

Draft P-12 Utah Core Mathematics Standards

Step 11 Revision February 2026

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Introduction

Utah's Core Mathematics Standards assist in opening doors of opportunity for all students. They set clear expectations and guide students through a focused and coherent progression of math concepts, considering age-level development and emphasizing proficiency of identified essential competencies. These standards establish a strong mathematical foundation in early grades that allows students to engage with complex mathematical concepts as they progress through secondary mathematics.

All students must have the opportunity to learn and meet high mathematics standards in order to understand and navigate the modern world. Engaging with mathematics helps students develop tools to analyze patterns, quantify relationships, and make informed decisions. Understanding mathematics provides students with skills that support success in college, career, and family life.

Mathematics plays a critical role in the development of cognitive abilities, significantly contributing to the architectural growth of a child's brain. Regarded as a unifying discipline, it assists students in a comprehensive understanding of the world around them. Additionally, it enhances logical reasoning, problem-solving abilities, and overall mental development.

The standards set grade-specific expectations but do not dictate curriculum or teaching methods, nor do they define the strategies necessary to meet diverse learners' needs ([53E-4-202](#)). Customized supports must be provided to ensure all learners have access to the grade-level content in *Utah's Core Mathematics Standards*.

Organization of the Standards

In Utah State Code [53E-4-202](#), the State Board is charged with establishing core standards for Utah Public Schools that identify the basic knowledge, skills, and competencies each student is expected to acquire or master as the student advances through the public education system. These basic knowledge, skills, and competencies increase in depth and complexity from year to year.

Strands

The *Utah Core Standards* are organized into *strands*, which represent significant areas of learning within content areas. In mathematics, the strands are organized by overarching themes but do not necessarily represent isolated topics. Mathematical concepts are connected across multiple strands and grade levels. The trajectory of the strands is designed to follow a logical learning progression.

Standards

Within each strand are *standards*. A standard articulates the knowledge and skills to be obtained by students. A standard represents the essential elements of the expected learning.

Utah Mathematical Practices

In each grade-level core, there is a collection of *Mathematical Practices*. Each skill is a fundamental mathematical practice that students build as they engage with mathematical content. The Practices are not written in hierarchical order, they are all important skills that students must engage in to build their mathematical understanding. To ensure students develop capacity with these skills, they have been embedded into every standard across all strands and can be *identified by the use of italic text*.

Although specific Mathematical Practices have been intentionally identified and connected to each standard, instruction and practice of other Mathematical Practices not referenced in an individual standard may be appropriate if other skills support student learning in mathematics learning experience.

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. The list of competencies is not exhaustive of every standard; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide additional instruction or interventions based on the standards that support these competencies to ensure students are proficient.

Shifts in the Standards

Changes in the standards were made to improve their consistency, clarity, and progression through the grade levels. These changes include:

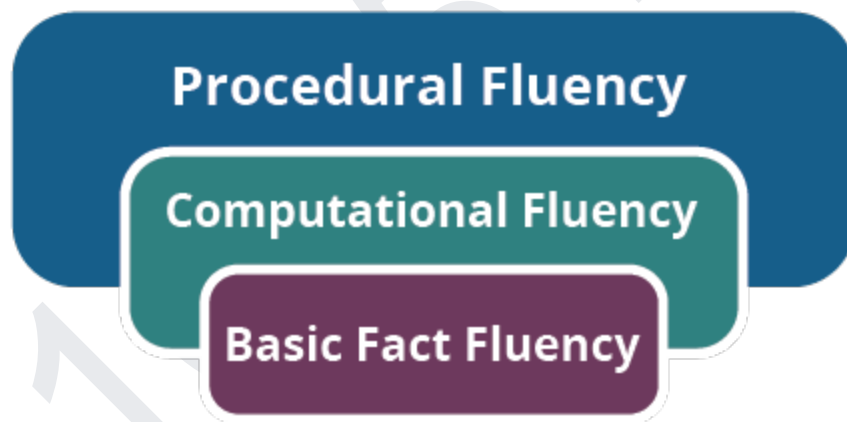
- Standards were clarified, and the overall number of standards was reduced.
- Mathematical Practices were identified and are highlighted throughout the standards P-12.
- Expectations were defined for the mathematical practices at each grade level.
- Essential Competencies were identified for each grade level specifying the success criteria associated with proficiently completing the major work of that grade level.
- Data Science standards were emphasized and incorporated into all grades.
 - The Data Science standards follow a cyclical structure where students iteratively model with mathematics by:
 - Asking statistical questions,
 - Collecting and exploring data,
 - Analyzing data,
 - And interpreting results
- In Secondary Math III, specific standards that are essential for all students were highlighted, and additional standards were developed to provide students and their families agency in choosing which math course pathways will best support their post-high school goals.

Building Conceptual Understanding & Procedural Fluency

These standards define what students should understand and be able to do in their study of mathematics. Throughout the progression of the Utah mathematics core standards, special attention was paid to ensuring that students develop a strong

foundation in conceptual understanding and procedural fluency. Procedural fluency is defined as the ability to apply procedures efficiently, flexibly, and accurately, transfer procedures to different problems and contexts, build or modify procedures from other procedures, and recognize when one strategy or procedure is more appropriate to apply than another. Student learning is supported when procedures and concepts are explicitly connected in ways that make sense to them. Conceptual foundations provide opportunities for students to develop reasoning strategies, which in turn deepens conceptual understanding and allows students to build procedural fluency.

Through the progression of the standards, students are provided opportunities to make sense of and build a conceptual understanding of the mathematical operations. This conceptual understanding supports children in becoming automatic in their retrieval of basic math facts. That basic fact fluency can then be applied as students develop computational and procedural fluency using a variety of numerical strategies and efficient algorithms.



Mathematical Tools and Technology

An excellent mathematics program integrates the use of mathematical tools and technology as resources to empower students to meet the learning intentions of the classroom. Software and online programs cannot replace authentic interactions between teachers and students, and the value of student-to-student discourse and collaboration. Technology and tools should be used in ways that promote mathematical reasoning and sense-making.

Utah Mathematical Practices

The mathematical practices included in *Utah's P-12 Core Mathematics Standards* describe mathematical skills that Utah students develop throughout their educational experiences. Students become proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills. Each grade level's standards have a description in their introductory material specifying the ways students will engage with these practices as they learn math concepts. Although specific Mathematical Practices have been intentionally identified and connected to each standard, instruction and practice of other Mathematical Practices not referenced in that standard are encouraged. Below, general descriptions are provided for each Mathematical Practice.

Descriptions of what the practices look like in each grade band are included in the grade level standards. More details about the practices and how they are integrated throughout the standards are included in each grade level's [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Mathematical Practice 1: Ask questions to explore mathematical concepts

Students examine mathematical concepts and formulate questions. During mathematical discourse, students ask probing questions to better understand math concepts.

Asking questions cultivates students' curiosity about mathematical relationships and helps them refine their understanding. As students build their capacity to ask probing questions, they engage in more meaningful discourse about mathematics. When students ask and explore rich questions, it advances the mathematical understanding of all learners.

Mathematical Practice 2: Add or remove context to make sense of mathematics

Students add or remove context to help make sense of mathematical concepts. Students add context by connecting numbers and operations to images, stories, objects, and events. They remove context from a situation by representing the mathematics using numbers, symbols, and equations.

When students contextualize and decontextualize mathematical concepts, they can make sense of underlying structures. Working in and out of context throughout a learning progression empowers students to make sense of mathematics and persevere as they examine abstract and real-world problems. This process allows students to take their knowledge of mathematics and meaningfully apply it to their lives.

Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments

Students construct arguments using the mathematical reasoning that underlies a strategy, solution, or conjecture. They justify their conjectures and clearly communicate their reasoning using concrete referents such as objects, drawings, diagrams, and actions. They justify their arguments with peers through discourse, refining their thinking to create more robust and precise arguments.

Constructing, justifying, and communicating arguments fosters a deep understanding of mathematical concepts. By defending their conjectures, students solidify their mathematical understanding and identify potential flaws in their reasoning. Clear communication enables students to articulate their thoughts effectively, enhancing their ability to explain their reasoning to others. Discourse with peers exposes students to diverse perspectives, promoting critical thinking and collaborative problem-solving.

Mathematical Practice 4: Build and use models

Mathematical models use structures to represent relationships. Students build and use verbal, contextual, visual, symbolic, and physical models to represent and make sense of the relationships between quantities in real-world situations and mathematical problems.

Using models enhances student learning by providing students with multiple ways

to represent and understand mathematical concepts, connect mathematical concepts to real-world situations, and develop problem-solving skills. Students explore the idea that various models can showcase mathematical thinking in valuable ways. Using multiple models helps students notice the connections between mathematical structures and promotes the development of mathematical generalizations that can be leveraged as tools to make sense of and persevere in solving novel problems.

Mathematical Practice 5: Select and use tools appropriately and strategically

Students strategically select and use relevant and valuable tools for mathematical tasks with which they engage. Tools can be physical objects (e.g., compasses or manipulatives), technological tools (e.g. graphing calculators or virtual manipulatives) or mathematical constructs (e.g., diagrams, strategies, or algorithms).

Students consider the available mathematical problem-solving tools and choose the most relevant and useful ones. When students have the agency to select mathematical tools strategically, it empowers them to visualize, represent, and understand mathematical concepts effectively, fostering critical thinking, problem-solving flexibility, perseverance, and engagement.

Mathematical Practice 6: Attend to precision and reasonableness

Students exercise mathematical precision and reasonableness in their communication, calculations, and processes. They calculate flexibly, accurately, and efficiently, using clear and concise notation to record their work. They regularly evaluate whether actions, ideas, processes, and solutions make sense and are reasonable.

Precision in communication and calculation fosters a deeper understanding of mathematical concepts. Using precise language, students articulate their thoughts clearly, enhancing their ability to explain their reasoning to others and understand others' ideas. Accurate and efficient calculations and precise notation enable students to solve problems effectively and minimize errors. The continuous

evaluation of reasonableness ensures that students improve their ability to identify and learn from mathematical errors.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

Students describe and represent structures, patterns, and relationships to make sense of mathematics. While engaging in mathematical tasks, students use their understanding to make generalizations about the structure of numbers and become more flexible and efficient in applying mathematical concepts.

Students demonstrate these skills when they identify patterns and make connections across mathematical ideas. Analyzing these relationships allows students to bridge the gap between concrete observations and abstract reasoning.

Mathematical Practice 8: Make conjectures and evaluate the results

Students make sense of mathematics when they engage in mathematical investigations. They use their understanding of structures, patterns, and relationships to speculate about the nature of a relationship and then test their conjecture to determine how to proceed. As they test their mathematical hypotheses and evaluate the outcomes of their investigation, they build mathematical generalizations.

Developing and assessing conjectures encourages active engagement and critical thinking. Conjecturing acts as an entry point to exploration for all students. When making conjectures, students have opportunities to continually revise their thinking. Students formulate conjectures, test them, and refine their understanding based on the outcomes. Students view mistakes as assets that help them better refine their mathematical understanding. This process fosters a deeper understanding of underlying mathematical principles. It promotes curiosity, students' ownership of their learning, and the development of positive mathematical identities.

Standards: Preschool Age 3

Introduction

In Preschool Age 3 (P3) and Preschool Age 4 (P4), the major work of both grades focuses on developing an understanding of the foundational concepts of counting and cardinality and developing the ability to describe and compare measurable attributes of objects and shapes.

In P3, instructional time should focus on three major works: (1) developing concepts of counting and cardinality; (2) beginning to recognize simple patterns; and (3) beginning to describe shapes and objects. Quality learning environments focus on hands-on experiences during play and interaction with others to incorporate well-designed mathematical experiences aligned to specific learning goals.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In grades P-2, students ask questions to help them develop understanding of numbers, place value, the properties of addition and subtraction, and the properties of shapes. Students begin to ask questions about data related to the classroom and their day-to-day lives.

Mathematical Practice 2: Add or remove context to make sense of mathematics

In grades P-2, students build an understanding of what the numbers, symbols, pictures, words, etc., in their work represent. Students develop flexibility in moving between concrete, pictorial, and abstract representations of mathematical ideas. They begin moving back and forth between a problem's context and its representation to use the form that best fits the situation.

Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments

In grades P3 and P4, students are learning to communicate their ideas and listen to the ideas of others. They develop strategies to agree and disagree respectfully. Students begin to explain their ideas about mathematics.

Mathematical Practice 4: Build and use models

In grades P-2, students use a variety of models to build their understanding of numbers with increasing complexity, leading to an understanding of the base ten place value system. They model the actions and comparisons represented by the operations of addition and subtraction. Consistent exposure to these models supports fluency and helps students connect concrete experiences to foundational mathematical ideas.

Mathematical Practice 5: Select and use tools appropriately and strategically

In grades P-2, students use tools when they help them make sense of mathematics. Tools can include, but are not limited to, physical tools, visual tools, strategies, and algorithms. Consistent, hands-on practice with tools builds confidence and prepares students for flexible application.

Mathematical Practice 6: Attend to precision and reasonableness

In grades P-K, students use informal language as they develop and learn accurate mathematical vocabulary. They begin to learn strategies for assessing their accuracy when counting, naming numbers, and describing shapes.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In grades P-K, students explore patterns and structures in counting and with shapes. Students begin to develop understanding of the relationship between addition and subtraction using small numbers.

Mathematical Practice 8: Make conjectures and evaluate the results

In grades P-2, students make and test conjectures related to numbers, place value, the properties of addition and subtraction, and the attributes of shapes. During this

process, students learn to evaluate the reasonableness of their solution path and adjust as needed. Providing students opportunities to revise their thinking helps students develop confidence in problem solving.

Standards: Preschool Age 3 (P3)

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Counting and Cardinality (CC)

Identify numerals by name, count to 10, use one-to-one correspondence, describe quantities of objects counted, use ordinal language, and subitize.

P3.CC.1

Recite number names in sequence to 10, attending to *structure* and *precise language*. (MP 6, 7)

P3.CC.2

Begin to recognize that written numbers represent a quantity. Use a variety of *tools* to *build and model* numbers. (MP 4, 5)

P3.CC.3

Demonstrate one-to-one correspondence by *accurately* counting and pointing/touching a line of 1-5 objects. (MP 6)

P3.CC.4

Begin to use *accurate* number names to identify the number of objects in a group of up to 5 objects, recognizing that the last number in a count represents the total number of objects. (MP 6)

P3.CC.5

With support, identify whether the number of objects in one group is the same as those in another group for sets of up to 4 objects, using *precise mathematical language*. (MP 6)

P3.CC.6

Identify whether an object is "first" or "second" in a sequence by noticing ordinal structure. Begin to recognize ordering words in everyday *contexts*. (MP 2)

P3.CC.7

Subitize quantities up to 3 using *patterns and structures*. (MP 7)

Operations and Algebraic Thinking (OA)

Identify and explore simple patterns.

P3.OA.1

Describe and *model* simple *patterns* in the environment and begin to duplicate and extend those patterns. (MP 4, 7)

Measurement and Geometry (MG)

Identify and describe measurable attributes of basic shapes.

P3.MG.1

With prompting and support, identify and *describe* measurable attributes (e.g., big, small, tall, short). (MP 7)

P3.MG.2

With prompting and support, identify, *describe*, and *represent* basic shapes and their attributes. (MP 7)

Data Science (D)

Begin asking questions that lead to collecting and analyzing data by sorting and counting objects.

P3.D.1

With prompting and support, *ask questions* about people or objects in the classroom that lead to data collection. (MP 1)

P3.D.2

With prompting and support, collect and sort data or objects by attribute (color, shape, size, etc), and *build data models*. (MP 4)

P3.D.3

With prompting and support, *accurately* count the number of objects or data points in each category (no more than five objects per category). (MP 6)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the knowledge and skills embedded within the standards, by the end of P3, children will be able to:

- Count up to 5 objects in a set; then tell how many objects are in the set without counting again. (CC)
- Count to 10 by ones and recognize that numbers have a known sequence (e.g., “1, 2, 3, 4, 5. What comes next?”). (CC)
- Quickly recognize small groups of up to 3 objects without counting. (CC)
- Identify and describe basic shapes and their measurable attributes. (MG)
- Begin sorting and counting objects. (D)

Standards: Preschool Age 4

Introduction

In Preschool Age 3 (P3) and Preschool Age 4 (P4), the major work of both grades focuses on developing an understanding of foundational concepts of counting and cardinality and developing the ability to describe and compare measurable attributes of objects and shapes.

In P4, instructional time should focus on three major works: (1) reinforcing concepts of counting and cardinality; (2) exploring concepts of addition and subtraction; and (3) exploring and describing shapes, patterns, and measurable attributes of objects. Quality learning environments focus on hands-on experiences during play and interaction with others to incorporate well-designed mathematical experiences aligned to specific learning goals.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In grades P-2, students ask questions to help them develop understanding of numbers, place value, the properties of addition and subtraction, and the properties of shapes. Students begin to ask questions about data related to the classroom and their day-to-day lives.

Mathematical Practice 2: Add or remove context to make sense of mathematics

In grades P-2, students build an understanding of what the numbers, symbols, pictures, words, etc., in their work represent. Students develop flexibility in moving between concrete, pictorial, and abstract representations of mathematical ideas. They begin moving back and forth between a problem's context and its representation to use the form that best fits the situation.

Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments

In grades P3 and P4, students are learning to communicate their ideas and listen to the ideas of others. They develop strategies to agree and disagree respectfully. Students begin to explain their ideas about mathematics.

Mathematical Practice 4: Build and use models

In grades P-2, students use a variety of models to build their understanding of numbers with increasing complexity, leading to an understanding of the base ten place value system. They model the actions and comparisons represented by the operations of addition and subtraction. Consistent exposure to these models supports fluency and helps students connect concrete experiences to foundational mathematical ideas.

Mathematical Practice 5: Select and use tools appropriately and strategically

In grades P-2, students use tools when they help them make sense of mathematics. Tools can include, but are not limited to, physical tools, visual tools, strategies, and algorithms. Consistent, hands-on practice with tools builds confidence and prepares students for flexible application.

Mathematical Practice 6: Attend to precision and reasonableness

In grades P-K, students use informal language as they develop and learn accurate mathematical vocabulary. They begin to learn strategies for assessing their accuracy when counting, naming numbers, and describing shapes.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In grades P-K, students explore patterns and structures in counting and with shapes. Students begin to develop understanding of the relationship between addition and subtraction using small numbers.

Mathematical Practice 8: Make conjectures and evaluate the results

In grades P-2, students make and test conjectures related to numbers, place value, the properties of addition and subtraction, and the attributes of shapes. During this

process, students learn to evaluate the reasonableness of their solution path and adjust as needed. Providing students opportunities to revise their thinking helps students develop confidence in problem solving.

Standards: Preschool Age 4 (P4)

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Counting and Cardinality (CC)

Count to 20, recognize numerals, demonstrate one-to-one correspondence, compare quantities, use ordinal language, and subitize.

P4.CC.1

With support, *accurately* recite number names and the counting sequence to 20. (MP 6)

P4.CC.2

Associate quantities with written numerals up to 10. Begin to form numerals and use a variety of *tools* to represent quantities. (MP 5)

P4.CC.3

Demonstrate one-to-one correspondence by accurately counting and pointing to or touching one object per count for an arrangement of up to 10 objects. (MP 6)

P4.CC.4

Use *accurate* number names to identify the number of objects in a group of up to 10, recognizing that the last number in a count represents the total number of objects. (MP 6)

P4.CC.5

Use mathematical language to *accurately* identify whether the number of objects in one group is more than, less than, or the same as the number of objects in another group for sets of up to 6 objects. (MP 6)

P4.CC.6

Identify and state whether an object is "first," "second," "third," or "last" in a sequence. Begin to apply ordering words to everyday *contexts*. (MP 2)

P4.CC.7

Use *patterns* and *structures* to subitize quantities of up to five. (MP 7)

Operations and Algebraic Thinking (OA)

Identify and manipulate simple patterns and model addition and subtraction problems involving numbers up to and including 5.

P4.OA.1

Build and use models to represent real-world situations involving addition (adding to or putting together) and subtraction (taking from or taking apart) with numbers up to and including 5. (MP 4)

P4.OA.2

Decompose numbers less than or equal to five *using a variety of models* and explain why a number or set of objects can be broken apart in multiple ways. (MP 4, 8)

P4.OA.3

Select and use tools to *model* sums of 5 using quantities from 0–5. (MP 4, 5)

P4.OA.4

Create, duplicate, and extend simple *patterns*. (MP 7)

Measurement and Geometry (MG)

Identify, describe, build, and compare basic shapes. Describe their measurable attributes.

