional Math Practices: Determine whether any MP(s) not named in the standard would be better aligned with the content or performance expectations in the standard.	Identify the practice(s) or mark "none".	

Appendix B. Illustrative tables Providing Specific Examples of Measurability and Clarity Concerns

Table B1. Illustrative Examples of Standards with Measurability Concerns Including Explanations and Suggested Revision

Standard Text	Rating	Explanation	Suggested Revision
P3.CC.2—Begin to recognize that written numbers represent a quantity. Use a variety of *tools to build and model* numbers. (MP 4, 5)	0—Not measurable or poorly specified	**Too broad; non-observable verb.** "Begin to recognize that written numbers represent a quantity" is an internal, non-observable process. "Use a variety of tools to build and model numbers" is overly broad and poorly specified.	Identify what specific, observable behavior would indicate to a teacher that a student has begun to recognize that written numbers represent a quantity and reword the standard accordingly. Reword the second statement to make it clear what tools students should using to build and model numbers.
K.MG.2—Describe structures of two-dimensional and three-dimensional shapes, including names and attributes, regardless of size or orientation. Describe their relative positions in the environment. (MP 7)	1—Partially measurable and/or well-specified	**Multiple expectations.** This standard includes two separate performance expectations that are difficult to assess well with a single performance. The first is focused on shape knowledge and geometry while the second is focused on spatial skills.	Separate these two performance expectations into two different standards.
1.MG.2—Construct a reasonable argument about the order of three objects sorted by length. Compare the lengths of two objects indirectly using a third object and justify the results.	1—Partially measurable and/or well-specified	**Poorly specified.** Teachers and assessors will need to make an inference about what is and is not "reasonable" and perhaps even about what counts as an argument for this grade level.	The measurability issue here can be addressed by separating the standard into two separate standards and by simplifying the language: "a) Order three objects by length and justify the choice. b) Compare the lengths of two objects indirectly using a third object and justify the results."

2.D.3—Analyze data sets and single-unit scale data visualizations using features such as titles, labels, and legends. *Ask questions* to make sense of the data. (MP 1)	1—Partially measurable and/or well-specified	N**on-observable verb; too vague; multiple expectations.** “Analyze” covers a great deal of ground and needs to be further specified. “Ask questions to make sense of the data” is also vague and high-inference and does not specify the type, quality, or purpose of the questions (e.g., comparison, interpretation, or inference). The phrase “make sense of the data” reflects an internal cognitive process, not a directly observable performance. The expectation also combines multiple skills (analysis + question generation), which makes it harder to assess with a single, focused performance.	Determine what, specifically, is meant by “Analyze data sets and single-unit scale data visualizations” and what observable behaviors a teacher or assessor would look for to determine proficiency. Determine what it means to “make sense of the data” in this situation and again what observable behaviors a teacher would be looking for. (While asking questions is an important mathematical behavior, it does not generally indicate proficiency with content.)
3.OA.6—Multiply and divide *flexibly, accurately, and efficiently*, with products, dividends, and divisors up to and including 100, and quotients up to and including 10. Know from memory all products of two one-digit numbers. (MP 6)	1—Partially measurable and/or well-specified	**Poorly specified.** “Flexibly” and “efficiently” are subjective and undefined, making them difficult to assess consistently without explicit criteria. Without further specification, teachers and item writers would be required to use their own judgment about what constitutes “flexibly” and “efficiently.”	Determine what, specifically, is meant by “flexibly” and “efficiently,” and what observable behaviors a teacher or assessor would look for to determine whether a students’ calculating counts as “flexible” and “efficient.”
4.MG.4—*Attend to precision and reasonableness* when measuring or sketching angles in whole-number degrees using a protractor. (MP 6)	0—Not measurable or poorly specified	**Non-observable verb.** “Attend to precision and reasonableness” is not a measurable behavior.	“Attend to precision and reasonableness” can be concisely captured by using a modifier such as “correctly” (e.g., “Correctly measure angles in whole-number degrees using a protractor, and correctly sketch angles of specified measure.”).