P4.MG.1

Describe measurable attributes of objects using *accurate* vocabulary (e.g., length [long/short], weight [heavy/light], size [big/small], and distance [near/far]). (MP 6)

P4.MG.2

Compare measurable attributes of objects by *making and testing conjectures*.

Directly compare two objects, using measurable attributes to *justify* the comparison. (MP 3, 8)

P4.MG.3

Identify, *describe, and represent* two- and three-dimensional shapes. Describe attributes of two dimensional shapes including size, number of sides, and number of corners. (MP 7)

P4.MG.4

Use names of shapes to describe objects in the environment. Identify the relative positions of these objects *accurately* using terms such as above, below, beside, in front of, behind, and next to. (MP 6)

P4.MG.5

Create basic shapes using a *variety of tools*. With support, combine basic shapes to create new shapes. (MP 5)

P4.MG.6

Recognize patterns in activities that occur during different times of day including morning, afternoon, and night. (MP 7)

Data Science (D)

Ask questions that lead to collecting, analyzing, and interpreting data by sorting and counting objects.

P4.D.1

With prompting and support, *ask questions* about a given topic that lead to

collecting, analyzing, and interpreting data gathered within the classroom. (MP 1)

P4.D.2

Collect and sort data or objects organized by attribute (color, shape, size, etc). With prompting and support, *build data models*. (MP 4)

P4.D.3

With prompting and support, analyze data by *accurately* counting the number of objects or data points in each category. (MP 6)

P4.D.4

With prompting and support, interpret data to identify and *justify* which groups contain more, less, or the same amount. (MP 3)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of P4, children will be able to:

- Count up to 10 objects in a set, then tell how many objects are in the set without counting again. (CC)
- Begin to connect number names and quantities with written numerals. (CC)
- Count to 20. (CC)
- Quickly recognize groups of up to 5 objects without counting. (CC)
- Model addition and subtraction with numbers up to and including 5 using concrete objects, fingers, movements, and simple drawings. (OA)

- Identify, describe, build, and compare basic shapes using their measurable attributes. (MG)
- Begin interpreting data when sorting and counting objects. (D)

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Standards: Kindergarten (K)

Introduction

In kindergarten through second grade, the major work of each grade focuses on developing an understanding of addition and subtraction and building an understanding of the base-ten place value number system. Students begin to develop an understanding of foundational concepts of linear measurement and explore the defining attributes of shapes.

In kindergarten, instructional time should focus on three major works: (1) mastering concepts of counting and cardinality; (2) representing, relating, and operating on whole numbers, initially with sets of objects; and (3) describing shapes and space. More learning time in kindergarten should be devoted to number sense than to other topics, as students build fluency through concrete experiences, visual models, and verbal expressions.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In grades P-2, students ask questions to help them develop understanding of numbers, place value, the properties of addition and subtraction, and the properties of shapes. Students begin to ask questions about data related to the classroom and their day-to-day lives.

Mathematical Practice 2: Add or remove context to make sense of mathematics

In grades P-2, students build an understanding of what the numbers, symbols, pictures, words, etc., in their work represent. Students develop flexibility in moving between concrete, pictorial, and abstract representations of mathematical ideas. They begin moving back and forth between a problem's context and its representation to use the form that best fits the situation.

Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments

In grades K-2, students are learning to communicate their ideas and listen to the ideas of others. They develop strategies to agree and disagree respectfully. They begin to justify their arguments with evidence and draw connections between mathematical representations and procedures. Students also begin to compare and contrast their strategies and solutions.

Mathematical Practice 4: Build and use models

In grades P-2, students use a variety of models to build their understanding of numbers with increasing complexity, leading to an understanding of the base ten place value system. They model the actions and comparisons represented by the operations of addition and subtraction. Consistent exposure to these models supports fluency and helps students connect concrete experiences to foundational mathematical ideas.

Mathematical Practice 5: Select and use tools appropriately and strategically

In grades P-2, students use tools when they help them make sense of mathematics. Tools can include, but are not limited to, physical tools, visual tools, strategies, and algorithms. Consistent, hands-on practice with tools builds confidence and prepares students for flexible application.

Mathematical Practice 6: Attend to precision and reasonableness

In grades P-K, students use informal language as they develop and learn accurate mathematical vocabulary. They begin to learn strategies for assessing their accuracy when counting, naming numbers, and describing shapes.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In grades P-K, students explore patterns and structures in counting and with shapes. Students begin to develop understanding of the relationship between addition and subtraction using small numbers.

Mathematical Practice 8: Make conjectures and evaluate the results

In grades P-2, students make and test conjectures related to numbers, place value, the properties of addition and subtraction, and the attributes of shapes. During this process, students learn to evaluate the reasonableness of their solution path and adjust as needed. Providing students opportunities to revise their thinking helps students develop confidence in problem solving.

Standards: Kindergarten (K)

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Counting and Cardinality (CC)

Know number names and the counting sequence. Count to tell the number of objects and represent a given quantity in a set. Identify and compare quantities of objects and numerals. Identify an object's ordinal position. Subitize quantities.

K.CC.1

Accurately recite numbers in sequence from 1 to 100 by ones and tens. *Accurately* count backward from 10. Attend to the counting *structure*. (MP 6, 7)

K.CC.2

Accurately count forward from a given number between 1 and 100. Attend to the counting *structure*. (MP 6, 7)

K.CC.3

Use one-to-one correspondence to *accurately* identify the quantity of objects in a group of 0–20. Understand that the last number stated represents the number of objects counted. (MP 6)

K.CC.4

Given a number 1-20, count out that many objects. When counting objects, *build*

models to show that each successive number refers to a quantity that is one greater than the previous number. (MP 4)

K.CC.5

Read and write numbers 0 to 20 using base-ten numerals *with precision*. Represent a quantity of 0 to 20 objects with written numerals. (MP 6)

K.CC.6

Identify and compare quantities of objects and numerals up to 10 using the words “greater than,” “less than,” and “equal to.” Use matching or counting strategies and reasoning to *justify* comparisons. (MP 3)

K.CC.7

Identify and *justify* an object's ordinal position, *using precise mathematical language*. Order quantities or numerals using the ordinal words “first” through “tenth.” (MP 3, 6)

K.CC.8

Use patterns and structures to subitize quantities up to 10. (MP 7)

Operations and Algebraic Thinking (OA)

Understand addition as putting together and adding to, and subtraction as taking apart and taking from.

K.OA.1

Represent addition and subtraction with sums and minuends up to and including 10 *using a variety of tools and models*. *Attend to the precision and reasonableness* of results. (MP 4, 5, 6)

K.OA.2

Build and use models to solve addition and subtraction story problems with sums or minuends up to and including 10. *Add or remove story context* to solve problem type situations of adding to and taking from with result unknown, putting together with the total unknown, and putting together with both addends unknown. (MP 2, 4)

K.OA.3

Compose and decompose numbers less than or equal to 10 in multiple ways *using a variety of models*. Make sums to 10 using numbers 1-9. *Make conjectures* about why a number or set of objects can be broken apart in multiple ways. (MP 4, 8)

K.OA.4

Use strategies to add and subtract *flexibly, accurately, and efficiently* using numbers less than or equal to 5. (MP 6)

Number and Operations in Base Ten (NBT)

Build and use models to compose and decompose numbers from 11-19 into ten ones and some more ones to conceptually understand place value.

K.NBT.1

Build and use models to compose and decompose numbers from 11-19 into ten ones and some more ones. *Represent structure* from those models with math language and an equation (e.g., "Eighteen is the same as 10 and 8"; $18=10+8$). *Describe the pattern* that teen numbers are always composed of ten ones and some more ones. (MP 4, 7)

Measurement and Geometry (MG)

Identify, describe, and compare measurable attributes of objects. Analyze, compare, create, and compose shapes.

K.MG.1

Describe several measurable attributes of a single object (e.g., length and weight), *attending to precision and reasonableness*. Compare two objects with a common measurable attribute to see which object has "more of"/ "less of" the attribute, and *justify the results*. (MP 3, 6)

K.MG.2

Describe structures of two-dimensional and three-dimensional shapes, including names and attributes, regardless of size or orientation. *Describe* their relative positions in the environment. (MP 7)

K.MG.3

Analyze, compare, and sort two- and three-dimensional shapes and objects regardless of size or orientation. *Justify* the sort using informal language to describe similarities, differences, and other attributes (e.g., color, size, shape, number of sides). (MP 3)

K.MG.4

Build and use models to create two- and three-dimensional shapes and objects. Use simple shapes to form larger shapes. (MP 4)

K.MG.5

Recognize that clocks are tools used to measure and show time. Connect the use of clocks to daily routines and events. *Recognize structures* and features of clocks. (MP 7)

Data Science (D)

Ask questions that lead to collecting, analyzing, and interpreting data. Sort, count, and compare objects in a data set.

K.D.1

Ask questions as a class about a given topic that will lead to collecting, analyzing, and interpreting data gathered within the classroom. (MP 1)

K.D.2

Collect and organize data. *Build data models* with up to three categories. (MP 4)

K.D.3

Analyze data by observing *patterns and structures* in data models. *Accurately* count the number of objects or data points in each category. (MP 6, 7)

K.D.4

Interpret data to identify *and justify* which groups contain more, less, or the same amount by comparing the number in each category. (MP 3)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of kindergarten, children will be able to:

- Count up to 20 objects in a set to answer questions like "how many." (CC)
- Read and write numbers 0-20. (CC)
- Count to 100 by ones. (CC)
- Compare quantities of objects and numerals up to 10. (CC)
- Quickly recognize groups of up to 10 objects without counting. (CC)
- Fluently add and subtract when the largest number in the equation is no greater than 5. (Basic fact fluency) (OA)
- Draw pictures or use objects to solve addition and subtraction story problems where the largest number in the story is no greater than 10. (OA)
- Break apart numbers less than or equal to 10 in multiple ways. (OA)
- Break apart numbers 11-19 into ten ones and some more ones. (NBT)

Standards: Grade 1

Introduction

In kindergarten through second grade, the major work of each grade focuses on developing an understanding of addition and subtraction and building an understanding of the base-ten place value number system. Students develop an understanding of foundational concepts of linear measurement and explore the defining attributes of shapes.

In Grade 1, instructional time should focus on four major works: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In grades P-2, students ask questions to help them develop understanding of numbers, place value, the properties of addition and subtraction, and the properties of shapes. Students begin to ask questions about data related to the classroom and their day-to-day lives.

Mathematical Practice 2: Add or remove context to make sense of mathematics

In grades P-2, students build an understanding of what the numbers, symbols, pictures, words, etc., in their work represent. Students develop flexibility in moving between concrete, pictorial, and abstract representations of mathematical ideas. They begin moving back and forth between a problem's context and its representation to use the form that best fits the situation.

Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments

In grades K-2, students are learning to communicate their ideas and listen to the ideas of others. They develop strategies to agree and disagree respectfully. They begin to justify their arguments with evidence and draw connections between mathematical representations and procedures. Students also begin to compare and contrast their strategies and solutions.

Mathematical Practice 4: Build and use models

In grades P-2, students use a variety of models to build their understanding of numbers with increasing complexity, leading to an understanding of the base ten place value system. They model the actions and comparisons represented by the operations of addition and subtraction. Consistent exposure to these models supports fluency and helps students connect concrete experiences to foundational mathematical ideas.

Mathematical Practice 5: Select and use tools appropriately and strategically

In grades P-2, students use tools when they help them make sense of mathematics. Tools can include, but are not limited to, physical tools, visual tools, strategies, and algorithms. Consistent, hands-on practice with tools builds confidence and prepares students for flexible application.

Mathematical Practice 6: Attend to precision and reasonableness

In grades 1-2, students refine informal language as they develop and learn precise mathematical vocabulary. Students use structure to accurately record, label, and share their thinking. They explore when a precise answer is necessary and when making an estimate, which can help them determine if a solution makes sense. Students use estimation strategies based on place value structure and the properties of operations to assess reasonableness throughout the problem-solving process.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In grades 1-2, students explore patterns and structures such as place value,

properties of operations, and composition of shapes. They use structures and patterns to develop a deep understanding of addition and subtraction. Students use structures and patterns to see complicated things as single objects or as composed of several objects.

Mathematical Practice 8: Make conjectures and evaluate the results

In grades P-2, students make and test conjectures related to numbers, place value, the properties of addition and subtraction, and the attributes of shapes. During this process, students learn to evaluate the reasonableness of their solution path and adjust as needed. Providing students opportunities to revise their thinking helps students develop confidence in problem solving.

Standards: Grade 1

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Operations and Algebraic Thinking (OA)

Represent and solve problems involving addition and subtraction with sums and minuends less than or equal to 20. Solve problem type situations with unknowns in all places. (Mastery of start-unknown problems is not expected until second grade.) Understand and apply properties of operations and the relationship between addition and subtraction. Understand the meaning of the equal sign.

1.OA.1

Build and use models to solve addition and subtraction story problems with sums and minuends less than or equal to 20. *Add or remove story context* to solve problem type situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions. (MP 2, 4)

1.OA.2

Build and use models to solve story problems involving adding three whole numbers with sums less than or equal to 20. *Add or remove context to make sense* of problems and solutions. (MP 2, 4)

1.OA.3

Make and test conjectures about properties of addition. *Add context* to make sense of these properties as strategies to add and subtract. (MP 2, 8)

1.OA.4

Describe and represent subtraction as an unknown-addend problem. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers, and *justify the results*. (MP 3, 7)

1.OA.5

Describe structures and relationships to connect counting to addition and subtraction. (MP 7)

1.OA.6

Add and subtract with sums and minuends less than or equal to 20. *Connect and justify* different strategies for addition and subtraction. (MP 3)

1.OA.7

Use strategies to add and subtract *flexibly, accurately, and efficiently* with sums and minuends less than or equal to 10. (MP 6)

1.OA.8

Build and use models to make sense of the meaning of the equal sign. *Make and test conjectures* regarding whether addition and subtraction equations are true or false. (MP 4, 8)

Number and Operations in Base Ten (NBT)

Count and represent numbers up to and including 120. Build and use models to conceptually understand place value. Use place value understanding to add and subtract larger numbers.

1.NBT.1

Count to 120 *accurately*, starting at any number. Represent a number of objects up to and including 120 with a written numeral. (MP 6)

1.NBT.2

Build and use models to represent and identify the number of tens and ones in a two-digit number. Show place value understanding by bundling ten ones to make a “ten.” (MP 4)

1.NBT.3

Compare two two-digit numbers using *models* and/or place value understanding. *Represent the relationship* between the two numbers using comparison symbols $<$, $=$, and $>$. (MP 4, 7)

1.NBT.4

Build and use concrete and pictorial models to determine the sum of two two-digit numbers, a two-digit number and a one-digit number, and a two-digit number and a multiple of ten (less than or equal to 90). Use the models *to represent and describe the structure* of combining tens with tens, ones with ones, and composing ten ones to make a ten. *Ask questions to compare* concrete and pictorial models. (MP 4, 7, 8)

1.NBT.5

Build and use concrete and pictorial models to subtract a multiple of ten from a larger multiple of ten. *Ask questions to compare* concrete and pictorial models. (MP 1, 7)

1.NBT.6

Mentally find ten more or ten less than a given two-digit number and *justify the result* using place value language. (MP 3)

1. NBT.7

Use place value understanding to count forward or backward by tens from any number, and *describe the pattern* in the ones place and tens place when counting by ten. (MP 7)

Measurement and Geometry (MG)

Measure lengths. Tell and write time. Identify the value of U.S. coins. Reason with shapes and their attributes.

1.MG.1

Select a tool to use as a non-standard length unit. Measure by repeating the non-standard length unit. Express the length of the object as a whole number. (MP 5)

1.MG.2

Construct a reasonable argument about the order of three objects sorted by length. Compare the lengths of two objects indirectly using a third object and *justify the results*. (MP 3)

1.MG.3

Tell and write time in hours and half-hours using analog and digital clocks, *attending to precision and reasonableness*. (MP 6)

1.MG.4

Describe attributes of and identify common U.S. coins. *Attend to precision* when representing their comparative values and use appropriate notation (5¢). (MP 6)

1.MG.5

Ask questions to distinguish between defining and non-defining attributes of shapes. Given defining attribute(s), build and draw shapes that possess the defining attribute(s). (MP 1)

1.MG.6

Use structures, patterns, and relationships to create a composite shape formed by two- and three-dimensional shapes and compose new shapes from the composite shape. (MP 7)

1.MG.7

Build and use models to partition circles and rectangles into two or four equal

shares. *Justify, compare, and evaluate* how decomposing a shape into more equal shares creates smaller shares. (MP 3, 4)

Data Science (D)

Ask investigative questions that lead to collecting, analyzing, and interpreting data. Build models to organize data, and interpret data visualizations.

1.D.1

Determine a topic question and *ask investigative questions* that will lead to collecting, analyzing, and interpreting data within the context of the classroom. (MP 1)

1.D.2

Collect and organize data. *Build data visualizations* including bar graphs and picture graphs. When given a data set, *build data models and visualizations*. (MP 4)

1.D.3

Analyze data sets and data visualizations with up to three categories. *Ask questions* to make sense of the data representation. (MP 1)

1.D.4

Interpret data to answer investigative questions. *Construct and justify arguments* about the data. (MP 3)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of first grade, children will be able to:

- Fluently add and subtract when the largest number in the equation is no

greater than 10. (Basic fact fluency) (OA)

- Draw pictures, use objects or write equations to solve addition and subtraction story problems where the largest number in the story is no greater than 20. (OA)
- Use a variety of strategies to add and subtract. (OA)
- Count, write, and recognize numbers to 120 (NBT)
- Make sense of place value and the meaning of tens and ones in two-digit numbers. (NBT)
- Add two-digit numbers using objects and pictures. (NBT)
- Measure length using non-standard measurement units. (MG)

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Standards: Grade 2

Introduction

In kindergarten through second grade, the major work of each grade focuses on developing an understanding of addition and subtraction and building an understanding of the base-ten place value number system. Students develop an understanding of foundational concepts of linear measurement and explore the defining attributes of shapes.

In Grade 2, instructional time should focus on four major works: (1) extending understanding of place value, including grouping in hundreds, tens and ones; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In grades P-2, students ask questions to help them develop understanding of numbers, place value, the properties of addition and subtraction, and the properties of shapes. Students begin to ask questions about data related to the classroom and their day-to-day lives.

Mathematical Practice 2: Add or remove context to make sense of mathematics

In grades P-2, students build an understanding of what the numbers, symbols, pictures, words, etc., in their work represent. Students develop flexibility in moving between concrete, pictorial, and abstract representations of mathematical ideas. They begin moving back and forth between a problem's context and its representation to use the form that best fits the situation.

Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments

In grades K-2, students are learning to communicate their ideas and listen to the

ideas of others. They develop strategies to agree and disagree respectfully. They begin to justify their arguments with evidence and draw connections between mathematical representations and procedures. Students also begin to compare and contrast their strategies and solutions.

Mathematical Practice 4: Build and use models

In grades P-2, students use a variety of models to build their understanding of numbers with increasing complexity, leading to an understanding of the base ten place value system. They model the actions and comparisons represented by the operations of addition and subtraction. Consistent exposure to these models supports fluency and helps students connect concrete experiences to foundational mathematical ideas.

Mathematical Practice 5: Select and use tools appropriately and strategically

In grades P-2, students use tools when they help them make sense of mathematics. Tools can include, but are not limited to, physical tools, visual tools, strategies, and algorithms. Consistent, hands-on practice with tools builds confidence and prepares students for flexible application.

Mathematical Practice 6: Attend to precision and reasonableness

In grades 1-2, students refine informal language as they develop and learn precise mathematical vocabulary. Students use structure to accurately record, label, and share their thinking. They explore when a precise answer is necessary and when making an estimate, which can help them determine if a solution makes sense. Students use estimation strategies based on place value structure and the properties of operations to assess reasonableness throughout the problem-solving process.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In grades 1-2, students explore patterns and structures such as place value, properties of operations, and composition of shapes. They use structures and

patterns to develop a deep understanding of addition and subtraction. Students use structures and patterns to see complicated things as single objects or as composed of several objects.

Mathematical Practice 8: Make conjectures and evaluate the results

In grades P-2, students make and test conjectures related to numbers, place value, the properties of addition and subtraction, and the attributes of shapes. During this process, students learn to evaluate the reasonableness of their solution path and adjust as needed. Providing students opportunities to revise their thinking helps students develop confidence in problem solving.

Standards: Grade 2

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Operations and Algebraic Thinking (OA)

Represent and solve problems involving addition and subtraction with unknowns in all locations. Add and subtract fluently with sums and minuends less than or equal to 20. Work with equal groups of objects to gain foundations for multiplication.

2.OA.1

Build and use models to solve one-step addition and subtraction story problems with sums and minuends less than or equal to 100, and two-step addition and subtraction story problems with sums and minuends less than or equal to 20. *Add or remove story context* to solve problem type situations with unknowns in all positions while attending to the meaning of the equal sign. (MP 2, 4)

2.OA.2

Use strategies and properties to add and subtract *flexibly, accurately, and efficiently* with sums and minuends less than or equal to 20. Know from memory all sums of

two one-digit numbers. (MP 6)

2.OA.3

Describe and represent structures and patterns to determine if a group of up to 20 objects has an odd or even number of items by pairing them or counting by twos. (MP 7)

2.OA.4

Build and use models involving repeated addition to determine the total number of objects in rectangular arrays with up to 5 rows and up to 5 columns. *Describe and represent the structures and patterns* of the arrays. *Make and evaluate conjectures* about whether the sums are the same when adding by row or column. (MP 4, 7, 8)

Number and Operations in Base Ten (NBT)

Build and use models to conceptually understand place value. Use place value understanding to add and subtract.

2.NBT.1

Show place value understanding by bundling ten 10s to make a “hundred.” Use a variety of *models to represent* the amount of hundreds, tens, and ones in a three-digit number. Identify the number of hundreds, tens, and ones in a three-digit number and describe the value of each digit. (MP 4)

2.NBT.2

Count within 1,000, *making conjectures* about counting patterns; Skip-count by twos, fives, tens, and hundreds. (MP 8)

2.NBT.3

Read three-digit whole numbers and *represent the place value structure* in those numbers using numerals, base 10 word form, and expanded form. (MP 7)

2.NBT.4

Compare two three-digit numbers *using models* and/or place value understanding. *Represent the relationship* between the two numbers using comparison symbols $<$, $=$,

and $>$. (MP 4, 7)

2.NBT.5

Use numerical strategies to determine the sum of up to four two-digit numbers, and the difference of two two-digit numbers *flexibly, accurately, and efficiently. Justify* a chosen strategy using the relationship between addition and subtraction, properties of addition, and/or place value language. (MP 3, 6)

2.NBT.6

Build and use concrete and pictorial models to determine the sums and differences of three-digit numbers with sums and minuends up to and including 999. Use the models to *represent and describe the structure of* combining hundreds with hundreds, tens with tens, ones with ones, and composing and decomposing tens and hundreds. *Ask questions to compare* concrete and pictorial models with efficient numerical strategies. (MP 1, 4, 7)

2.NBT.7

Mentally add and subtract multiples of 10 or 100 from a three-digit number and *justify the result* using place value language. (MP 3)

Measurement and Geometry (MG)

Measure and estimate lengths in standard units and relate addition and subtraction to length. Work with time and money. Analyze and partition shapes to build a foundation for multiplication, area, and fractions.

2.MG.1

Select and use appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes to measure the length of objects to the nearest whole unit. (MP 5)

2.MG.2

Describe and represent the length of an object using two different units of measure. *Describe the relationship* of the two measurements and relate them to the units chosen. (MP 7)

2.MG.3

Construct, justify, and communicate clear and reasonable arguments when estimating lengths using inches, feet, centimeters, and meters. (MP 3)

2.MG.4

Measure with *precision* to determine how much longer one object is than another. Express the difference in length using a standard unit. (MP 6)

2.MG.5

Build and use a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2,... as a *model* to represent whole numbers as lengths from zero. *Strategically use an open number line as a tool* to model and solve whole-number addition and subtraction equations and word problems within 100. (MP 4, 5)

2.MG.6

Attend to precision when telling and writing time to the nearest five minutes. *Add context* to determine when to use a.m. and p.m., *attending to reasonableness*. (MP 2, 6)

2.MG.7

Add or remove context when solving problems involving dollar bills, quarters, dimes, nickels, and pennies. (MP 2)

2.MG.8

Ask questions to identify and describe shapes having specific attributes. *Build and draw* shapes that possess specified attributes. (MP 1)

2.MG.9

Build and use models to partition a rectangle into rows and columns of same-size squares and count to find the total number of squares. (MP 4)

2.MG.10

Describe and represent structures, patterns and relationships when partitioning two-dimensional shapes into two, three, or four equal shares; describe the shares using the words halves, thirds, a fourth of, quarters, etc. Recognize that equal parts of

identical wholes need not have the same shape. (MP 7)

Data Science (D)

Ask investigative questions that lead to collecting, analyzing, and interpreting data. Organize and build data visualizations using features such as titles, labels, and legends. Consider the impact missing data may have on a data set.

2.D.1

Determine a topic question and *ask investigative questions* that will lead to collecting, analyzing, and interpreting data within the context of classroom and grade. (MP 1)

2.D.2

Collect and organize data. *Build data visualizations* including bar graphs and pictographs with single-unit scales. *Ask questions* to consider a given data set about a classroom or grade level context, and organize that data using different variables. (MP 1, 4)

2.D.3

Analyze data sets and single-unit scale data visualizations using features such as titles, labels, and legends. *Ask questions* to make sense of the data. (MP 1)

2.D.4

Interpret data to answer investigative questions. *Construct and justify arguments* about the data. Make conjectures about the impact missing data may have on interpretation of the data. (MP 3, 8)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of second grade,

children will be able to:

- Fluently add and subtract when the largest number in the equation is no greater than 20. (Basic fact fluency) (OA)
- Connect numerical strategies to concrete and pictorial models to solve one- and two-step addition and subtraction story problems. (OA)
- Fluently add and subtract when the largest number in the equation is no greater than 100. (Computational fluency) (NBT)
- Make sense of place value and the meaning of hundreds, tens, and ones in three-digit numbers using visual and numerical models. (NBT)
- Add and subtract three-digit numbers using objects, pictures, and numerical strategies. (NBT)
- Relate addition and subtraction to length. (MG)
- Analyze and partition shapes and begin using fractional language (e.g., halves, thirds and fourths). (MG)

Standards: Grade 3

Introduction

In third through fifth grade, the major work of each grade focuses on representing and understanding multiplication and division and developing an understanding of fractions. Students generalize and expand their place value understanding. Measurement concepts are expanded to include two- and three-dimensional measurements.

In Grade 3, instructional time should focus on four major works: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In grades 3-5, students ask questions to help them understand multi digit numbers, fractions, decimals, the properties of all four operations, and the classification of shapes. Students ask questions to help them make sense of data and different data representations.

Mathematical Practice 2: Add or remove context to make sense of mathematics

In grades 3-5, students understand what the numbers, symbols, pictures, words, etc. in their work represent when using the four operations. Students flexibly move between concrete, pictorial, and abstract representations of mathematical ideas. They move back and forth between a problem's context and its representation to use the form that best fits the situation.

Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments

In grades 3-5, students use mathematical vocabulary to communicate their ideas and listen to others' ideas. They utilize strategies to agree and disagree respectfully. They back their argument up with evidence and draw connections between mathematical representations, comparing and contrasting their strategy and solution to others.

Mathematical Practice 4: Build and use models

In grades 3-5, students use a variety of models to extend their understanding of the base ten place value system, including decimals. They use models to build their understanding of fractions. They model the actions and comparisons represented by all four operations. In the upper grades, students focus on models that represent multiplication and division, including area models, number lines, and other multiplicative comparisons.

Mathematical Practice 5: Select and use tools appropriately and strategically

In grades 3-5, students use tools when they help them understand mathematics. Tools can include, but are not limited to, physical tools, visual tools, strategies, and algorithms.

Mathematical Practice 6: Attend to precision and reasonableness

In grades 3-5, students refine informal language as they develop and learn precise mathematical vocabulary. Students use structure to accurately record, label, and share their thinking. They determine when a precise answer is necessary and when making an estimate can help them determine if a solution makes sense. Students use estimation strategies such as rounding, utilizing benchmark fractions, and properties of operations to assess reasonableness through the problem solving process.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In grades 3-5, students use structures and patterns to develop a deep understanding of the operations of multiplication and division. They extend their

understanding of patterns, structures, and relationships through work with fractions, decimals, and larger whole numbers. Students use structures and patterns to see complicated things as single objects or composed of several objects.

Mathematical Practice 8: Make conjectures and evaluate the results

In grades 3-5, students make and test conjectures related to multi digit numbers, fractions, decimals, the properties of all four operations, and the properties of shapes. As students test their conjectures, they evaluate the reasonableness of their solution path and adjust as needed.

Standards: Grade 3

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Operations and Algebraic Thinking (OA)

Represent and solve problems involving multiplication and division with whole number products and quotients. Demonstrate understanding of the properties of multiplication and the relationship between multiplication and division. Utilize the associative, commutative, distributive, identity, and zero properties. Use the four operations to solve multistep problems and to explain patterns in arithmetic.

3.OA.1

Build and use models to find and interpret the products of whole numbers. (MP 4)

3.OA.2

Build and use models to find and interpret whole-number quotients. (MP 4)

3.OA.3

Solve multiplication and division story problems with products or dividends less than or equal to 100 and whole number quotients, *using a variety of models. Add or*

remove context to solve situations involving equal groups, arrays, and measurement quantities. (MP 2, 4)

3.OA.4

Evaluate the unknown whole number in a multiplication or division equation. *Represent* division as an unknown-factor problem, and *describe the relationship* between multiplication and division. (MP 7)

3.OA.5

Make and assess conjectures about the properties of multiplication. *Add context* to make sense of these properties as strategies to multiply and divide with products and dividends less than or equal to 100. (MP 2, 8)

3.OA.6

Multiply and divide *flexibly, accurately, and efficiently*, with products, dividends, and divisors up to and including 100, and quotients up to and including 10. Know from memory all products of two one-digit numbers. (MP 6)

3.OA.7

Build and use models to solve two-step story problems involving whole numbers and having whole-number answers using the four operations, and attend to *the reasonableness* of answers. *Remove context* to represent situations as equations with a variable standing in for the unknown quantity. (MP 2, 4, 6)

Number and Operations in Base Ten (NBT)

Use efficient numerical strategies (founded in place value thinking and properties of operations) to perform multi-digit addition, subtraction, and multiplication.

3.NBT.1

Describe and represent place value structure by bundling ten 100s to make a thousand. *Represent* the amount of thousands, hundreds, tens, and ones in a 4-digit number, and *describe* the value of each. (MP 7)

3.NBT.2

Use *models* to round 2- or 3-digit whole numbers to the nearest 10 or 100 *in and out of context*. Determine if rounding is appropriate for a given situation and *justify* the reasoning. (MP 2, 3, 4)

3.NBT.3

Determine sums and differences of three-digit whole numbers using numerical strategies *flexibly, accurately, and efficiently*. *Justify* a chosen strategy with place value language. (MP 3, 6)

3.NBT.4

Find the product of a one-digit whole number and a multiple of 10 (10-90). *Make and test conjectures* to generalize patterns that occur when multiplying by a multiple of ten. (MP 8)

Number and Operations – Fractions (NF)

Develop an understanding of fractions including whole numbers. Represent fractions, find equivalent fractions, and make sense of fractions in context. Denominators are limited to 2, 3, 4, 6, and 8 in third grade.

3.NF.1

Describe and represent a fraction, a/b , as a numerator and a non-zero denominator, including unit fractions and fractions equal to whole numbers. Compose and decompose a fraction using unit fractions. Identify the meaning of the whole for a given context. (MP 7)

3.NF.2

Build and use number line models to represent fractions, using unit fractions as intervals. Connect number line models with area models to make sense of fractions. (MP 4)

3.NF.3

Build and use models to make sense of and generate equivalent fractions including whole numbers. *Justify* why equivalent fractions must reference the same whole to be equivalent. (MP 3, 4)

3.NF.4

Compare fractions with like numerators or like denominators, and *justify* the comparisons *using models* and/or by reasoning about their size. (MP 3, 4)

Measurement and Geometry (MG)

Solve problems involving estimated and precise measurements. Develop an understanding of area and perimeter. Relate area to multiplication and division. Reason with shapes and their attributes.

3.MG.1

Make reasonable estimates about length, weight, and liquid volume. *Select and use tools appropriately and strategically* to find *precise* measurements. (MP 5, 6)

3.MG.2

Attend to precision when telling and writing time to the nearest minute. (MP 6)

3.MG.3

Add or remove context to solve one-step story problems involving length, liquid volume, mass, and time. (MP 6)

3.MG.4

Recognize area and perimeter as attributes of plane figures. *Build and use models* to understand the concept of unit squares and area. *Select and use tools strategically and appropriately* to distinguish between linear (perimeter) and area measures. Use tiling to show that the area of a rectangle is the same as would be found by multiplying side lengths. (MP 4, 5)

3.MG.5

Build and use area models to represent the distributive property and relate the representations to the operations of addition and multiplication. (MP 4)

3.MG.6

Add and remove context to solve real-world and mathematical problems involving

perimeters of simple polygons and rectangular areas. *Make and evaluate conjectures* about rectangles with the same perimeter and different areas or with the same area and different perimeters. Describe how area and perimeter are related. (MP 4, 8)

3.MG.7

Describe and represent shared attributes of shapes in a category, and recognize that the shared attributes define the larger category. *Build models* of quadrilaterals that do not belong to a given subcategory. (MP 4, 7)

3.MG.8

Attend to precision when partitioning shapes into parts with equal areas and expressing the area of each part as a unit fraction of the whole. (MP 6)

Data Science (D)

Identify whether investigative questions will result in quantitative or categorical data collection. Represent data sets with three or more categories using visualizations with scales of 2, 5, and 10. Determine the benefits and drawbacks of different data visualizations and scales. Identify patterns, trends, and outliers in a data set.

3.D.1

Determine a topic question and *ask investigative questions* that will lead to collecting, analyzing, and interpreting data. Identify whether the question asked will result in quantitative or categorical data. (MP 1)

3.D.2

Select and use tools strategically to collect and organize data. Build data visualizations using scales of 2s, 5s, and 10s. Represent a data set with three or more categories. *Ask questions* to consider a given data set. Identify data as quantitative or categorical. (MP 1, 5)

3.D.3

Analyze data sets and *bar graphs and pictographs* with scales of 2s, 5s, and 10s.

Make and evaluate conjectures about benefits and drawbacks of different visualizations and scales used to analyze a set of data. (MP 6, 8)

3.D.4

Interpret data, including scaled visualizations, to answer investigative questions. *Construct and justify arguments* about the data. Identify patterns, trends, and outliers in the data. Identify ways to refine investigative questions, data collection, visualization, or analysis. (MP 3)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of third grade, children will be able to:

- Use pictures, objects, and equations to solve multiplication and division story problems when the largest number in the story is no greater than 100. (OA)
- Fluently multiply two one-digit numbers. (Basic fact fluency) (OA)
- Fluently divide numbers when the dividend is no greater than 100 and the quotient is no greater than 10. (Basic fact fluency) (OA)
- Fluently add and subtract when the largest number in the equation is no greater than 999. (Computational fluency) (NBT)
- Describe and represent a fraction as a numerator and denominator, a fraction as a part of a whole, and as a location on a number line. (NF)
- Build models and use reasoning strategies to compare fractions. (NF)
- Build models to generate equivalent fractions. (NF)
- Use models to determine the area of rectangles. (MG)

Standards: Grade 4

Introduction

In third through fifth grade, the major work of each grade focuses on representing and understanding multiplication and division and developing an understanding of fractions. Students generalize and expand their place value understanding.

Measurement concepts are expanded to include two- and three-dimensional measurements.

In Grade 4, instructional time should focus on four major works: (1) generalizing understanding of place value to 1,000,000, understanding the multiplicative relationship between the value of each place; (2) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (3) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (4) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In grades 3-5, students ask questions to help them understand multi digit numbers, fractions, decimals, the properties of all four operations, and the classification of shapes. Students ask questions to help them make sense of data and different data representations.

Mathematical Practice 2: Add or remove context to make sense of mathematics

In grades 3-5, students understand what the numbers, symbols, pictures, words, etc. in their work represent when using the four operations. Students flexibly move between concrete, pictorial, and abstract representations of mathematical ideas.

They move back and forth between a problem's context and its representation to use the form that best fits the situation.

Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments

In grades 3-5, students use mathematical vocabulary to communicate their ideas and listen to others' ideas. They utilize strategies to agree and disagree respectfully. They back their argument up with evidence and draw connections between mathematical representations, comparing and contrasting their strategy and solution to others.

Mathematical Practice 4: Build and use models

In grades 3-5, students use a variety of models to extend their understanding of the base ten place value system, including decimals. They use models to build their understanding of fractions. They model the actions and comparisons represented by all four operations. In the upper grades, students focus on models that represent multiplication and division, including area models, number lines, and other multiplicative comparisons.

Mathematical Practice 5: Select and use tools appropriately and strategically

In grades 3-5, students use tools when they help them understand mathematics. Tools can include, but are not limited to, physical tools, visual tools, strategies, and algorithms.

Mathematical Practice 6: Attend to precision and reasonableness

In grades 3-5, students refine informal language as they develop and learn precise mathematical vocabulary. Students use structure to accurately record, label, and share their thinking. They determine when a precise answer is necessary and when making an estimate can help them determine if a solution makes sense. Students use estimation strategies such as rounding, utilizing benchmark fractions, and properties of operations to assess reasonableness through the problem solving process.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In grades 3-5, students use structures and patterns to develop a deep understanding of the operations of multiplication and division. They extend their understanding of patterns, structures, and relationships through work with fractions, decimals, and larger whole numbers. Students use structures and patterns to see complicated things as single objects or composed of several objects.

Mathematical Practice 8: Make conjectures and evaluate the results

In grades 3-5, students make and test conjectures related to multi digit numbers, fractions, decimals, the properties of all four operations, and the properties of shapes. As students test their conjectures, they evaluate the reasonableness of their solution path and adjust as needed.

Standards: Grade 4

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Operation and Algebraic Thinking (OA)

Understand multiplication as a comparison. Use the four operations to solve multistep problems in which remainders must be interpreted. Demonstrate understanding of the properties of operations. Explore the relationship between factors and multiples. Generate and interpret multiplicative patterns.

4.OA.1

Build and use models to visualize a multiplication equation as a comparison. *Represent* verbal statements of multiplicative comparisons as multiplication equations. (MP 4, 7)

4.OA.2

Solve story problems involving multiplicative comparison by *building and using models*, including drawings and equations with a symbol for the unknown number. *Make and evaluate conjectures* about the difference between multiplicative and additive comparison. (MP 4, 8)

4.OA.3

Use and name properties of operations to *justify* strategies used to solve problems involving the four operations. (MP 3)

4.OA.4

Build and use models to solve multi-step story problems involving whole numbers using the four operations, including problems in which remainders must be interpreted. *Attend to the reasonableness* of answers. *Remove context* to represent situations as equations with a variable standing in for the unknown quantity. (MP 2, 4, 6)

4.OA.5

Find factors and multiples of whole numbers. *Make and test conjectures* about the relationship between any whole number and its factors. (MP 8)

4.OA.6

Generate a number or shape pattern based on a given rule. *Describe and represent* the features of the pattern that are not explicitly stated in the rule itself. *Make and test conjectures* about how the pattern will continue. (MP 7, 8)

Number and Operations in Base Ten (NBT)

Use place value understanding to make multiplicative comparisons. Write, compare, and round whole numbers less than or equal to 1,000,000. Use efficient algorithms for multi-digit addition and subtraction. Use efficient numerical strategies, place value understanding, and properties of operations to perform multi-digit multiplication and division.