5.MG.3—*Build and use a model* of the coordinate plane to represent real-world problems. Understand the numbers in the coordinate pair indicate distance from the origin in the direction of the horizontal and vertical axes. Graph points in the first quadrant of the coordinate plane Interpret coordinate values based on a given context. (MP 2, 4)	1—Partially measurable and/or well-specified	**Multiple expectations; non-observable verb.** "Understand" is not an observable verb. The standard also seems to include up to four distinct PEs (building and using a model to represent real-world problems; understanding how coordinate pairs communicate distance from the origin; graph points in the first quadrant; interpret coordinate values in context).	a) Determine what observable behaviors would indicate that a student "Understand[s] the numbers in the coordinate pair indicate distance from the origin in the direction of the horizontal and vertical axes." b) The measurability issues with this standard are compounded by clarity issues—specifically, the phrase "*Build and use a model* of the coordinate plane" is not mathematically clear or precise; this issue needs to be addressed first, before measurability can be addressed.
6.NS.3—*Attend to precision and reasonableness* when adding, subtracting, multiplying, and dividing multi-digit whole numbers and decimals *flexibly, accurately, and efficiently*. (MP 6)	0—Not measurable or poorly specified	**Non-observable verb; poorly specified.** "Attend to precision and reasonableness" is not a measurable behavior. "Flexibly" and "efficiently" are subjective and undefined, making them difficult to assess consistently without explicit criteria. Without further specification, teachers and item writers would be required to use their own judgment about what constitutes "flexibly" and "efficiently."	a) Determine what, specifically, is meant by "flexibly" and "efficiently," and what observable behaviors a teacher or assessor would look for to determine whether a students' calculating counts as "flexible" and "efficient." b) "Attend to precision and reasonableness" can be concisely captured by using a modifier such as "fluently" (e.g., "Fluently add, subtract, multiply, and divide multi-digit whole numbers …").
7.RP.7—*Interpret* the meaning of a point (***x***, ***y***) on the graph of a proportional relationship in a given context. Connect and *interpret the relationship* between the points (0, 0) and (1, ***r***) where ***r*** is the unit rate. (MP 4, 7).	1—Partially measurable and/or well-specified	**Too vague.** It is not clear what is meant by "connect … the relationship between the points (0,0) and (1, *r*) …"	The word "connect" can probably be removed altogether, for example: "*Interpret* the meaning of a point (*x*, *y*) on the graph of a proportional relationship in a given context. *Interpret the relationship* between the points (0, 0) and (1, *r*) where r is the unit rate. (MP 4, 7)."

8.D.1—Ask questions about bivariate data sets (quantitative and categorical) to investigate patterns of associations between two quantities. (MP 1)	1—Partially measurable and/or well-specified	**Too vague; poorly specified.** "Ask questions" is poorly specified in terms of quantity, quality, or type of questions. "Investigate patterns of associations" is broad and does not clearly define what evidence of investigation looks like. It is also unclear whether students will need to be able to both 1) ask questions that could be used to investigate and 2) to actually complete the investigation.	Determine specifically a) what patterns of associations, specifically, students should be investigating and how, and b) what mathematical behaviors a teacher or assessor would need to see in order to determine proficiency. (While asking questions is an important mathematical behavior, it does not generally indicate proficiency with content.)
S1.F.10/A1.BF.4—*Use structures and patterns* to identify the effect of k on the graph when replacing f(x) by f(x) + k. Explore specific values of k for linear and exponential functions in and out of context. (MP 7)	1—Partially measurable and/or well-specified	**Multiple expectations; non-observable verb; poorly specified.** This standard includes two distinct performance expectations (PEs). The first is poorly specified in that it is not clear what structures and patterns a student should be using, or how. The second uses the non-observable verb "explore"; this sentence describes an activity which is better suited for classroom instruction than for assessment. It is not clear what knowledge or skill students are expected to show mastery of.	For the first PE, determine whether the expected performance is indeed the use of structures or patterns, or identifying the effect of k on the graph. If it is the first, specify which patterns and structures students should be using and how. If it is the second, the language "use structures and patterns to" can be removed. For the second PE, consider whether this statement belongs in the standards since it seems to be describing an instructional/ learning activity rather than a mastery behavior.
S2.NS.1/A1.NS.1—Extend *understanding of the structure* of integer exponent properties to rational exponents. (MP 8)	0—Not measurable or poorly specified	**Non-observable verb; poorly specified.** In order to be able to measure proficiency, we need to know what observable behavior would indicate "understanding" as well as what is meant by "extending" that understanding.	One possible way retain the substance of this standard while improving its specificity and identifying an observable behavior: "Explain how the meaning and properties of rational exponents follow from extending the properties of integer exponents, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5."

S3.F.2/A2.F.2—*Build and use models* of periodic phenomena with sine and cosine functions that connect real world contexts with amplitude, frequency, and midline. *Use structures and patterns* to support understanding of a domain extended beyond 0 to 90 degrees, or 0 to π / 2 radians. (MP 4, 7)	0—Not measurable or poorly specified	**Multiple expectations; non-observable verb; poorly specified.** This standard includes two distinct PEs that are unlikely to be assessable with a single performance (“Build and use models of periodic phenomena ...” and “Use structures and patterns to support ...”). Because “model” is defined so broadly in MP 4 (essentially used as a synonym for “representation”), it is not clear what type of model or models are expected. “Support understanding” is also unobservable; it is not clear what specific behavior an educator or assessor would be looking for to determine proficiency.	Use sine and cosine to construct trigonometric functions that model real-world periodic phenomena, including amplitude, frequency, and midline. Explain in context what it means to extend the domain beyond 0 to 90 degrees, or 0 to π / 2 radians. (MP 4, 7)

Table B2. Illustrative Examples of Standards With Clarity Concerns Including Explanations and Suggested Revision

Standard Text	Rating	Explanation	Suggested Revision
P3.CC.2—Begin to recognize that written numbers represent a quantity. Use a variety of *tools to build and model* numbers. (MP 4, 5)	1—Partially clear and accessible	**Vague/ambiguous language.** The phrase "begin to recognize" is imprecise and does not clearly define what level of understanding is expected. "A variety of tools" is broad and not specified, which may leave teachers and parents unsure of expectations.	This clarity issue is highly related to the measurability issue described above, so the recommendations are similar: a) Determine what level of understanding is expected in order to count as proficient, and b) what tools students should be using to build and model numbers, and how.
K.CC.8—Use patterns and *structures* to subitize quantities up to 10. (MP 7)	1—Partially clear and accessible	**Vague/ambiguous language.** The term "subitize" is technical and may not be familiar to all users, especially parents. The phrase "patterns and structures" is abstract and does not clearly describe what students should do (e.g., recognizing groups like dots on a ten-frame or dice).	Rewrite in plain language and include more guidance around what it means to "use patterns and structures." For instance, it could read as "Use spatial patterns to efficiently determine how many in a collection. For example, rather than counting one-by-one, determine that there are six dots in the pattern below by mentally breaking it into two rows of three." 
1.D.1—Determine a topic question and ask *investigative questions* that will lead to collecting, analyzing, and interpreting data within the context of the classroom. (MP 1)	1—Partially clear and accessible	**Insufficient guidance to understand PE.** The phrase "determine a topic question" is vague and may not clearly distinguish between a general topic and a specific question. "Investigative questions" is somewhat also abstract and may not be easily understood without examples.	Add examples to clarify for teachers, parents, and others what is meant by a "topic question" and "investigative questions."