4.NBT.1

Describe the structure of place value by explaining how the value of one place

represents ten times more than the place to its right. *Represent those patterns* with a statement of multiplicative comparison. (MP 7)

4.NBT.2

Read multi-digit whole numbers up to and including 1,000,000. *Represent* multi-digit whole numbers using numerals and expanded form. (MP 7)

4.NBT.3

Compare two whole numbers with up to 6 digits and *represent the relationship* between two numbers using comparison symbols $<$, $=$, and $>$. (MP 7)

4.NBT.4

Apply knowledge of place value structure to round whole numbers up to and including six digits *in and out of context*. *Justify* whether rounding is appropriate in a given situation. (MP 2, 3)

4.NBT.5

Use efficient algorithms to determine sums and differences of multi-digit whole numbers *in real-world contexts and equations flexibly, accurately, and efficiently*. Determine if sums and differences are *reasonable*. (MP 2, 6)

4.NBT.6

Develop visual and numerical models based on place value and/or the properties of multiplication to multiply a whole number of up to four digits by a one-digit whole number, and to multiply two two-digit numbers. (MP 4)

4.NBT.7

Develop visual and numerical models based on place value and/or the relationship between multiplication and division to divide up to a three-digit number by a one-digit number. (MP 4)

Number and Operations–Fractions (NF)

Extend understanding of equivalence and ordering fractions. Build on previous understanding of fractions to begin operating with fractions and explore the

relationship between fractions and decimals. Understand decimal notation to the hundredths. Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100 in fourth grade.

4.NF.1

Compose and decompose fractions, mixed numbers and fractions greater than one *by building and using models*. Represent fractions as the sum of fractions with the same denominator in more than one way. *Justify* the equivalence with equations. Recognize that a value can be expressed as a fraction greater than one and an equivalent mixed number. (MP 3, 4)

4.NF.2

Recognize and generate equivalent fractions, connecting numerical strategies to visual fraction models. Make and test conjectures about what happens to a fraction when the numerator and the denominator are multiplied by the same number. (MP 8)

4.NF.3

Compare and order fractions with different numerators and denominators. *Construct and communicate arguments that justify* the order of the fractions. (MP 3)

4.NF.4

Build and use models to add and subtract fractions with like denominators, including mixed numbers. Solve problems involving addition and subtraction of fractions *in real world contexts and equations*. (MP 2, 4)

4.NF.5

Build and use models to extend understanding of multiplication to fractions. Connect repeated addition to multiplication when multiplying a fraction by a whole number. Multiply a fraction by a whole number using numerical strategies. Solve problems involving multiplying a fraction by a whole number *in real world contexts*. (MP 2, 4)

4.NF.6

Build and use models to express a fraction with denominator of 10 as an equivalent

fraction with denominator of 100, and use this technique to add two fractions with respective denominators of 10 and 100. (MP 4)

4.NF.7

Connect decimal notation to fractions with denominators of 10 or 100. *Construct clear and reasonable arguments to justify* how the decimal notation and fraction are equivalent. (MP 3)

4.NF.8

Compare two decimal numbers to the hundredths place by reasoning about their size. *Justify* the comparison. *Represent the relationship* between two numbers using comparison symbols $<$, $=$, and $>$. (MP 3, 7)

Measurement and Geometry (MG)

Solve problems involving measurement and conversion of measurements. Solve real-world area and perimeter problems. Understand concepts of angles and angle measurement. Draw and identify lines and angles, and classify shapes by the properties of their lines and angles.

4.MG.1

Add and remove context to solve real-world and mathematical problems involving the area and perimeter of rectangles and rectilinear shapes. *Build and use models* to make sense of the area and perimeter formulas for rectangles. (MP 2, 4)

4.MG.2

Draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. *Describe these structures and their relationships* within two-dimensional figures. (MP 7)

4.MG.3

Describe angles as geometric figures formed by two rays with a common endpoint. Understand that an angle is measured with reference to a circle. *Describe the relationship* between an angle's measure and the fraction of the circular arc that it represents. (MP 7)

4.MG.4

Attend to precision and reasonableness when measuring or sketching angles in whole-number degrees using a protractor. (MP 6)

4.MG.5

Understand and demonstrate the additive nature of angle measures. Find and *justify* unknown angles on a diagram using addition and subtraction. (MP 3)

4.MG.6

Compare and classify two-dimensional figures including quadrilaterals and triangles based on the presence or absence of parallel lines, perpendicular lines, and angles of specified sizes. *Justify and evaluate* the classification. (MP 3)

Data Science (D)

Investigate and analyze situations with up to two variables. Identify the impact of the sampling process and sample size on the collection, visualization, and interpretation of data.

4.D.1

Determine a topic question and *ask investigative questions* that will lead to collecting, analyzing, and interpreting data with up to two variables. *Make conjectures* about the impact of the sampling process and sample size on results. (MP 1, 8)

4.D.2

Select and use tools strategically to collect and organize data. Build data visualizations. *Ask questions* to consider a given data set with up to two variables. (MP 1, 5)

4.D.3

Analyze data and data visualizations. *Evaluate conjectures* about how different sampling processes or data visualizations might influence interpretation of data. (MP 6, 8)

4.D.4

Interpret data and data visualizations to answer investigative questions. *Construct and justify arguments* about the data. Discuss ways that the sampling process and sample size may affect how people draw conclusions from data. Identify ways to refine investigative questions, data collection, visualization, or analysis. (MP 3)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of fourth grade, children will be able to:

- Understand multiplication as a comparison. (OA)
- Solve whole number, multi-step word problems involving addition, subtraction, multiplication, and division. (OA)
- Generalize place value understanding for multi-digit whole numbers (less than 1,000,000) by analyzing patterns, representing whole numbers in a variety of ways, and making multiplicative comparisons. (NBT)
- Use efficient algorithms to fluently add and subtract. (Computational and procedural fluency) (NBT)
- Use models to represent and make sense of two-digit by two-digit multiplication and division with one-digit divisors (ex, $125 \div 5 = 25$ where 5 is the divisor). (NBT)
- Make connections between numerical representations and visual models to compare, order, and determine if fractions have equal values. (NF)
- Build and use models to add and subtract fractions with like denominators and to multiply fractions by a whole number. (NF)
- Build and use models to connect decimal values to fractions with

denominators of tenths and hundredths. (NF)

- Apply knowledge of area to solve real-world and mathematical problems.
(MG)

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Standards: Grade 5

Introduction

In third through fifth grade, the major work of each grade focuses on representing and understanding multiplication and division and developing an understanding of fractions. Students generalize and expand their place value understanding. Measurement concepts are expanded to include two- and three-dimensional measurements.

In Grade 5, instructional time should focus on three major works: (1) extending understanding of addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) master place value understanding to develop understanding of operations with decimals to hundredths and with whole number and decimal operations; and (3) developing an understanding of volume.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In grades 3-5, students ask questions to help them understand multi digit numbers, fractions, decimals, the properties of all four operations, and the classification of shapes. Students ask questions to help them make sense of data and different data representations.

Mathematical Practice 2: Add or remove context to make sense of mathematics

In grades 3-5, students understand what the numbers, symbols, pictures, words, etc. in their work represent when using the four operations. Students flexibly move between concrete, pictorial, and abstract representations of mathematical ideas. They move back and forth between a problem's context and its representation to use the form that best fits the situation.

Mathematical Practice 3: Construct, justify, and communicate clear and

reasonable arguments

In grades 3-5, students use mathematical vocabulary to communicate their ideas and listen to others' ideas. They utilize strategies to agree and disagree respectfully. They back their argument up with evidence and draw connections between mathematical representations, comparing and contrasting their strategy and solution to others.

Mathematical Practice 4: Build and use models

In grades 3-5, students use a variety of models to extend their understanding of the base ten place value system, including decimals. They use models to build their understanding of fractions. They model the actions and comparisons represented by all four operations. In the upper grades, students focus on models that represent multiplication and division, including area models, number lines, and other multiplicative comparisons.

Mathematical Practice 5: Select and use tools appropriately and strategically

In grades 3-5, students use tools when they help them understand mathematics. Tools can include, but are not limited to, physical tools, visual tools, strategies, and algorithms.

Mathematical Practice 6: Attend to precision and reasonableness

In grades 3-5, students refine informal language as they develop and learn precise mathematical vocabulary. Students use structure to accurately record, label, and share their thinking. They determine when a precise answer is necessary and when making an estimate can help them determine if a solution makes sense. Students use estimation strategies such as rounding, utilizing benchmark fractions, and properties of operations to assess reasonableness through the problem solving process.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In grades 3-5, students use structures and patterns to develop a deep

understanding of the operations of multiplication and division. They extend their understanding of patterns, structures, and relationships through work with fractions, decimals, and larger whole numbers. Students use structures and patterns to see complicated things as single objects or composed of several objects.

Mathematical Practice 8: Make conjectures and evaluate the results

In grades 3-5, students make and test conjectures related to multi digit numbers, fractions, decimals, the properties of all four operations, and the properties of shapes. As students test their conjectures, they evaluate the reasonableness of their solution path and adjust as needed.

Standards: Grade 5

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Operations and Algebraic Thinking (OA)

Explore prime and composite numbers. Write and interpret numerical expressions. Justify and generalize understanding of properties of operations. Analyze patterns and relationships.

5.OA.1

Use structures, patterns, and relationships to determine whether a given whole number between 1 and 100 is prime or composite. (MP 7)

5.OA.2

Add context to make sense of numerical expressions involving multiple operations with parentheses, brackets, or braces. Write and/or evaluate numerical expressions. (SMP 2)

5.OA.3

Generate two numerical patterns using two given rules. *Describe and represent relationships* between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. (MP 7)

Number and Operations in Base Ten (NBT)

Understand structures and patterns in the place value system including decimals to thousandths and powers of ten. Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.1

Describe the structure of place value by explaining that the value of one place represents ten times as much as the place to its right and $\frac{1}{10}$ of the place to its left. *Represent those patterns* with a statement of multiplicative comparison. (MP 7)

5.NBT.2

Represent powers of 10 with whole-number exponents. *Make and test conjectures about* a decimal's placement when multiplying by powers of 10. (MP 7, 8)

5.NBT.3

Describe and represent decimals to the thousandths place using models, base-ten numerals, number names, and expanded forms. (MP 7)

5.NBT.4

Compare two decimal numbers and *represent the relationship* between the two numbers using comparison symbols $<$, $=$, and $>$. (MP 7)

5.NBT.5

Apply knowledge of place value structure to round decimal numbers to the nearest whole, tenth, or hundredth *in and out of context*. Determine and *justify* situations where rounding is appropriate. (MP 2, 3)

5.NBT.6

Use efficient numerical strategies based on properties of multiplication and/or place value to multiply a whole number of up to four digits by a one digit whole number and to multiply two two-digit numbers *flexibly, accurately, and efficiently in real-world contexts and equations*. Determine if products are *reasonable*. (MP 2, 6)

5.NBT.7

Use efficient numerical strategies based on properties of multiplication and/or place value to divide up to a four digit number by a one digit number. *Communicate* the meaning of the remainder *in real world contexts*. (MP 2, 3)

5.NBT.8

Build and use models to find sums and differences of decimals to hundredths *in real-world contexts and equations*. Use the models to *represent and describe the structure* of adding tenths with tenths and hundredths with hundredths. (MP 2, 4, 7)

5.NBT.9

Build and use models to find products of decimals to hundredths *in real-world contexts and equations*. (MP 2, 4)

5.NBT.10

Build and use models to find quotients of decimals limited to a whole number dividend with a decimal divisor of tenths between 0 and 1, or a decimal dividend with a single-digit whole number divisor *in real-world contexts and equations*. (MP 2, 4)

Number and Operations–Fractions (NF)

Use equivalent fractions as a strategy to add and subtract fractions. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.1

Add and subtract fractions with unlike denominators *with precision*. Solve problems involving addition and subtraction with unlike denominators in *real world contexts and equations*. Determine if the solution is *reasonable*. (MP 2, 6)

5.NF.2

Interpret a fraction as division (a/b is the same as $a \div b$). Solve problems involving division *in real world contexts* using *visual models and/or equations*. (MP 2, 4)

5.NF.3

Build and use models, including area models, to multiply fractions, whole numbers, and mixed numbers. Connect the properties of multiplication to multiplying fractions. Solve problems involving multiplication of fractions and mixed numbers *using real world contexts and equations* (fraction by a fraction, fraction by a mixed number, mixed number by mixed number). (MP 2, 4)

5.NF.4

Interpret multiplication as scaling. *Construct, justify, and communicate clear and reasonable arguments* about the size of the resulting product when multiplying a given number by a fraction greater than one, a fraction less than one, a fraction equal to one. *Make and test conjectures* about the size of the product based on the size of the factors without computing. (MP 3, 8)

5.NF.5

Build and use models that extend understanding of division to divide unit fractions by whole numbers and whole numbers by unit fractions. Solve problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions *using real world contexts and equations*. *Make conjectures* about the relationship between multiplication and division when operating with fractions *and evaluate the results*. (MP 2, 4, 8)

Measurement and Geometry (MG)

Convert like measurement units. Develop an understanding of volume. Understand and use quadrant one in the coordinate plane. Classify two-dimensional figures based on their attributes.

5.MG.1

Recognize volume as an attribute of solid figures. *Build and use models* to understand the concept of volume. Demonstrate that unit cubes can be packed into rectangular prisms. Determine volume by counting unit cubes and expressing the volume in cubic units. (MP 4)

5.MG.2

Build and use models to make sense of and apply the formulas for finding the volume of right rectangular prisms. *Add or remove context to solve real-world problems* involving the volume of a right rectangular prisms with whole-number side lengths. (MP 2, 4)

5.MG.3

Build and use a model of the coordinate plane to represent real-world problems. Understand the numbers in the coordinate pair indicate distance from the origin in the direction of the horizontal and vertical axes. Graph points in the first quadrant of the coordinate plane Interpret coordinate values based on a given context. (MP 2, 4)

5.MG.4

Describe and represent structure, patterns, and relationships when classifying quadrilaterals in a hierarchy based on attributes. (MP 7)

Data Science (D)

Investigate and analyze situations with up to four variables. Interpret data to answer investigative questions. Consider the impact of data visualizations, bias, and sampling approaches. Explore features of visible distribution.

5.D.1

Determine a topic question and *ask investigative questions* that will lead to collecting, analyzing, and interpreting data with up to four variables. *Ask questions* to identify potential sources of bias in data collection, representation, and interpretation. (MP 1)

5.D.2

Select and use tools strategically, to collect and organize data. Build data visualizations. *Ask questions* to consider a given data set with up to four variables. *Make conjectures about* potential sampling approaches and how those approaches impact results. (MP 1, 5, 8)

5.D.3

Analyze data sets with up to four variables and multiple data visualizations *Evaluate* sampling approaches and data visualizations to identify potential data bias. (MP 6, 8)

5.D.4

Interpret data and data visualizations to answer investigative questions while considering the impact of data bias, choice of data visualizations, and sample size. *Construct and justify arguments* about the data based on visible features of distribution such as center and range. Identify ways to refine investigative questions, data collection, visualization, or analysis. (MP 3)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of fifth grade, children will be able to:

- Generalize place value understanding for decimals to the thousandths place by analyzing patterns, representing numbers in a variety of ways, and making multiplicative comparisons. (NBT)
- Fluently multiply multi-digit whole numbers by a one-digit whole number and two two-digit whole numbers. (Computational and procedural fluency) (NBT)

- Use efficient numerical strategies to solve division problems. (NBT)
- Build and use models to solve addition, subtraction, multiplication and division problems involving decimals. (NBT)
- Add and subtract fractions with unlike denominators. (NF)
- Build and use models to multiply fractions, whole numbers, and mixed numbers. (NF)
- Recognize volume as an attribute of a solid shape and explain how multiplication and addition relate to volume. (MG)

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Standards: Grade 6

Introduction

In sixth through eighth grade, the major work of each grade focuses on applying and using operations with rational numbers, understanding ratio concepts and applying proportional reasoning, and simplifying expressions and solving equations.

In sixth grade, instructional time should focus on four major works: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) interpreting data to answer investigative questions.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In grades 6-8, students formulate questions to explore mathematical ideas involving real-world problems that use rate, ratio, and percents; in the context of data analysis, students interpret quantitative bivariate data to answer investigative questions, asking questions about patterns, associations, and potential outliers within the data.

Mathematical Practice 2: Add or remove context to make sense of mathematics

In grades 6-8, students understand what the numbers, symbols, pictures, words, etc. in their work represent when working with algebraic equations and expressions. Students flexibly move between concrete, pictorial, and abstract representations of mathematical ideas like linear functions. They move back and forth between a problem's context and its representation to use the form that best fits the situation.

Mathematical Practice 3: Construct, justify, and communicate clear and

reasonable arguments

In grades 6-8, students construct, justify, and communicate arguments in various mathematical contexts such as: solving problems involving ratios and rates and explaining their thinking about proportional relationships, to justify and explain the logical steps needed to solve multi-step equations and inequalities, and to communicate informal arguments to establish angle relationships for parallel lines cut by a transversal and angle relationships for triangles.

Mathematical Practice 4: Build and use models

In grades 6-8, students build visual models of fractions to extend their understanding of multiplication and division. They build a variety of models to extend their understanding of proportional relationships and linear functions.

Mathematical Practice 5: Select and use tools appropriately and strategically

In grades 6-8, students focus on building foundational skills through the diverse use of mathematical tools. Students use algebraic tools to generate equivalent expressions and solve multi-step equations involving positive and negative numbers. For geometry, students engage with geometric concepts by using both classical tools like a straight edge and compass and dynamic geometry software to explore, construct, and analyze shapes. Students use computer based tools to explore data, utilizing graphical displays and numerical summaries as statistical tools to analyze and interpret data.

Mathematical Practice 6: Attend to precision and reasonableness

In grades 6-8, students attend to mathematical precision and reasonableness in their communication, calculations and processes. They calculate flexibly, accurately, and efficiently, using clear and concise notation to record their work. Students apply this skill when adding, subtracting, multiplying and dividing multi-digit whole numbers and decimals. Students attend to the reasonableness of solutions when solving multi-step, real world problems using positive and negative rational numbers, and when finding rational approximations of irrational numbers.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In grades 6-8, students describe and represent their mathematical understanding of structures, pattern and relationships to extend their understanding of multiplication and division in order to divide fractions by fractions; to develop and apply properties of integer exponents to generate equivalent numerical expressions and to use the structure of similar triangles to explain why the slope of a non-vertical line is constant between any two points on the coordinate plane.

Mathematical Practice 8: Make conjectures and evaluate the results

In grades 6-8, students develop and assess conjectures related to proportional relationships and apply the concepts of unit rate and percent. They also develop and assess conjectures about the properties of irrational numbers by comparing them to the set of rational numbers, and make conjectures about dilations and congruency of geometric figures.

Standards: Grade 6

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Algebra (A)

Build on previous learning in Operations and Algebraic Thinking to reason with and solve one-variable equations and inequalities.

6.A.1

Use structure to write and evaluate numerical expressions involving whole numbers exponents. (MP 7)

6.A.2

Describe and represent patterns and structures of expressions to write, read, and evaluate algebraic expressions. (MP 7)

6.A.3

Use properties of operations *as tools* to generate equivalent expressions. Use properties of operations and understanding of arithmetic *to justify* why two expressions are equivalent. (MP 3, 5)

6.A.4

Use substitution to reason about whether a given number in a specified set makes an equation or inequality true. Provide *justification* to support that reasoning. (MP 3)

6.A.5

Describe, represent and solve real-world and mathematical problems using variables within the *structure* of one-step equations and inequalities. (MP 7)

6.A.6

Create equations to *model* relationships between two quantities using variables, both *in and out of context*. (MP 2, 4)

Number Systems (NS)

Build on previous work involving operations to compute fluently with multi-digit numbers, decimal and fractions. Understand that there is a set of numbers called integers and create models to show and describe integers and their relationships.

6.NS.1

Extend prior understanding of multiplication and division to divide fractions by fractions. *Build and use visual fraction models* to make sense of and solve problems dividing fractions by fractions. *Add and remove context with* problems involving division of fractions by fractions and interpret the quotients. (MP 2, 4)

6.NS.2

Describe and represent structures and patterns to apply an algorithm to divide multi-digit decimals *flexibly, accurately, and efficiently*. Limit division to a whole number

dividend with a decimal divisor or a decimal dividend with a whole number divisor. Determine and *justify* situations where estimation is appropriate. (MP 3, 6, 7)

6.NS.3

Attend to precision and reasonableness when adding, subtracting, multiplying, and dividing multi-digit whole numbers and decimals *flexibly, accurately, and efficiently*. (MP 6)

6.NS.4

Find the greatest common factor between two whole numbers 1 to 100 and the least common multiples of two whole numbers 1 to 12 by making use of *structures and patterns*. Relate greatest common factors to the distributive property. (e.g., re-express $36 + 8$ as $4(9 + 2)$). (MP 7)

6.NS.5

Use positive and negative numbers together to represent quantities in *real world contexts*, explain the meaning of zero in *context*. *Communicate* understanding that positive and negative numbers are used together to describe quantities having opposite directions or values. (MP 2, 3)

6.NS.6

Build and use models to show positive and negative numbers on a number line and coordinate plane. Use models to order positive and negative numbers. (MP 4)

6.NS.7

Build and use models to develop the definition of absolute value. Work with absolute value *in and out of context*. (MP 2, 4)

Ratio and Proportional Relationships (RP)

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1

Clearly communicate understanding of ratio relationships by using ratio language. (MP 3)

6.RP.2

Clearly communicate understanding of the concept of a unit rate associated with a ratio and use rate language to *represent* a unit rate associated with a ratio relationship. (MP 7)

6.RP.3

Build and use models including tables, double number lines, and/or tape diagrams to represent and reason *about* real-world ratio relationships with whole number quantities. (MP 4)

6.RP.4

Use ratio and rate reasoning *as a tool* to make sense of and solve problems in context. (MP 5)

6.RP.5

Use ratio reasoning *as a tool* to convert measurement units when given a unit conversion chart. (MP 5)

Geometry (G)

Solve real-world and mathematical problems involving area, surface area, and volume. Draw polygons on the coordinate plane.