Standard Text	Rating	Explanation	Suggested Revision
2.OA.3—*Describe and represent structures and patterns* to determine if a group of up to 20 objects has an odd or even number of items by pairing them or counting by twos. (MP 7)	1—Partially clear and accessible	**Jargon; insufficient guidance to understand PE.** The phrase "describe and represent structures and patterns" is abstract and does not clearly define what students should do. It is unclear what "structures and patterns" refers to in this specific context (e.g., pairs, leftover objects).	Since the language of the standard already includes "by pairing them or counting by twos," the additional language at the front ("Describe and represent structures and patterns") is repetitive and unnecessary. The PE would be much clearer left as "Determine if a group of up to 20 objects has an odd or even number of items by pairing them or counting by twos. (MP 7)"
3.OA.7— *Build and use models* to solve two-step story problems involving whole numbers and having whole-number answers using the four operations, and attend to *the reasonableness* of answers. *Remove context* to represent situations as equations with a variable standing in for the unknown quantity. (MP 2, 4, 6)	1—Partially clear and accessible	**Vague/ambiguous language.** It may not be clear to teachers and parents what kinds of models students should be building and using to solve story problems. The phrases "attend to the reasonableness of answers" and "remove context" are also abstract and not clearly defined. In general, the statement is long and combines multiple expectations (modeling, solving problems, checking reasonableness, translating to equations). The inclusion of the language "Remove context" contributes to this issue and is repetitive.	This is another example of closely related measurability and clarity issues. The language can be simplified and clarified by specifying what types of models students should be using and by removing language that is repetitive (e.g., representing situations as equations with a variable *is* removing context, so "removing context" is not needed). For example, it could read, "Represent situations as equations with a variable standing in for the unknown quantity, and use these equations to solve two-step story problems with whole-number quantities and answers, using the four operations."

Standard Text	Rating	Explanation	Suggested Revision
4.NF.5—*Build and use models* to extend understanding of multiplication to fractions. Connect repeated addition to multiplication when multiplying a fraction by a whole number. Multiply a fraction by a whole number using numerical strategies. Solve problems involving multiplying a fraction by a whole number *in real world contexts*. (MP 2, 4)	1—Partially clear and accessible	**Vague/ambiguous language.** Students selecting and developing models with the intention of extending their understanding is not developmentally appropriate at grade 4. We suspect that this is not the intent of the standard, so the phrasing of “Build and use models to extend understanding of multiplication to fractions” is ambiguous. Some of the other language use in the standard is also confusing (e.g., “Connect repeated addition to multiplication when multiplying a fraction by a whole number”).	The word “apply” may be the most appropriate here, e.g., “Apply and extend previous understandings of multiplication to fractions: a) Explain why the fraction a or b is the product of *a* and the unit fraction 1/*b*; b) Use properties of number and operation to find the product of *a* and 1/*b*; and c) Solve real-world problems involving multiplying a fraction by a whole number.”
5.NBT.6—Use efficient numerical strategies based on properties of multiplication and/or place value to multiply a whole number of up to four digits by a one digit whole number and to multiply two two-digit numbers *flexibly, accurately, and efficiently in real-world contexts and equations*. Determine if products are reasonable. (MP 2, 6)	1—Partially clear and accessible	**Vague/ambiguous language.** The terms “flexibly, accurately, and efficiently” are subjective, making the PE unclear without examples. Also, there is some redundancy about efficiency: "use efficient numerical strategies to multiply numbers efficiently"	A possible revision could be: “a) Find products of four-digit and single-digit integers using strategies based on place value and the properties of operations, and determine their reasonableness. b) Find products of two-digit integers using strategies based on place value and the properties of operations, and determine their reasonableness.”
6.NS.7—Build and use models to develop the definition of absolute value. Work with absolute value in and out of context. (MP 2, 4)	1—Partially clear and accessible	**Vague/ambiguous language; Insufficient guidance to understand the performance expectation.** It is not clear what is meant by “Build and use models to develop the definition of absolute value.” “Work with absolute value” is also vague, ambiguous, and needs to be described in a more mathematically precise way.	Determine specifically what the performance expectation is for students. What does it mean to “Build and use models to develop the definition of absolute value”? Include that precise language in the standard. Similarly, clarify what is meant by “Work with absolute value” and include mathematically precise language in the standard to make it clear what is expected of students.

Standard Text	Rating	Explanation	Suggested Revision
7.D.1—Formulate statistical investigative questions that anticipate variability in data collected from a random sample of a population. (MP 1)	1—Partially clear and accessible	**Vague/ambiguous language.** It may not be clear to teachers, parents, or others what is meant by "statistical investigative questions."	Include examples to help teachers, parents, and others better understand what types of questions students are required to come up with in order to demonstrate proficiency.
8.NS.4—*Justify* why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational. *Build and use models* to connect to physical situations. (MP 3, 4)	1—Partially clear and accessible	**Insufficient guidance to understand PE; Other.** The standard is dense and combines multiple distinct expectations (proof/ justification, properties of number systems, and modeling applications) in a single statement. While the mathematical meaning is precise, the phrasing requires unpacking for instructional and assessment use. In particular, "build and use models to connect to physical situations" is broad and does not specify what counts as an acceptable model or level of connection.	This standard could be rewritten as two standards, one of which has three subparts: 1) Explain a) why sums and products of rational numbers are rational, b) why the sum of a rational number and an irrational number is irrational, and c) why the product of a nonzero rational number and an irrational number is irrational (MP 3). 2) [A more specific version of the building and using models piece which makes the PE clear.] (MP 3, 4)
S1.G.2/G.CO.2—*Construct and clearly communicate definitions* of the rigid transformations to show that these transformations are functions that take points in the plane as inputs and give corresponding points as outputs. (MP 3)	1—Partially clear and accessible	**Vague/ambiguous language; Insufficient guidance to understand PE.** The clause "*Construct and clearly communicate definitions* of the rigid transformations" is mathematically imprecise and thus unclear, and so it is not clear what is expected of students.	We wonder if the intent was something like "Represent rigid transformations in the plane and use their definitions to show that these transformations are functions that take points in the plane as inputs and give other points as outputs."

Standard Text	Rating	Explanation	Suggested Revision
S2.G.9/G.C.3—*Describe and represent*, using similarity, the definition of a radian as the ratio of arc length to radius. Create a distinction between arc length and arc measure and be able to *attend to precisions* while relating each of these to radian measure or the formula for the area of a sector. (MP 6, 7)	1—Partially clear and accessible	**Vague/ambiguous language; Insufficient guidance to understand PE; Unclear grammar/typo(s).** The phrase "describe and represent" creates confusion here where the word "derive" would suffice (and is much clearer). "Create a distinction" is also unclear (vs for example "Explain the distinction). "Attend to precisions" is both grammatically incorrect (typo?) and is unclear in terms of the precise PE.	Consider rewording the first PE to be clearer, for example: “Derive, using similarity, the definition…” For the second PE, determine the specific behavior that is expected of students (for example, is “Explain the distinction between…”). For the third PE, decide what specific, observable behavior is expected of student and reword accordingly.
S3.F.1/A2.F.1— Understand and interpret the features of polynomial, radical or logarithmic functions represented graphically using interval notation and inequalities by applying and removing context. Focus on these key features: domain, range, intervals of increase and/or decrease, location of absolute maximum and/or absolute minimum, intercepts and end behavior. Build graphic models that satisfy given key features. (MP 2, 4)	1—Partially clear and accessible	**Vague and ambiguous language; insufficient guidance to understand PE.** The first sentence, “Understand and interpret the features of ... functions ... by applying and removing context,” is unclear in terms of what it means, specifically, in terms of a PE for this standard. Part of this clarity derives from the measurability that arises from the non-observable verb “understand.” The other part arises from the language “by applying and removing context.” It is not clear specifically how applying or removing context supports understanding or interpreting features of functions.	Reflect on what mathematical knowledge, skill, or understanding is intended by the first sentence; then, determine precisely what observable evidence a teacher would be looking for to determine proficiency. The word “understand” very well may not be needed at all.