6.G.1

Find the area of right triangles, other triangles, special quadrilaterals, and polygons by *composing and decomposing into rectangles, triangles and/or other shapes*. (MP 7)

6.G.2

Build and use nets to represent 3-dimensional figures composed of rectangles and triangles. *Use nets as tools* to find the surface area of the figure. (MP 4, 5)

6.G.3

Select and use tools appropriately to solve *real-world and decontextualized mathematical problems* involving surface area and volume of right rectangular

prisms with fractional edge lengths. (MP 2, 5)

6.G.4

Construct polygons on the coordinate plane given coordinates for the vertices. Use coordinates to find the length of a side joining points with the same x coordinate or the same y coordinate. Apply these techniques to *solve problems with and without real-world context*. (MP 2)

Data Science (D)

Investigate and analyze situations with large data sets. Describe and represent structures and patterns of statistical variability of data. Interpret data to answer investigative questions about distributions, considering the interpretation of the following features of distributions: center, variability, overall shape, and deviations. Identify ways to refine investigative questions, data collection, visualization, or analysis.

6.D.1

Formulate statistical investigative questions that anticipate variability in data and how the data may impact the investigation question. (MP 1)

6.D.2

Select and use tools appropriately and strategically to generate, collect, and organize data. *Ask questions* about how the data were collected and whether the data are useful to answer the statistical question of interest. (MP 1, 5)

6.D.3

Create *precise and reasonable* visual representations, including histograms (with or without technology), dot plots, and box plots. *Justify* which graph or plot best represents the data. Analyze and interpret visual representations and numerical summaries for measures of center (mean and/or median) and measures of variability (mean absolute deviation and/or interquartile range). (MP 3, 6)

6.D.4

Construct, justify, and communicate answers to investigative questions about

distributions. Consider the interpretation of the following features of distributions: center (mean and/or median), variability (mean absolute deviation and/or inter quartile range), and overall shape (symmetric or non-symmetric). Describe any striking deviations (outliers). Identify ways to refine investigative questions, data collection, visualization, or analysis. (MP 3)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of sixth grade, students will be able to:

- Apply previous understanding of arithmetic to generate equivalent algebraic expressions. (A)
- Reason with and solve one-variable equations and inequalities and analyze the relationship between two quantities in a real-world context. (A)
- Fluently add, subtract, multiply, and divide with rational numbers. (Computational and procedural fluency) (NS)
- Define, identify, and explain the meaning of integers on number lines, coordinate grids, and in real-world context. (NS)
- Understand rate and ratio relationships. (RP)
- Build and use models to represent real-world ratio relationships and solve ratio/rate problems. (RP)
- Solve real-world and mathematical problems involving area, surface area, and volume. (G)
- Answer investigative questions about distributions, considering the interpretation of the following features: center, variability, overall shape, and

deviations. (D)

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Standards: Grade 7

Introduction

In sixth through eighth grade, the major work of each grade focuses on applying and using operations with rational numbers, understanding ratio concepts and applying proportional reasoning, and simplifying expressions and solving equations.

In seventh grade, instructional time should focus on four major works: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In grades 6-8, students formulate questions to explore mathematical ideas involving real-world problems that use rate, ratio, and percents; in the context of data analysis, students interpret quantitative bivariate data to answer investigative questions, asking questions about patterns, associations, and potential outliers within the data.

Mathematical Practice 2: Add or remove context to make sense of mathematics

In grades 6-8, students understand what the numbers, symbols, pictures, words, etc. in their work represent when working with algebraic equations and expressions. Students flexibly move between concrete, pictorial, and abstract representations of mathematical ideas like linear functions. They move back and forth between a problem's context and its representation to use the form that best fits the situation.

Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments

In grades 6-8, students construct, justify, and communicate arguments in various mathematical contexts such as: solving problems involving ratios and rates and explaining their thinking about proportional relationships, to justify and explain the logical steps needed to solve multi-step equations and inequalities, and to communicate informal arguments to establish angle relationships for parallel lines cut by a transversal and angle relationships for triangles.

Mathematical Practice 4: Build and use models

In grades 6-8, students build visual models of fractions to extend their understanding of multiplication and division. They build a variety of models to extend their understanding of proportional relationships and linear functions.

Mathematical Practice 5: Select and use tools appropriately and strategically

In grades 6-8, students focus on building foundational skills through the diverse use of mathematical tools. Students use algebraic tools to generate equivalent expressions and solve multi-step equations involving positive and negative numbers. For geometry, students engage with geometric concepts by using both classical tools like a straight edge and compass and dynamic geometry software to explore, construct, and analyze shapes. Students use computer based tools to explore data, utilizing graphical displays and numerical summaries as statistical tools to analyze and interpret data.

Mathematical Practice 6: Attend to precision and reasonableness

In grades 6-8, students attend to mathematical precision and reasonableness in their communication, calculations and processes. They calculate flexibly, accurately, and efficiently, using clear and concise notation to record their work. Students apply this skill when adding, subtracting, multiplying and dividing multi-digit whole numbers and decimals. Students attend to the reasonableness of solutions when solving multi-step, real world problems using positive and negative rational numbers, and when finding rational approximations of irrational numbers.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In grades 6-8, students describe and represent their mathematical understanding of structures, pattern and relationships to extend their understanding of multiplication and division in order to divide fractions by fractions; to develop and apply properties of integer exponents to generate equivalent numerical expressions and to use the structure of similar triangles to explain why the slope of a non-vertical line is constant between any two points on the coordinate plane.

Mathematical Practice 8: Make conjectures and evaluate the results

In grades 6-8, students develop and assess conjectures related to proportional relationships and apply the concepts of unit rate and percent. They also develop and assess conjectures about the properties of irrational numbers by comparing them to the set of rational numbers, and make conjectures about dilations and congruency of geometric figures.

Standards: Grade 7

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Algebra (A)

Use properties of operations to generate equivalent expressions. Represent and solve mathematical problems using algebraic expressions and equations.

7.A.1

Apply properties of operations as *tools* to add, subtract, factor and expand linear expressions with rational coefficients in different forms. (MP 5)

7.A.2

Use *algebraic tools* to solve equations and inequalities representing real-world problems of the form $px+q=r$ and $p(x+q)=r$ using positive and negative rational numbers. *Attend to the reasonableness* of the solutions. (MP 5, 6)

7.A.3

Given a context, construct equations and inequalities to *model* real-world problems of the form $px+q=r$ and $p(x+q)=r$ using positive and negative rational numbers. (MP 4)

7.A.4

Build and use models including tables, graphs, and equations to represent and understand the relationship between independent and dependent variables. (MP 4)

The Number System (NS)

Build upon understanding of fractions and decimals to define the set of rational numbers. Apply understanding of rational numbers and operations to solve problems.

7.NS.1

Describe and represent structures, patterns and relationships to define rational numbers and make connections between fraction, decimal and percent representations. of a rational number. Flexibly use the various representations of rational numbers *while attending to precision*. (MP 6, 7)

7.NS.2

Build and use models including a horizontal or vertical number line diagram to extend previous understanding of addition and subtraction to add and subtract rational numbers *in and out of context* including signed decimals and fractions. (MP 2, 4)

7.NS.3

Extend previous understanding of multiplication and division to multiply and divide rational numbers *in and out of context*. (MP 2)

7.NS.4

Flexibly, accurately, and efficiently add, subtract, multiply and divide rational numbers including signed decimals and fractions *in and out of context*. Attend to *precision and reasonableness* of solutions. (MP 5, 6)

Ratio and Proportional Relationships (RP)

Use ratio and rate understanding to develop, recognize, and model proportional relationships. Use patterns in these relationships to reason about and solve real-world problems. Apply proportional reasoning to understand and solve percent problems.

7.RP.1

Calculate unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities. Apply unit rate as a tool to *make and evaluate conjectures* about proportional relationships (MP 8)

7.RP.2

Understand percent of a quantity as a rate per 100. Describe and *represent structures and patterns* that connect percent to ratio. (MP 7)

7.RP.3

Build and use models including tables, graphs, double number lines, or tape diagrams to find a percent of a quantity as a rate per 100. Solve problems involving finding the whole, the part, or the percent. (MP 4)

7.RP.4

Recognize *structures and patterns* when analyzing real world contexts to determine whether two quantities have a proportional relationship. Use tables, graphs, equations, diagrams and verbal descriptions to justify whether a relationship is proportional. (MP 3, 7)

7.RP.5

Build and use models (tables, graphs, equations, diagrams and verbal descriptions) to represent proportional relationships and identify and interpret the constant of proportionality. (MP 4)

7.RP.6

Use proportional relationships *as a tool* to solve multi-step and percent problems such as simple interest, tax, markups and markdowns, gratuities and commissions, fees, and percent increase and decrease. (MP 5)

7.RP.7

Interpret the meaning of a point (x,y) on the graph of a proportional relationship in a given context. Connect and *interpret the relationship* between the points $(0, 0)$ and $(1, r)$ where r is the unit rate. (MP 4, 7)

Geometry (G)

Draw, construct, and describe geometrical figures from given criteria. Use and connect 2D and 3D geometrical figures to real-life situations and use them to solve real-world and mathematical problems.

7.G.1

Select and use tools appropriately and strategically (e.g., a compass, straightedge, dynamic geometry software) to draw and accurately describe scaled geometrical figures. *Utilize the underlying structure* of scale drawings to understand scale factor and solve real world and mathematical problems. (MP 5, 7)

7.G.2

Select and use tools appropriately and strategically to create triangles from given conditions to determine whether a unique triangle, more than one triangle, or no triangle can be formed. (MP 5)

7.G.3

Flexibly, accurately and efficiently solve real-world and mathematical problems involving perimeter and area of circles, triangles, quadrilaterals and polygons as well as surface area of right prisms. (MP 2, 6)

7.G.4

Describe and use the relationship between the area of a base of a right-prism and its

volume to solve real world and mathematical problems. (MP 7)

7.G.5

Describe and represent structures, patterns, and relationships related to supplementary, complementary, vertical, and adjacent angles to solve for unknown angle measures in a figure. (MP 7)

Data Science (D)

Investigate and analyze situations with large data sets. Make inferences about populations based on samples. Draw informal comparative inferences about two populations. Investigate chance processes and develop, use, and evaluate probability models. Identify ways to refine investigative questions, data collection, visualization, or analysis.

7.D.1

Formulate statistical investigative questions that anticipate variability in data collected from a random sample of a population. (MP 1)

7.D.2

Generate, collect, and organize randomly sampled data that is representative of the population *using tools appropriately*. Explain why random sampling is more likely to produce representative samples and support valid inferences *using clear and reasonable arguments*. (MP 3, 5)

7.D.3

Create *precise and reasonable* visual representations, including histograms (with technology), dot plots, and box plots. Analyze and interpret visual representations and numerical summaries for measures of center (mean and/or median) and measures of variability (mean absolute deviation and/or inter-quartile range). (MP 6)

7.D.4

Construct, justify, and communicate answers to investigative questions about a population and comparisons about two different populations. Compare features

such as center (mean and median), variability (mean absolute deviation and inter-quartile range) and overall shape (symmetric or nonsymmetric). (MP 3)

7.D.5

Communicate clear understanding that the probability of a chance event is the likelihood of an event occurring and is expressed as a number between 0 and 1. (MP 3)

7.D.6

Estimate the probability of a chance event by observing its long-run relative frequency. Given the probability, predict and *evaluate the reasonableness* of approximate relative frequencies. (MP 8)

7.D.7

Build and use probability models such as organized lists, tables, tree diagrams, and simulations to find or estimate probabilities of simple and compound events. (MP 4)

7.D.8

Build a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies. If the agreement is not good, explain possible sources for discrepancy. (e.g., find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the simulation appear to be equally likely based on the observed frequencies?) (MP 4)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of seventh grade, students will be able to:

- Use properties of operations to generate equivalent expressions and solve real-world problems using algebraic equations and inequalities. (A)
- Add, subtract, multiply, and divide rational numbers including negative and positive numbers to solve problems with and without context. (NS)
- Flexibly use various representations of fractions, percents, and decimals. (NS)
- Recognize proportional relationships with and without context. Use patterns in proportional relationships to reason about and solve problems. (RP)
- Understand the meaning of percent connected to ratios, and solve percent problems. (RP)
- Find the perimeter and area of triangles, quadrilaterals, and polygons. Find the surface area and volume of right rectangular prisms. Find the area and circumference of a circle. (G)
- Solve for unknown angles measures using angle relationships (G)
- Make inferences about populations based on samples, supported by measures of center and measures of variability. (D)
- Investigate chance processes and develop, use, and evaluate probability models. (D)

Standards: Grade 8

Introduction

In sixth through eighth grade, the major work of each grade focuses on applying and using operations with rational numbers, understanding ratio concepts and applying proportional reasoning, and simplifying expressions and solving equations.

In eighth grade, instructional time should focus on three major works: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In grades 6-8, students formulate questions to explore mathematical ideas involving real-world problems that use rate, ratio, and percents; in the context of data analysis, students interpret quantitative bivariate data to answer investigative questions, asking questions about patterns, associations, and potential outliers within the data.

Mathematical Practice 2: Add or remove context to make sense of mathematics

In grades 6-8, students understand what the numbers, symbols, pictures, words, etc. in their work represent when working with algebraic equations and expressions. Students flexibly move between concrete, pictorial, and abstract representations of mathematical ideas like linear functions. They move back and forth between a problem's context and its representation to use the form that best fits the situation.

Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments

In grades 6-8, students construct, justify, and communicate arguments in various mathematical contexts such as: solving problems involving ratios and rates and explaining their thinking about proportional relationships, to justify and explain the logical steps needed to solve multi-step equations and inequalities, and to communicate informal arguments to establish angle relationships for parallel lines cut by a transversal and angle relationships for triangles.

Mathematical Practice 4: Build and use models

In grades 6-8, students build visual models of fractions to extend their understanding of multiplication and division. They build a variety of models to extend their understanding of proportional relationships and linear functions.

Mathematical Practice 5: Select and use tools appropriately and strategically

In grades 6-8, students focus on building foundational skills through the diverse use of mathematical tools. Students use algebraic tools to generate equivalent expressions and solve multi-step equations involving positive and negative numbers. For geometry, students engage with geometric concepts by using both classical tools like a straight edge and compass and dynamic geometry software to explore, construct, and analyze shapes. Students use computer based tools to explore data, utilizing graphical displays and numerical summaries as statistical tools to analyze and interpret data.

Mathematical Practice 6: Attend to precision and reasonableness

In grades 6-8, students attend to mathematical precision and reasonableness in their communication, calculations and processes. They calculate flexibly, accurately, and efficiently, using clear and concise notation to record their work. Students apply this skill when adding, subtracting, multiplying and dividing multidigit whole numbers and decimals. Students attend to the reasonableness of solutions when solving equations, real world problems using positive and negative rational numbers, and when finding rational approximations of irrational numbers.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In grades 6-8, students describe and represent their mathematical understanding of structures, pattern and relationships to extend their understanding of multiplication and division in order to divide fractions by fractions; to develop and apply properties of integers of exponents to generate equivalent numerical expressions and to use the structure of similar triangles to explain why the slope of a non-vertical line is constant between any two points on the coordinate plane.

Mathematical Practice 8: Make conjectures and evaluate the results

In grades 6-8, students develop and assess conjectures related to proportional relationships and apply the concepts of unit rate and percent. They also develop and assess conjecture about the properties of irrational numbers by comparing them to the set of rational numbers, and make conjectures about dilations and congruency of geometric figures.

Standards: Grade 8

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Algebra (A)

Build on prior understanding of expressions and equations to include work with integer exponents and radicals. Through the use of multiple representations, make connections between linear relationships and solve single-variable equations and inequalities.

8.A.1

Use structure and patterns to develop and apply the properties of integer exponents to generate equivalent expressions. (MP 7)

8.A.2

Use structure and patterns of operations to simplify and evaluate algebraic expressions, including those involving rational coefficients, exponents, and polynomial terms. (MP 7)

8.A.3

Use algebraic tools strategically to represent solutions to equations in the form $x^2 = p$ and $x^3 = p$. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. (MP 5)

8.A.4

Represent the structure and patterns of numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and express how many times as much one is than the other. (MP 7)

8.A.5

Apply structures, properties and patterns of exponents to perform all four operations with scientific notation. Interpret scientific notation in terms of a context. (MP 2, 7)

8.A.6

Represent patterns by graphing proportional relationships and interpret the unit rate as the slope of the graph. *Use the structure* of similar triangles to explain why the slope is the same between any two distinct points on a non-vertical line in the coordinate plane. (MP 7)

8.A.7

Represent and compare linear relationships using graphs, tables, and equations *with or without context*. Interpret slope and intercepts in *context*. (MP 2)

8.A.8

Use structures, patterns and properties of equivalence to solve symbolic and real-world single-variable equations and inequalities with rational number coefficients and justify algebraically. Solve absolute value equations in the form $|px| + q = c$ or $|px + q| = c$. (MP 7)

8.A.9

Build and use graphic models to analyze and solve systems of two linear equations in two variables and interpret their solutions. Identify systems that have zero, one, and infinite solutions with or without context. (MP 2, 4)

The Number System (NS)

Understand that there is a set of numbers which are not rational, called irrational numbers. Create models to show and describe irrational numbers and their relationships to rational numbers.

8.NS.1

Describe and represent structures and patterns comparing the set of irrational numbers to the set of rational numbers, in order to determine properties of irrational numbers. (MP 7)

8.NS.2

Find rational approximations of irrational numbers, *attending to precision and reasonableness*. Use a number line diagram as a *model* to compare values of irrational numbers. (MP 4, 6)

8.NS.3

Estimate the value of expressions containing irrational numbers, *with and without a context*. (MP 2)

8.NS.4

Justify why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational. *Build and use models* to connect to physical situations. (MP 3, 4)

Functions (F)

Describe functions and their features. Use functions to model relationships between two variables.

8.F.1

Construct and communicate clear arguments as to whether a relation is a function. (MP 3)

8.F.2

Given a *representation* (verbal description, table, two coordinate pairs, or an equation of $y=mx+b$), determine and interpret the rate of change and the initial value *in and out of context*. (MP 2, 4)

8.F.3

Construct a function *to model* a linear relationship between two quantities when given a verbal description, a table, and a graph. (MP 4)

8.F.4

Given a graph of two quantities (linear and non-linear), *describe and represent* features of the relationship, including intervals where the function is increasing or decreasing, the location of its maximum and minimum values. *Build graphic models* that satisfy given qualitative features. (MP 4, 7)

8.F.5

Use multiple representations (tables, graphs, equations, diagrams and verbal descriptions) to compare the properties of linear and nonlinear functions with and without a context. *Justify* whether a function is linear or non-linear. (MP 3)

Geometry (G)

Understand and describe both congruence and similarity in terms of transformations. Explore and verify angle relationships connected to parallel lines cut by a transversal. Understand and use the Pythagorean Theorem.

8.G.1

Use rigid transformations to establish a definition of congruent figures. *Justify* that two shapes are congruent through the use of rigid transformations. *Make and evaluate conjectures* about attributes of, rotations, reflections, and translations with

and without coordinates. (MP 3, 8)

8.G.2

Use transformations (dilation, translation, reflection and rotation) to establish a definition of similar figures. *Justify* that two shapes are similar through the use of transformations. *Make and examine conjectures* about attributes of dilations, with and without coordinates. (MP 3, 8)

8.G.3

Ask questions to determine the difference between congruence and similarity in terms of transformations. *Clearly communicate* the distinction between congruent and similar figures. (MP 1, 3)

8.G.4

Create and communicate informal arguments that establish angle relationships for parallel lines cut by a transversal and angle relationships for triangles, including the angle sum and exterior angle. (MP 3)

8.G.5

Use the Pythagorean theorem when solving problems with right triangles and finding distance between two coordinate points. *Use geometric structures and numerical patterns* to understand and explain proofs of the Pythagorean Theorem. (MP 7)

8.G.6

Solve real-world and mathematical problems *with or without a context* involving right triangles, distance, and volume of cones, cylinders, and spheres. (MP 2)

Data Science (D)

Investigate and analyze situations with large data sets. Investigate patterns of association in bivariate data. Identify ways to refine investigative questions, data collection, visualization, or analysis.

8.D.1

Ask questions about bivariate data sets (quantitative and categorical) to investigate patterns of associations between two quantities. (MP 1)

8.D.2

Select and use tools appropriately and strategically to identify existing data sets or generate, collect, and organize data to investigate associations between two quantities. (MP 5)

8.D.3

Explore, describe and compare variability in bivariate data for quantitative variables by creating scatter plots. For scatter plots that suggest a linear association, *represent the relationship* with a linear equation, and informally assess the line of fit by judging the closeness of the data points to the line. (MP 7)

8.D.4

Describe, represent and explain patterns of association such as clustering, positive or negative association, linear or nonlinear association and effects of outliers. (MP 7)

8.D.5

Answer investigative questions by interpreting quantitative bivariate data. *Construct, justify, and communicate clear and reasonable arguments regarding the data. Use the equation of a linear model* to solve problems in the context of bivariate measurement data, including interpreting the slope and intercept. Identify ways to refine the question, data collection, visualization, or analysis. (MP 3, 4)

8.D.6

Explore, describe and compare variability in bivariate data for categorical variables by constructing two-way tables. Calculate and interpret relative frequencies for rows or columns in a two-way table. *Attend to precision and reasonableness.* (MP 6)

8.D.7

Construct, analyze, and communicate clear and reasonable arguments to answer investigative questions about the association between categorical bivariate data. Use the relative frequencies calculated for rows or columns as evidence to justify

and describe possible associations between the two variables. (MP 3)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of eighth grade, students will be able to:

- Work with integer exponents and radicals. (A)
- Analyze and solve linear equations and inequalities. (A)
- Analyze graphs to solve systems of two linear equations in two variables and interpret their solutions. (A)
- Extend understanding of the number line to include rational and an approximation of irrational numbers. (NS)
- Use linear functions to model relationships between two variables. (F)
- Describe key features of linear relationships and compare the properties of linear and nonlinear functions. (F)
- Understand and describe both congruence and similarity in terms of transformations. (G)
- Explore and verify angle relationships connected to parallel lines cut by a transversal. (G)
- Understand and use the Pythagorean Theorem. (G)
- Interpret data to answer investigative questions about quantitative bivariate data. (D)

Standards: Secondary Math I

Introduction

In Secondary Math I through Secondary Math III the major works of each grade focus on learning to create, interpret, manipulate, and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations and building functions. Students learn to define functions, describe their features and transformations. Students understand, apply and prove congruence of similarity as defined in terms of geometric transformations. Students also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

In Secondary Math I instructional time should focus on five major works: (1) solving algebraic equations (linear and exponential); (2) understanding, comparing and representing linear and exponential functions; (3) describing characteristics of linear and exponential functions; (4) understanding and applying congruence as defined in terms of geometric transformations; and (5) analyzing and interpreting data that can be used to answer statistical questions of interest related to comparing multiple distributions or exploring associations between two quantitative variables.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In high school, students formulate and ask questions to highlight the difference between various function types. Students ask targeted questions about univariate data displays to explore mathematical ideas. Students ask questions about the underlying structure of geometric objects revealed by geometric constructions.

Mathematical Practice 2: Add or remove context to make sense of

mathematics

In high school, students contextualize and decontextualize functions to interpret their properties, connecting symbolic representations to meaningful situations. Students interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to distinguish between situations that can be modeled by various function types. They will construct arguments to justify claims such as why exponential growth eventually surpasses linear, quadratic or the growth in any other polynomial function.

Mathematical Practice 4: Build and use models

In high school, students build and use models in two primary ways. One of these is to use models to connect mathematical concepts to one another. Another way that students use models in high school is to model real-world phenomena.

Students create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. In high school students use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas between them.

To model real-world phenomena, students engage in mathematical and statistical modeling to represent, analyze, and predict real-world situations. In high school students use geometric shapes, their measures, and their properties to model real world objects and apply geometric methods to solve design problems.

Mathematical Practice 5: Select and use tools appropriately and strategically

In high school, students learn to select and leverage multiple types of tools while engaging in and applying mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential, or quadratic equations for a variable. Students select and use appropriate tools to build representations for various function types with and without technology. Students use mathematical ideas as a tool to validate the appropriateness and usefulness of a model.

Mathematical Practice 6: Attend to precision and reasonableness

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In high school, students describe the structures of linear and exponential expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define dilations and similar shapes, and then use these structures to solve problems. Students use models to explore and describe attributes of the structure, patterns and relationships of inverses for various function types.

Mathematical Practice 8: Make conjectures and evaluate the results

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to

answer investigative questions. Students make conjectures about the reasonableness and strength of the model they choose to describe the association between two quantitative variables. Students develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

Standards: Secondary Math I

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Algebra (A)

Interpret the structure and patterns of linear and exponential expressions. Create inequalities to describe linear and exponential relationships. Understand solving equations/inequalities or systems of equations as a process of reasoning and having a knowledge of the tools that can be used to create arguments or justifications for solutions.

S1.A.1

Use the structure of linear expressions and exponential expressions with integer exponents to identify parts of the expression and interpret their meaning in context. *Interpret* complicated expressions (such as $P(1+r)^n$) by viewing one or more of their parts as a single entity. (MP 7)

S1.A.2

Build symbolic models of linear equations and inequalities, and use them to solve problems *in and out of context*. (MP 3, 4)

S1.A.3

Use algebraic properties of equality *as tools* to rearrange formulas to isolate a quantity of interest. (MP 5)

S1.A.4

Solve linear equations and inequalities *using algebraic tools*, starting from the assumption that a solution exists. Communicate clear arguments for equations that do not have one unique solution and justify the results. Interpret solutions and use both inequality and interval notation. (MP 3, 5)

S1.A.5

Solve systems of linear equations *using algebraic tools*. Understand and *justify* how equations are manipulated while maintaining the consistency of the system. (MP 3, 5)

S1.A.6

Interpret whether a solution of a system of linear equations and inequalities is viable or non-viable *using graphic and analytic models*. Use *models* of systems of linear equations and inequalities to interpret whether a solution is viable or non-viable. (MP 4)

S1.A.7

Build and use graphic models to analyze and solve systems of two linear equations as well as systems of one linear equation and one exponential equation in two variables and interpret their solutions. (MP 4)

S1.A.8

Build and use graphic models to show the solutions to a linear inequality in two variables as a half-plane, and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (MP 4)

Functions (F)

Describe and represent arithmetic and geometric sequences with linear and exponential functions using function notation. Analyze and contextualize functions and their features across different representations. Develop, assess, and combine existing models to construct functions that relate two quantities. Construct and compare linear and exponential models and solve problems.

S1.F.1

Construct and communicate clear arguments to justify whether a relation is a function. (MP 3)

S1.F.2

Understand the structure of function notation, if f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$. (MP 7)

S1.F.3

Evaluate functions for inputs in their domains and interpret statements that use function notation *in terms of a context*. (MP 2)

S1.F.4

Build and use explicit and recursive equations to *model* arithmetic and geometric sequences using function notation. *Describe and represent* the domain of these models as a subset of the integers. (MP 4, 7)

S1.F.5

Understand and interpret the features of a function represented graphically, numerically and symbolically using interval notation and inequalities by *applying and removing context*. Focus on these key features: domain, range, intervals of increase and/or decreasing, location of absolute maximum and/or absolute minimum, and intercepts. *Build graphic models* that satisfy given key features. (MP 2, 4)

S1.F.6

Build representations that show key features of linear and exponential functions expressed symbolically. *Select and use appropriate tools* to model with and without technology. (MP 4, 5)

S1.F.7

Compare and contrast the properties of linear and exponential functions (represented in different ways). *Formulate and ask questions* to highlight the

similarities and differences between the functions. (MP 1)

S1.F.8

Build a function that *models* linear or exponential relationships. Appropriately use explicit and recursive rules to model these relationships and translate between the two forms. (MP 4)

S1.F.9

Construct arguments that *use the structures and patterns* of relationships to connect arithmetic sequences to linear functions and geometric sequences to exponential functions. (MP 3, 7)

S1.F.10

Use structures and patterns to identify the effect of k on the graph when replacing $f(x)$ by $f(x) + k$. Explore specific values of k for linear and exponential functions in and out of context. (MP 7)

S1.F.11

Communicate clear and reasonable arguments to distinguish between situations that can be modeled with linear functions, exponential functions, or neither. Connect the *structure* of linear and exponential functions with their parameters, ($y=mx+b$, $y=b^x(a)$). *Justify* that a quantity increasing exponentially eventually exceeds a quantity increasing linearly. (MP 3, 7)

Geometry (G)

Use prior knowledge of rigid motions to establish and justify the concept of congruence. Apply rigid motions and their properties to prove triangle congruence criteria and derive geometric theorems. Understand geometric constructions as both a tool for understanding transformations and as a foundation for logical reasoning. Connect algebraic and geometric processes by justifying figure attributes on the coordinate plane.

S1.G.1

Describe and represent the underlying structure of translations, rotations and

reflections, in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (MP 7)

S1.G.2

Construct and clearly communicate definitions of the rigid transformations to show that these transformations are functions that take points in the plane as inputs and give corresponding points as outputs. (MP 3)

S1.G.3

Select and use tools to strategically perform rigid transformations. Attend to precision when identifying and describing the transformation or the sequence of transformations to determine whether or not two figures are congruent. Understand that the image created from a rigid transformation or a sequence of rigid transformations is congruent to the pre-image. (MP 5, 6)

S1.G.4

Construct and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Use reflections and rotations to informally *justify* attributes of parallelograms including rhombus, rectangles and squares. (MP 3)

S1.G.5

Precisely construct geometric objects including equilateral triangle, rhombus, square, parallel line, perpendicular bisector, and angle bisector. *Communicate clearly* about the use of geometric construction tools and analyze the underlying structure of geometric objects as revealed by the tools. (MP 3)

S1.G.6

Construct, justify and communicate clear arguments that show how the criteria for triangle congruence (ASA, SAS, SSS) can be established using the definition of congruence in terms of rigid motions. (MP 3)

S1.G.7

Justify whether or not a quadrilateral has the attributes of a square, rectangle,

rhombus or parallelogram. Connect the Pythagorean Theorem to the distance formula and utilize slope along with distance as a means for *justification*. (MP 3)

Data Science (D)

Investigate and analyze situations with large data sets. Describe, represent, and interpret data on a single quantitative variable and on two quantitative variables and evaluate results. Identify ways to refine investigative questions, data collection, visualization, or analysis.

S1.D.1

Ask statistical questions to investigate situations that can be explored using random samples from populations to make inferences about differences between two populations or associations between two quantitative variables. Explain and *justify* the distinction between correlation and causation. (MP 1, 3)

S1.D.2

Select the appropriate statistical tools to create a data collection plan. Collect and organize primary data for the statistical investigative question of interest or determine the validity that the data collected is useful to answer the statistical question of interest when using secondary data. (MP 5)

S1.D.3

Compare two or more distributions with *precision* by selecting graphical displays (dot plots, histograms, and modified box plots to show outliers) that highlight features of interest. Analyze data by comparing measures of center (mean and median) and measures of variability (range, inter-quartile range and standard deviation) that are appropriate for the shape (skew, symmetry, outliers, modes) of the data distribution. Recognize how standard deviation builds on the mean absolute deviation and is another measure of how data values vary in a distribution. (MP 6)

S1.D.4

Analyze data to explore the association between two quantitative variables by using a scatterplot to determine and describe patterns of association such as

clustering, positive or negative association, linear or nonlinear association, and effects of outliers. Use technology to find and interpret the correlation coefficient and use it to assess the strength of the linear relationship. *Attend to precision and reasonableness* with calculations. (MP 6)

S1.D.5

Analyze and use statistical evidence to *evaluate results* and answer statistical investigative questions about the differences between two or more population parameters. Identify ways to refine the question, data collection, visualization, or analysis. (MP 8)

S1.D.6

Justify statistical reasoning to *evaluate results* from analyses to answer the statistical investigative question about associations between two quantitative variables. For scatter plots that suggest a linear association, represent the relationship with a line of best fit and use the model of the linear equation to solve problems, including interpreting the slope and intercept in the context of the data. *Justify* statistical reasoning and results to others in a variety of formats including verbal, written, and visual. (MP 3, 8)

Secondary Math 1: Extended Topics

Matrices

Perform operations on matrices and use matrices in applications. Solve systems of equations.

S1.M.1

Add, subtract, and multiply matrices of appropriate dimensions using *algebraic tools* for smaller matrices and *technology* for larger matrices. (MP 5)

S1.M.2

Use matrices to *represent* and manipulate data (e.g., to represent payoffs or incidences relationships in a network). Multiply matrices by scalars to produce new

matrices and *make sense* of this product by connecting it *with and without a context*. (MP 2, 4)

S1.M.3

Communicate understanding that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. (MP 3)

S1.M.4

Communicate understanding that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. (MP 3)

S1.M.5

Use 2×2 matrices to *model* transformations of the plane, and *interpret* the absolute value of the determinant in terms of area. (MP 4, 7)

S1.M.6

Model a system of linear equations as a single-matrix equation in a vector variable. (MP 4)

S1.M.7

Solve systems of linear equations up to three variables using matrix row reduction and technology *as tools*. (MP 5)

S1.M.8

Find the inverse of a matrix if it exists *using technology or algebraic tools*, and use the inverse to solve systems of linear equations (using technology for matrices of dimensions 3×3 or greater). (MP 5)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in

future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of Secondary I students will be able to:

- Create and solve equations and inequalities. (A)
- Solve systems of equations and inequalities with pairs of linear equations and inequalities in two variables. (A)
- Create explicit and recursive equations to model arithmetic and geometric sequences. (F)
- Interpret and analyze key features of graphs that model relationships between two variables for linear and exponential functions. (F)
- Build and interpret multiple representations of linear and exponential functions. (F)
- Establish triangle congruence criteria which will be used to prove other theorems. (G)
- Describe the transformation or the sequence of transformations to prove two figures are congruent. (G)
- Describe, represent, and interpret data on a single variable and on two quantitative variables and evaluate results. (D)

Standards: Secondary Math II

Introduction

In Secondary Math I through Secondary Math III the major works of each grade focus on learning to create, interpret, manipulate, and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations and building functions. Students learn to define functions, describe their features and transformations. Students understand, apply and prove congruence of similarity as defined in terms of geometric transformations. Students also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

In Secondary Math II instructional time should focus on five major works: (1) solving algebraic equations (linear, exponential and quadratic); (2) understanding, comparing and representing functions; (3) describing characteristics of functions; (4) proving congruence and similarity in terms of geometric transformations; and (5) analyzing and interpreting data that can be used to answer statistical questions of interest related associations between two quantitative variables.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In high school, students formulate and ask questions to highlight the difference between various function types. Students ask targeted questions about univariate data displays to explore mathematical ideas. Students ask questions about the underlying structure of geometric objects revealed by geometric constructions.

Mathematical Practice 2: Add or remove context to make sense of mathematics

In high school, students contextualize and decontextualize functions to interpret their properties, connecting symbolic representations to meaningful situations. Students interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to distinguish between situations that can be modeled by various function types. They will construct arguments to justify claims such as why exponential growth eventually surpasses linear, quadratic or the growth in any other polynomial function.

Mathematical Practice 4: Build and use models

In high school, students build and use models in two primary ways. One of these is to use models to connect mathematical concepts to one another. Another way that students use models in high school is to model real-world phenomena.

Students create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. In high school students use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas between them.

To model real-world phenomena, students engage in mathematical and statistical modeling to represent, analyze, and predict real-world situations. In high school students use geometric shapes, their measures, and their properties to model real world objects and apply geometric methods to solve design problems. design problems.

Mathematical Practice 5: Select and use tools appropriately and strategically

In high school, students learn to select and leverage multiple types of tools while engaging in and applying mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential, or quadratic equations for a variable. Students select and use appropriate tools to build representations for various function types with and without technology. Students use mathematical ideas as a tool to validate the appropriateness and usefulness of a model.

Mathematical Practice 6: Attend to precision and reasonableness

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In high school, students describe the structures of linear and exponential expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define dilations and similar shapes, and then use these structures to solve problems. Students use models to explore and describe attributes of the structure, patterns and relationships of inverses for various function types.

Mathematical Practice 8: Make conjectures and evaluate the results

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to

answer investigative questions. Students make conjectures about the reasonableness and strength of the model they choose to describe the association between two quantitative variables. Students develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

Standards: Secondary Math II

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Algebra (A)

Interpret the structure and patterns of expressions, extending analysis to highlight different aspects of quadratic functions. Build and manipulate linear and quadratic polynomials using basic operations. Create and solve inequalities describing linear, exponential, and quadratic relationships. Justify solutions to equations, inequalities, and systems as a process of reasoning and tool knowledge.

S2.A.1

Use the structure of quadratic and exponential expressions by identifying parts of the expression and interpret in context (ie. Use vertex form of a quadratic to identify the maximum height of an object flying through the air). Use algebraic tools to produce equivalent forms that reveal different properties. (MP 2, 5, 7)

S2.A.2

Use addition, subtraction and multiplication to build new linear and quadratic functions. (MP 4)

S2.A.3

Build symbolic models of linear and quadratic equations and inequalities, and use them to solve problems in and out of context. (MP 3, 4)

S2.A.4

Select and use algebraic and technologic tools to strategically solve quadratic equations and inequalities in one variable. Algebraic tools include taking square roots, completing the square, the quadratic formula, and factoring. (MP 5)

S2.A.5

Select and use algebraic and technologic tools to strategically solve a system of equations consisting of a linear equation and a quadratic equation in two variables. Algebraic tools, including factoring and the quadratic formula, should be used in simpler cases, and technologic tools in more complicated cases. (MP 5)

Number System (NS)

Extend the understanding of integer exponents to encompass rational exponents and explore how this relationship can be used to see patterns, reason about, and solve real world problems. Extend the number system to include all complex numbers. Build on the understanding of operations of real numbers to include imaginary numbers. Emphasize the essential role of complex numbers in solving quadratic equations by satisfying the conditions for the Fundamental Theorem of Algebra.

S2.NS.1

Extend *understanding of the structure* of integer exponent properties to rational exponents. (MP 8)

S2.NS.2

Use the properties of exponents as *tools* to rewrite expressions involving radicals and rational exponents in equivalent forms. (MP 5)

S2.NS.3

Use the Fundamental Theorem of Algebra to *construct arguments* that verify that the solutions to any quadratic equation will always be numbers within the set of complex numbers and will have the form $a + bi$ where a and b are real numbers. (MP 3)

S2.NS.4

Build and use models to solve quadratic equations with real coefficients that have complex solutions, rewriting x^2+4 as $(x + 2i)(x - 2i)$. (MP 4)

Functions (F)

Describe and represent functional relationships and their features. Analyze functions using different representations and contextualize functions and their features when appropriate. Building a function that models a quadratic relationship between two quantities. Develop an understanding of how existing models can be combined to build new functions. Construct and compare linear, quadratic, and exponential models.

S2.F.1

Understand and interpret the features of a function represented graphically, numerically and symbolically using interval notation and inequalities by *applying and removing context*. Focus on these key features: domain, range, intervals of increase and/or decrease, location of absolute maximum and/or absolute minimum, and intercepts. *Build graphic models* that satisfy given key features. (MP 2, 4)

S2.F.2

Determine the average rate of change of a function over a closed interval while attending to precision and reasonableness. Connect the average rate of change with *context* appropriately. (MP 2, 6)

S2.F.3

Build graphic representations that show key features of quadratic, linear, exponential, absolute value and piecewise functions expressed symbolically. *Select and use appropriate tools* to model with and without technology. (MP 4, 5)

S2.F.4

Compare and contrast the properties of quadratic, linear, exponential, absolute value and piecewise functions (represented in different ways), including zeros, extreme values, symmetry and percent rate, when appropriate. *Formulate and ask questions* to highlight the similarities and differences between function types. (MP 1)

S2.F.5

Select and use algebraic tools to create different but equivalent forms of functions that highlight different properties of the function. *Contextualize and decontextualize* these functions to interpret these properties. (MP 2, 5)

S2.F.6

Build a function that models a quadratic relationship. Use *structures and patterns* to build an equation that models explicit reasoning, or a recursive process. *Attend to the precision* of notation and labeling while building and using these models. (MP 4, 6, 7)

S2.F.7

Identify the effect on the graph when replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(x+k)$ by exploring specific values of k for quadratic and absolute value functions in and out of context. *Attend to precision* of language and notation while *constructing arguments* and carefully evaluate whether the ideas and processes observed during exploration are *reasonable*. (MP 3, 6)

S2.F.8

Communicate clear and reasonable arguments to distinguish between situations that can be modeled with quadratic, exponential, and linear functions. Interpret the *structure* of quadratic functions written in different forms with the features of their graphs. (MP 3, 7)

Geometry (G)

Prove geometric theorems using rigid motions and triangle congruence criteria. Develop and apply dilation and similarity to establish right triangle trigonometric ratios and solve real-world problems. Understand all circles are similar to justify relationships between angles, lines, and segments, and define radian measure. Describe circles algebraically on the coordinate grid, connecting to the Pythagorean Theorem and transformations. Explain how area and volume formulas scale with similarity transformations.

S2.G.1

Construct, justify and communicate clear and reasonable arguments to prove geometric theorems. Support refinement of reasoning by asking questions. Develop flexibility in creating logical arguments and proofs of various formats, including: narrative paragraphs, flow diagrams, two-column format, and diagrams without words. (MP 1, 3)

Proofs should focus on:

- a. theorems about lines and angles; vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
- b. theorems about triangles; measures of interior angles of a triangle sum to 180° ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- c. theorems about parallelograms; opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

S2.G.2

Describe and represent the underlying structures and properties that define dilations and similar shapes. Use the structures and properties of dilation and similarity to solve problems. (MP 7)

S2.G.3

Build and use models that include congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. Construct, justify and communicate a clear and reasonable proof of the Pythagorean Theorem using triangle similarity. (MP 3, 4)

S2.G.4

Describe and represent side ratios in right triangles based upon the underlying

structure of similarity. Connect consistency across similar right triangles to definitions of trigonometric ratios for acute angles. (MP 7)

S2.G.5

Verify, justify and communicate the relationships that exist between trigonometric ratios and trigonometric identities. Keep the focus on the relationship between sine and cosine of complementary angles, the Pythagorean Identity, and the tangent ratio in terms of sine and cosine. (MP 3)

S2.G.6

Flexibly, accurately, and efficiently use trigonometric ratios and the Pythagorean Theorem to solve *real-world and mathematical problems* that can be *modeled* with right triangles. (MP 2, 4)

S2.G.7

Construct, justify and communicate a clear and reasonable argument that all circles are similar. *Connect the underlying structures* of similarity to relationships between angles, segments and arcs within circles. (MP 3, 7)

S2.G.8

Describe and represent structures and patterns between inscribed angles, radii, and chords. Relationships include the relationships between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (MP 7)

S2.G.9

Describe and represent, using similarity, the definition of a radian as the ratio of arc length to radius. Create a distinction between arc length and arc measure and be able to *attend to precisions* while relating each of these to radian measure or the formula for the area of a sector. (MP 6, 7)

S2.G.10

Use structures and patterns to work with equations of circles, find different forms of the equations and different defining features of the circle, including the center,

radius, and points located on the circle. Use right triangles with the same hypotenuse length, and the Pythagorean Theorem to derive the equation of a circle. (MP 7)

S2.G.11

Attend to precision and reasonableness with respect to area and volume formulas (circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone) by applying informal arguments about how area and volume scale under similarity transformations. When scale factor k is applied to a figure, area scales by k^2 and volume scales by k^3 . (MP 6)

S2.G.12

Attend to precision and reasonableness while using volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (MP 6)

Data Science (D)

Investigate and analyze situations with large data sets. Describe, represent, and interpret data on two quantitative variables and evaluate results. Use linear, exponential and quadratic functions as models. Identify ways to refine investigative questions, data collection, visualization, or analysis.

S2.D.1

Ask statistical questions to investigate linear, exponential, and quadratic associations between two quantitative variables. (MP 1)

S2.D.2

Select the appropriate statistical tools to create a data collection plan. Collect and organize primary data to investigate the association between two variables and make conjectures. For secondary data, determine if data collected are useful to answer the statistical question of interest. (MP 5)

S2.D.3

Analyze data to explore the association between two quantitative variables. Use scatter plots to determine and explain patterns of association. Use technology to find and interpret the correlation coefficient and *evaluate the reasonableness* and

strength of the model used. Determine whether a linear, exponential or quadratic function is the most *reasonable* model for the data. (MP 6, 8)

S2.D.4

Use the statistical evidence to *evaluate results* from analyses to answer questions about associations between two quantitative variables. Identify ways to refine investigative questions, data collection, visualization, or analysis. Represent the relationship with a curve of best fit and use the model of the equation to solve problems. *Justify* statistical reasoning and results to others in a variety of formats including verbal, written, and visual. (MP 3, 8)

Secondary Math II: Extended Topics

Vectors

Represent and model with vector quantities. Perform operations on vectors.

S2.V.1

Recognize vector quantities as having both magnitude and direction. *Represent* vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes. (MP 7)

S2.V.2

Find and *represent* the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. (MP 7)

S2.V.3

Add and subtract vectors and multiply a vector by a scalar. *Make sense* of the operations by connecting *with and without a context*. (MP 2)

S2.V.4

Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. *Make sense* of this product by connecting it *with and without a context*. (MP 2)

S2.V.5

Solve *real world and mathematical problems* involving velocity and other quantities that can be represented by vectors. (MP 2)

Number System

Perform arithmetic operations with complex numbers. Represent complex numbers and their operations on the complex plane.

S2.NS.5

Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. *Represent* the conjugate and moduli on the complex plane. (MP 7)

S2.NS.6

Represent complex numbers on the complex plane in rectangular form, and *explain* why the rectangular form of a given complex number represents the same number. (MP 3, 4)

S2.NS.7

Represent addition, subtraction, and multiplication geometrically on the complex plane; use properties of this representation for computation. (MP 4)

Geometry

Translate between the geometric description and the equation for a conic section.

S2.G.13

Derive the equation of a parabola given a focus and directrix. (MP 3)

S2.G.14

Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. (MP 3)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of Secondary II, students will be able to:

- Create and interpret equivalent forms of quadratic expressions to reveal different properties of interest. (A)
- Solve quadratic equations and inequalities. (A)
- Compare different representations for linear, exponential and quadratic functions. (F)
- Build multiple representations of quadratic functions. (F)
- Interpret and analyze key features of quadratic functions. (F)
- Use the definition of congruence and triangle congruence criteria to create logical arguments for geometric theorems. (G)
- Develop properties of segments and angles related to circles (G).
- Develop trigonometric ratios for sides of right triangles. (G)
- Describe, represent, and interpret data on two quantitative variables and evaluate results. (D)

Utah's Secondary Mathematics Pathways

The Secondary III course has been divided into two distinct parts to better meet the needs of every student in the state than what a single course can provide.

The first part of Secondary III includes the mathematics that all students would benefit from engaging with (Math for All). These standards are intentionally designed to build the core mathematical knowledge—specifically in algebra, functions, and data science—that ensures all students have the critical and essential skills necessary regardless of their plans after high school. The second part of Secondary III provides students and families with a choice that can better prepare them for their post-secondary aspirations.

The Calculus Pathway is Utah's first option, designed specifically to prepare students for careers in business, biological and physical sciences, engineering and/or mathematics. This course focuses heavily on the core algebraic and trigonometric concepts necessary to equip students with the conceptual understanding and procedural skills necessary for the study of Calculus and other advanced mathematics and science courses. It is also aligned with the courses Utah Institutions of Higher Education (IHE) offer for the above majors, including Math 1050/1060 and Calculus (which can be taken concurrently enrolled or once the student reaches University). AP precalculus and AP Calculus courses are also aligned to this pathway.

Intermediate Algebra is an alternative course in this pathway for students interested in these same careers who prefer a more measured pace before tackling advanced mathematics. This course strengthens the foundational algebra skills necessary to prepare students for AP Pre-Calculus or Math 1050/1060 in the subsequent year.

The data science pathway is the next available option, specifically designed to prepare students for careers in fields such as history, nursing, psychology and the social sciences. This course concentrates on the descriptive and statistical methods essential for understanding and interpreting data. It provides the foundation necessary for students to pursue concurrent enrollment in Stat 1040 or AP Statistics.

The final option involves the creation of a new pathway that builds on the Math for All content towards another career goal. This pathway will eventually include math courses from a variety of CTE backgrounds, alongside the current Applied and Advanced Foundational courses currently acceptable as graduation requirements when a student opts out of Secondary III. The vision going forward is that these Board-approved versions of Secondary III would replace the opt out provision currently in place for the Secondary III course.

LEAs across the state will have flexibility in how to design their schedules around these Secondary III courses, running them as full year experiences or two half year classes, depending on their needs. USBE staff will work with USHE and Utah Institutions of Higher Education to find further alignment and explore the possibility of extended versions of these Secondary III courses that would allow students to earn additional credit or gain access to accelerated coursetaking options upon completion.

What follows are the standards our writing committee has developed for the core pathways. The USBE Mathematics team will work with LEA math communities across the state over the next few years to develop further Secondary III course pathways related to the third option described above.

Standards: Secondary Math III (Math for All Students)

Introduction

In Secondary Math I through Secondary Math III the major works of each grade focus on learning to create, interpret, manipulate, and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations and building functions. Students learn to define functions, describe their features and transformations. Students understand, apply and prove congruence of similarity as defined in terms of geometric transformations. Students also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

The Secondary Math III: Math for All Students standards are foundational content embedded within every Secondary III pathway. These standards are intentionally designed to build the core mathematical knowledge—specifically in algebra, functions, and data science—that ensures all students have the essential skills to access and succeed in any subsequent pathway.

This core foundation equips every student with the mathematical literacy necessary to be an informed and productive citizen.

In Secondary Math III instructional time should focus on four major works: (1) solving algebraic equations (polynomial, logarithmic, and radical); (2) understanding, comparing and representing functions - polynomial, exponential, logarithmic, and trig functions (sine and cosine); (3) describing characteristics of functions; (4) drawing and justifying conclusions from sample surveys, experiments, and observational studies.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In high school, students formulate and ask questions to highlight the difference between various function types. Students ask targeted questions about univariate data displays to explore mathematical ideas. Students ask questions about the underlying structure of geometric objects revealed by geometric constructions.

Mathematical Practice 2: Add or remove context to make sense of mathematics

In high school, students contextualize and decontextualize functions to interpret their properties, connecting symbolic representations to meaningful situations. Students interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to distinguish between situations that can be modeled by various function types. They will construct arguments to justify claims such as why exponential growth eventually surpasses linear, quadratic or the growth in any other polynomial function.

Mathematical Practice 4: Build and use models

In high school, students build and use models in two primary ways. One of these is to use models to connect mathematical concepts to one another. Another way that students use models in high school is to model real-world phenomena.

Students create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. In high school

students use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas between them.

To model real-world phenomena, students engage in mathematical and statistical modeling to represent, analyze, and predict real-world situations. In high school students use geometric shapes, their measures, and their properties to model real world objects and apply geometric methods to solve design problems.

Mathematical Practice 5: Select and use tools appropriately and strategically

In high school, students learn to select and leverage multiple types of tools while engaging in and applying mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential, or quadratic equations for a variable. Students select and use appropriate tools to build representations for various function types with and without technology. Students use mathematical ideas as a tool to validate the appropriateness and usefulness of a model.

Mathematical Practice 6: Attend to precision and reasonableness

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In high school, students describe the structures of linear and exponential expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define

dilations and similar shapes, and then use these structures to solve problems. Students use models to explore and describe attributes of the structure, patterns and relationships of inverses for various function types.

Mathematical Practice 8: Make conjectures and evaluate the results

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to answer investigative questions. Students make conjectures about the reasonableness and strength of the model they choose to describe the association between two quantitative variables. Students develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

Standards: Secondary Math III (Math for All Students)

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Algebra (A)

Interpret the structure of expressions. Extend to polynomial expressions. Create equations and inequalities that describe relationships, using all available types of functions to create such equations. Understand solving equations as a process of reasoning and explain the reasoning. Represent and solve equations and inequalities graphically.

S3.A.1

Interpret polynomial expressions given in factored form or standard form that represent a quantity in terms of a context. *Represent* polynomial expressions by using *technological tools* to produce equivalent forms (standard or factored) that reveal different properties, and *interpret one or more of their parts as a single entity*. (MP 2, 5, 7)

S3.A.2

Create equations and inequalities in one variable to *model a context* and use them to solve problems. Include equations arising from linear, quadratic, simple cubic (e.g., $f(x)=2x^3$), simple exponential, square root and cube root functions. (MP 3)

S3.A.3

For exponential models, express as a logarithm the solution to $f(t)=ab^{ct} = d$ where a , c , and d are numbers, and the base b is 2 or 10; evaluate the logarithm using *technological tools*. (MP 2)

S3.A.4

Solve an equation of the form $f(x)=c$ for a simple function f that has an inverse. *Use the attributes of inverse relationships* to write inverse expressions. Include linear, quadratic, simple cubic (e.g., $f(x)=2x^3$), simple exponential, logarithmic, square root, and cube root functions. (MP7)

Functions (F)

Describe, represent, and analyze functional relationships and their features across different representations, contextualizing and interpreting function expressions in terms of the modeled situation. Build, refine, and assess models relating two quantities. Construct and compare linear, quadratic, polynomial, and exponential functions to solve problems. Model periodic behavior with sine and cosine functions.

S3.F.1

Understand and interpret the features of polynomial, radical or logarithmic functions represented graphically using interval notation and inequalities by *applying and removing context*. Focus on these key features: domain, range, intervals of increase and/or decrease, location of absolute maximum and/or absolute minimum, intercepts and end behavior. *Build graphic models* that satisfy given key features. (MP 2, 4)

S3.F.2

Build and use models of periodic phenomena with sine and cosine functions that connect real world contexts with amplitude, frequency, and midline. *Use structures and patterns* to support understanding of a domain extended beyond 0 to 90 degrees, or 0 to $\frac{\pi}{2}$ radians. (MP 4, 7)

S3.F.3

Understand and interpret key features (period, midline and amplitude) of sine and cosine functions represented graphically. *Build graphic models* that satisfy the given key features. (MP 4)

S3.F.4

Build graphical representations that show key features of polynomial, square root, cube root, exponential, logarithmic, and trigonometric (sine and cosine) functions expressed symbolically. *Select and use appropriate tools* to graph, by hand for simple cases, and using technology for more complicated cases. (MP 4, 5)

S3.F.5

Identify the effect on the graph when replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(x+k)$ by exploring specific values of k for polynomial, square root, cube root, exponential, logarithmic, and trigonometric functions in and out of context. *Attend to precision* of language and notation while *constructing arguments* and carefully evaluate whether the ideas and processes observed during exploration are *reasonable*. (MP 3, 6)

S3.F.6

Use the attributes of inverse relationships to build models (tables, graphs, verbal descriptions and equations) for the inverses of linear, exponential, and quadratic functions. (MP 4, 7)

S3.F.7

Determine which function type (polynomial, square root, cubic root, exponential, or trigonometric) best models a given real world context. (MP 4)

S3.F.8

Justify that a quantity increasing exponentially eventually exceeds a quantity

increasing linearly or quadratically, or (more generally) as a polynomial function. (MP 3)

Geometry (G)

Apply geometric reasoning to real world modeling situations.

S3.G.1

Use geometric shapes, their measures, and their properties to *model* real world objects and solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost). *Use these models to ask targeted and probing questions* with respect to area and volume *in context*. (MP 1, 2, 4)

S3.G.2

Apply concepts of density based on area and volume in *modeling* situations. (MP4)

S3.G.3

Attend to precision and reasonableness while using volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (MP 6)

Data Science (D)

Use probability models to interpret data and compute probabilities of compound and conditional events. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages using technology. Summarize, represent, and interpret data on a single categorical or quantitative variable. Draw and justify conclusions from sample surveys, experiments, and observational studies. Employ simulation to compute p-values and determine statistical significance.

S3.D.1

Describe and represent structures and patterns of data on two categorical variables. *Attend to precision and reasonableness* when summarizing categorical data in two-way frequency models, and *construct, justify, and communicate clear and reasonable arguments* about possible associations and trends in the data. (MP 3, 6, 7)

S3.D.2

Build and use probability models including organized lists, venn-diagrams, tree diagrams, and two-way tables to find or estimate probabilities of compound or conditional events. (MP 4)

S3.D.3

Use multiple representations as tools to describe and represent structure to build understanding of the addition rule, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Understand the conditional probability of A given B as $P(A \cap B)/P(B)$. Interpret independence of A and B as saying that the conditional probability of B given A is the same as the probability of B. (MP 7)

S3.D.4

Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. *Select and use physical and/or tech tools* to estimate areas under the normal curve. (MP 5)

S3.D.5

Formulate statistical investigative questions that are either summary-based or comparative for surveys, observational studies, or experiments using primary and/or secondary data. *Select appropriate statistical tools* to develop a plan for data collection and analysis to address these questions. (MP 1, 5)

S3.D.6

Select appropriate statistical tools to implement a data collection plan for primary data related to an investigative question. When using secondary data, assess its validity and relevance to answering the question. *Construct an argument* justifying the chosen data collection method (survey, observational study or experiment). Understand and apply best practices for designing sample surveys, experiments, and observational studies. (MP 3, 5)

S3.D.7

Attend to precision and reasonableness when summarizing and organizing collected

data to answer an investigative question using tables, graphs, and numerical summary statistics. *Use models* from sample data to estimate population means or proportions. Employ simulation models for random sampling to determine approximate sampling distributions and compute p-values from those distributions. (MP 4, 6)

S3.D.8

Use statistical evidence from the analyses to *evaluate results* and answer questions. Identify ways to refine investigative questions, data collection, visualization, or analysis. *Justify* outcomes or estimates of population characteristics, considering their plausibility compared to chance variation, in a variety of formats (verbal, written, visual). Communicate the interpretation of simulated p-values and determine statistical significance. (MP 3, 8)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of Secondary III (Math for All), students will be able to:

- Solve simple equations (square root, cube root and polynomial). (A)
- Understand, distinguish between and represent functions including polynomial, trigonometric, logarithmic, and inverse functions. (F)
- Interpret and analyze key features of linear, quadratic, exponential, polynomial, logarithmic and trigonometric functions (sine and cosine). (F)
- Use congruency, similarity and right triangle ratios to extend the domain of trigonometric functions (sine and cosine). (F)
- Compute probabilities for compound and conditional events and use them to

interpret data. (D)

- Draw and justify conclusions from sample surveys, experiments, and observational studies. (D)

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Standards: Secondary Math III (Intermediate Algebra Pathway)

Introduction

In Secondary Math I through Secondary Math III the major works of each grade focus on learning to create, interpret, manipulate, and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations and building functions. Students learn to define functions, describe their features and transformations. Students understand, apply and prove congruence of similarity as defined in terms of geometric transformations. Students also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

The Secondary Math III Intermediate Algebra Pathway is designed for students who plan to pursue college degrees in STEM and business fields (including engineering, physical and biological sciences, and mathematics) but prefer a more deliberate pace before advancing to Calculus.

This course concentrates on the essential foundational algebra concepts needed to build a strong mathematical base. It positions students for success in AP Pre-Calculus or College Algebra/Trigonometry the following academic year.

The Intermediate Algebra Pathway shares but does not encompass all standards from the Calculus Pathway course. As a result, some of the Intermediate Algebra Pathway standards are nonsequential. Standards with the prefix S3 represent standards from the *Math for All Students* content. Standards with the prefix CS3 are specific to the Calculus Pathway course.

In the Secondary Math III Intermediate Algebra Pathway, instructional time should focus on four major works: (1) understanding, comparing, and representing polynomial and rational functions; (2) understanding, comparing, and representing

exponential and logarithmic functions; (3) understanding, and representing trigonometric functions; and (4) creating, interpreting, manipulating, and solving algebraic equations.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In high school, students formulate and ask questions to highlight the difference between various function types. Students ask targeted questions about univariate data displays to explore mathematical ideas. Students ask questions about the underlying structure of geometric objects revealed by geometric constructions.

Mathematical Practice 2: Add or remove context to make sense of mathematics

In high school, students contextualize and decontextualize functions to interpret their properties, connecting symbolic representations to meaningful situations. Students interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to distinguish between situations that can be modeled by various function types. They will construct arguments to justify claims such as why exponential growth eventually surpasses linear, quadratic or the growth in any other polynomial function.

Mathematical Practice 4: Build and use models

In high school, students build and use models in two primary ways. One of these is to use models to connect mathematical concepts to one another. Another way that students use models in high school is to model real-world phenomena.

Students create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. In high school students use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas between them.

To model real-world phenomena, students engage in mathematical and statistical modeling to represent, analyze, and predict real-world situations. In high school students use geometric shapes, their measures, and their properties to model real world objects and apply geometric methods to solve design problems.

Mathematical Practice 5: Select and use tools appropriately and strategically

In high school, students learn to select and leverage multiple types of tools while engaging in and applying mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential, or quadratic equations for a variable. Students select and use appropriate tools to build representations for various function types with and without technology. Students use mathematical ideas as a tool to validate the appropriateness and usefulness of a model.

Mathematical Practice 6: Attend to precision and reasonableness

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In high school, students describe the structures of linear and exponential expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define dilations and similar shapes, and then use these structures to solve problems. Students use models to explore and describe attributes of the structure, patterns and relationships of inverses for various function types.

Mathematical Practice 8: Make conjectures and evaluate the results

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to answer investigative questions. Students make conjectures about the reasonableness and strength of the model they choose to describe the association between two quantitative variables. Students develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

Standards: Secondary III (Intermediate Algebra Pathway)

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Algebra (A)

Create equations that describe numbers or relationships, using all types of functions to create such equations. Interpret the structure of polynomial and rational expressions. Write expressions in equivalent forms to solve problems. Write expressions in equivalent forms to solve problems. Perform arithmetic operations on polynomials and understand the relationship between zeros and factors of polynomials. Rewrite rational expressions.

CS3.A.5 (Extension of S3.A.2)

Create equations and inequalities in one variable to *model a context* and use them to solve problems. Include equations arising from polynomial, exponential, rational and radical functions. (MP 3)

CS3.A.6 (Extension of S3.A.3)

For exponential models, express as a logarithm the solution to $f(t)=ab^{ct} = d$ where a , b , c , and d are numbers, $b>0$; evaluate the logarithm using *technological tools*. *Justify* and use the properties of logarithms to generate equivalent expressions, connecting them to the properties of exponents. (MP 2, 3)

CS3.A.7

Understand the inverse relationship between exponents and logarithms. Use this *relationship* to solve problems involving exponents and logarithms. (MP 7)

CS3.A.10

Add, subtract, multiply and divide polynomial and rational functions. *Justify* whether these operations result in new polynomial or rational functions. (MP 3)

CS3.A.11

Identify zeros of polynomial functions *using algebraic tools*, including factoring or the Remainder Theorem. (MP 5)

CS3.A.12

Expand and simplify polynomial sums, products, and powers using *structures and patterns*. (MP 7)

CSS3.A.13

Solve compound inequalities in one variable, including absolute value inequalities *using algebraic or technological tools*. (MP 5)

Number System (NS)

Perform operations with complex numbers and represent them on the complex plane. Apply complex numbers to polynomial identities and equations, emphasizing their essential role in satisfying the Fundamental Theorem of Algebra.

CS3.NS.2

Using the Fundamental Theorem of Algebra, *construct arguments* that verify the solutions to polynomial equations will always be numbers within the set of complex numbers and will have the form $a + bi$ where a and b are real numbers. (MP 3)

CS3.NS.3

Find solutions for polynomials that are limited to integer coefficients *using algebraic or technological tools*. (MP 5)

Functions (F)

Analyze functional relationships and their features using different representations, contextualizing and interpreting function expressions in terms of the modeled situation. Build, refine, and compare polynomial and exponential models to solve problems. Build new functions from existing functions. Extend the domain of trigonometric functions using the unit circle. Model periodic phenomena with trigonometric functions.

CS3.F.9 (Extension of S3.F.1)

Understand and interpret the features of polynomial, radical, rational or logarithmic functions represented graphically using interval notation and inequalities *by applying and removing context*. Focus on these key features: domain, range, intervals of increase and/or decreasing, location of absolute maximum and/or absolute minimum, intercepts, asymptotes, and end behavior. *Build graphic models* that satisfy given key features. (MP 2, 4)

CS3.F.10 (Extension of S3.F.2)

Build and use models of periodic phenomena with trigonometric functions that connect real world contexts with amplitude, frequency, and midline. *Use structures and patterns* to support understanding of a domain extended beyond 0 to 90 degrees, or 0 to $\frac{\pi}{2}$ radians. (MP 4, 7)

CS3.F.11 (Extension of S3.F.3)

Understand and interpret key features (period, midline and amplitude/asymptote)

of trigonometric functions represented graphically. *Build graphic models* that satisfy the given key features. (MP 4)

CS3.F.12 (Extension of S3.F.4)

Build graphical representations that show key features of polynomial, radical, exponential, logarithmic, rational and trigonometric functions expressed symbolically. *Select and use appropriate tools* to graph, by hand for simple cases, and use technology for more complicated cases. (MP 4, 5)

CS3.F.13 (Extension of S3.F.5)

Identify the effect on the graph when replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, $f(x+k)$ by exploring specific values of k for polynomial, radical, exponential, logarithmic, rational, and trigonometric functions *in and out of context*. *Attend to precision* of language and notation while constructing arguments and carefully evaluate whether the ideas and processes observed during exploration are *reasonable*. (MP 3, 6)

CS3.F.16

Build a new function from a composition of functions that *models a relationship* between two quantities. (MP 4)

CS3.F.17

Verify by composition that one function is the inverse of another function. *Build an invertible function* from a non-invertible function by restricting the domain. (MP 4)

Geometry/Trigonometry (G)

Apply geometric reasoning to real world modeling situations. Apply prior understanding of trigonometric ratios to all types of triangles using the Law of Sines and Cosines. Build and use the unit circle to extend the domain of trigonometric functions.

CS3.G.4

Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles *with and without context*. (MP 2)

CS3.G.5

Use special triangles as tools to construct the unit circle with degrees and radians. Describe and represent patterns in the unit circle and use them as tools to evaluate the trigonometric functions at multiples of $\frac{\pi}{4}$ and $\frac{\pi}{6}$. (MP 5, 7)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of Secondary III (Intermediate Algebra), students will be able to:

- Solve equations for all families of functions (A)
- Describe, represent and build polynomials, using zeros and factors. (A)
- Solve equations (rational and radical) as well as inequalities including compound inequalities. (A)
- Understand, distinguish between, and represent functions including polynomial, rational, trigonometric, logarithmic, and inverse functions. (F)
- Interpret and analyze key features of linear, quadratic, exponential, polynomial, rational, logarithmic and trigonometric functions. (F)
- Use congruence, similarity and right triangle ratios to work with the unit circle, and trigonometric identities. (F)

Standards: Secondary Math III (Calculus Pathway)

Introduction

In Secondary Math I through Secondary Math III the major works of each grade focus on learning to create, interpret, manipulate, and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations and building functions. Students learn to define functions, describe their features and transformations. Students understand, apply and prove congruence of similarity as defined in terms of geometric transformations. Students also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

The Secondary Math III Calculus Pathway is designed to establish the essential mathematical foundation for students pursuing college degrees in STEM and business fields, including engineering, physical and biological sciences, and mathematics.

The standards focus heavily on the core algebraic concepts necessary to equip students with the conceptual understanding and procedural skills necessary for the study of calculus and other advanced mathematics and science courses. Completion of this pathway directly prepares students for introductory concurrent enrollment Calculus, AP Pre-Calculus, AP Calculus, or IB Math courses.

Within each domain, the standard numbering begins where the Secondary Math III *Math for All Students* standards conclude. Standards with the prefix *S3* represent standards from the *Math for All Students* content. Standards with the prefix *CS3* are specific to the Calculus Pathway course.

In the Secondary Math III Calculus Pathway, instructional time should focus on five major works: (1) understanding, comparing, and representing polynomial and

rational functions; (2) understanding, comparing, and representing exponential and logarithmic functions; (3) understanding and representing trigonometric functions; (4) creating, interpreting, manipulating, and solving algebraic equations; and (5) understanding and representing parametric and polar functions.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In high school, students formulate and ask questions to highlight the difference between various function types. Students ask targeted questions about univariate data displays to explore mathematical ideas. Students ask questions about the underlying structure of geometric objects revealed by geometric constructions.

Mathematical Practice 2: Add or remove context to make sense of mathematics

In high school, students contextualize and decontextualize functions to interpret their properties, connecting symbolic representations to meaningful situations. Students interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to distinguish between situations that can be modeled by various function types. They construct arguments to justify claims such as why exponential growth eventually surpasses linear, quadratic or the growth in any other polynomial function.

Mathematical Practice 4: Build and use models

In high school, students build and use models in two primary ways. One of these is to use models to connect mathematical concepts to one another. Another way that students use models in high school is to model real-world phenomena.

Students create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. In high school students use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas between them.

To model real-world phenomena, students engage in mathematical and statistical modeling to represent, analyze, and predict real-world situations. In high school students use geometric shapes, their measures, and their properties to model real world objects and apply geometric methods to solve design problems.

Mathematical Practice 5: Select and use tools appropriately and strategically

In high school, students learn to select and leverage multiple types of tools while engaging in and applying mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential, or quadratic equations for a variable. Students select and use appropriate tools to build representations for various function types with and without technology. Students use mathematical ideas as a tool to validate the appropriateness and usefulness of a model.

Mathematical Practice 6: Attend to precision and reasonableness

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In high school, students describe the structures of linear and exponential expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define dilations and similar shapes, and then use these structures to solve problems. Students use models to explore and describe attributes of the structure, patterns and relationships of inverses for various function types.

Mathematical Practice 8: Make conjectures and evaluate the results

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to answer investigative questions. Students make conjectures about the reasonableness and strength of the model they choose to describe the association between two quantitative variables. Students develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

Standards: Secondary Math III (Calculus Pathway)

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Algebra (A)

Analyze and interpret the structure and patterns of polynomial, rational and radical expressions, and rewrite them in equivalent form to simplify and solve problems. Construct and represent series, rational expressions, and higher-degree polynomials using zeros and factors. Use a variety of algebraic and technologic tools to solve rational, radical, and compound inequalities.

CS3.A.5 (Extension of S3.A.2)

Create equations and inequalities in one variable to *model a context* and use them to solve problems. Include equations arising from polynomial, exponential, rational and radical functions. (MP 3)

CS3.A.6 (Extension of S3.A.3)

For exponential models, express as a logarithm the solution to $f(t)=ab^{ct} = d$ where a , b , c , and d are numbers, $b>0$; evaluate the logarithm using *technological tools*. *Justify* and use the properties of logarithms to generate equivalent expressions, connecting them to the properties of exponents. (MP 2, 3)

CS3.A.7

Understand the inverse relationship between exponents and logarithms. Use this *relationship* to solve problems involving exponents and logarithms. (MP 7)

CS3.A.8

Model arithmetic and geometric series, including infinite geometric series algebraically, graphically, and numerically. (MP 4)

CS3.A.9

Build and use expressions including sigma notation to represent the sum of a finite arithmetic or geometric series. (MP 4)

CS3.A.10

Add, subtract, multiply and divide polynomial and rational functions. *Justify* whether these operations result in new polynomial or rational functions. (MP 3)

CS3.A.11

Identify zeros of polynomial functions *using algebraic tools*, including factoring or the Remainder Theorem. (MP 5)

CS3.A.12

Expand and simplify polynomial sums, products, and powers using *structures and patterns*. (MP 7)

CS3.A.13

Solve compound inequalities in one variable, including absolute value inequalities *using algebraic or technological tools*. (MP 5)

Number System (NS)

Build on understanding of real numbers and quadratic equations to include complex solutions of polynomial equations and satisfy the conditions for the Fundamental Theorem of Algebra. Use and perform operations with imaginary and complex numbers and represent them on the complex plane.

CS3.NS.1

Represent addition, subtraction, and multiplication geometrically on the complex plane. Perform algebraic operations with complex numbers on the complex plane. (MP 7)

CS3.NS.2

Using the Fundamental Theorem of Algebra, *construct arguments* that verify the solutions to polynomial equations will always be numbers within the set of complex numbers and will have the form $a + bi$ where a and b are real numbers. (MP 3)

CS3.NS.3

Find solutions for polynomials that are limited to integer coefficients *using algebraic or technological tools*. (MP 5)

Functions (F)

Analyze functional relationships and their features using different representations, contextualizing and interpreting function expressions in terms of the modeled situation. Build, refine, and compare polynomial and exponential models to solve problems. Construct and use parametric and graphic models. Build new functions from existing functions. Extend the domain of trigonometric functions using the unit circle. Model periodic phenomena with trigonometric functions. Use inverse trigonometric functions to solve equations and inequalities.

CS3.F.9 (Extension of S3.F.1)

Understand and interpret the features of polynomial, radical, rational or logarithmic functions represented graphically using interval notation and inequalities by *applying and removing context*. Focus on these key features: domain, range, intervals of increase and/or decreasing, location of absolute maximum and/or absolute minimum, intercepts, asymptotes, and end behavior. *Build graphic models* that satisfy given key features. (MP 2, 4)

CS3.F.10 (Extension of S3.F.2)

Build and use models of periodic phenomena with trigonometric functions that connect real world contexts with amplitude, frequency, and midline. *Use structures and patterns* to support understanding of a domain extended beyond 0 to 90 degrees, or 0 to $\frac{\pi}{2}$ radians. (MP 4, 7)

CS3.F.11 (Extension of S3.F.3)

Understand and interpret key features (period, midline and amplitude/asymptote) of trigonometric functions represented graphically. *Build graphic models* that satisfy the given key features. (MP 4)

CS3.F.12 (Extension of S3.F.4)

Build graphical representations that show key features of polynomial, radical, exponential, logarithmic, rational and trigonometric functions expressed symbolically. *Select and use appropriate tools* to graph, by hand for simple cases, and using technology for more complicated cases. (MP 4, 5)

CS3.F.13 (Extension of S3.F.5)

Identify the effect on the graph when replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, $f(x+k)$ by exploring specific values of k for polynomial, radical, exponential, logarithmic, rational, and trigonometric functions in and out of context. *Attend to precision* of language and notation while *constructing arguments* and carefully evaluate whether the ideas and processes observed during exploration are *reasonable*. (MP 3, 6)

CS3.F.14

Define a curve parametrically, and *create a graphical model*. (MP 4)

CS3.F.15

Build a new function from a composition of functions that *models* a relationship between two quantities. (MP 4)

CS3.F.16

Verify by composition that one function is the inverse of another function. *Build* an invertible function from a non-invertible function by restricting the domain. (MP 3, 4)

CS3.F.17

Use inverse trigonometric functions to solve equations and inequalities *with and without a context*. Determine the *reasonableness* of solutions using *context*. (MP 2, 6)

Geometry/Trigonometry (G)

Apply geometric reasoning to real world modeling situations. Apply prior understanding of trigonometric ratios to all types of triangles using the Law of Sines and Cosines. Build and use the unit circle to extend the domain of trigonometric functions.

CS3.G.4

Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles *with and without context*. (MP 2)

CS3.G.5

Use special triangles as *tools* to construct the unit circle with degrees and radians. *Describe and represent patterns* in the unit circle and use them as *tools* to evaluate the trigonometric functions at multiples of $\frac{\pi}{4}$ and $\frac{\pi}{6}$. (MP 5, 7)

CS3.G.6

Prove the addition and subtraction formulas for sine, cosine and tangent and use them *as tools* to solve problems. (MP 5)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in

future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of Secondary III (Calculus), students will be able to:

- Understand, distinguish between, and represent functions including polynomial, rational, trigonometric, logarithmic, and inverse functions. (F)
- Interpret and analyze key features of linear, quadratic, exponential, polynomial, rational, logarithmic, and trigonometric functions. (F)
- Use congruence, similarity, and right triangle ratios to work with the unit circle and trigonometric identities. (F)
- Describe, represent, and build polynomials using zeros and factors. (A)
- Solve equations (rational and radical) as well as inequalities including compound inequalities. (A)
- Perform operations with complex numbers. (NS)
- Apply trigonometric identities and inverse trigonometric functions in solving equations and inequalities. (F)

Standards: Secondary Math III (Data Science Pathway)

Introduction

In Secondary Math I through Secondary Math III, the major works of each grade focus on learning to create, interpret, manipulate, and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations, and building functions. Students learn to define functions and describe their features and transformations. Students understand, apply and prove congruence of similarity as defined in terms of geometric transformations. Students also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

The Secondary Math III Data Science Pathway prepares students for a diverse range of college majors and careers that rely on understanding, interpreting, and communicating data. This includes fields like nursing, psychology, history, and the social sciences.

This course focuses on descriptive and statistical methods—the essential tools used to analyze, model, and draw insights from data. Completion of this pathway provides students with the foundational skills necessary to successfully enroll in introductory university statistics courses or AP Statistics.

Within each domain the standard numbering begins where the Secondary Math III *Math for All Students* standards conclude. Standards with the prefix *S3* represent standards from the *Math for All Students* content. Standards with the prefix *DS3* are specific to the Data Science Pathway course.

In the Secondary Math III Data Science Pathway instructional time should focus on five major works: (1) data acquisition and preparation; (2) exploratory data analysis; (3) computational thinking; (4) data driven decision making; and (5) model

evaluation and validation.

These five pillars are the foundations of the mathematics students explore in the Data Science Pathway.

- Data Acquisition and Preparation:
 - Create usable datasets by collecting, filtering, cleaning, and scaling data, while prioritizing data privacy.
- Exploratory Data Analysis:
 - Describe patterns and variations within data by analyzing visualizations and identifying trends and features. Provide contextual interpretations of visual and numeric findings. Explore relationships between variables and formulate questions to guide further analysis.
- Computational Thinking:
 - Frame problems for computational solutions. Apply algorithmic thinking to decompose complex problems into manageable parts, enabling reuse of components across applications.
- Data-Driven Decision Making:
 - Build and apply models relevant to specific contexts for real-world applications. Critically evaluate the ethical implications of generalizations. Justify conclusions based on rigorous data analysis.
- Model Evaluation and Validation:
 - Assess the validity and potential bias of models. Consider the role of randomness in inference and computation. Evaluate the representativeness of data in relation to model validity. Distinguish between statistical significance and practical importance. Conduct hypothesis testing to validate findings.

Mathematical Practices

Mathematical Practice 1: Ask questions to explore mathematical ideas

In high school, students formulate and ask questions to highlight the difference between various function types. Students ask targeted questions about univariate data displays to explore mathematical ideas. Students ask questions about the underlying structure of geometric objects revealed by geometric constructions.

Mathematical Practice 2: Add or remove context to make sense of mathematics

In high school, students contextualize and decontextualize functions to interpret their properties, connecting symbolic representations to meaningful situations. Students interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to distinguish between situations that can be modeled by various function types. They construct arguments to justify claims such as why exponential growth eventually surpasses linear, quadratic or the growth in any other polynomial function.

Mathematical Practice 4: Build and use models

In high school, students build and use models in two primary ways. One of these is to use models to connect mathematical concepts to one another. Another way that students use models in high school is to model real-world phenomena.

Students create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. In high school students use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas between them.

To model real-world phenomena, students engage in mathematical and statistical modeling to represent, analyze, and predict real-world situations. In high school students use geometric shapes, their measures, and their properties to model real world objects and apply geometric methods to solve design problems.

Mathematical Practice 5: Select and use tools appropriately and strategically

In high school, students learn to select and leverage multiple types of tools while engaging in and applying mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential, or quadratic equations for a variable. Students select and use appropriate tools to build representations for various function types with and without technology. Students use mathematical ideas as a tool to validate the appropriateness and usefulness of a model.

Mathematical Practice 6: Attend to precision and reasonableness

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

Mathematical Practice 7: Describe and represent structures, patterns, and relationships

In high school, students describe the structures of linear and exponential expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define dilations and similar shapes, and then use these structures to solve problems. Students use models to explore and describe attributes of the structure, patterns

and relationships of inverses for various function types.

Mathematical Practice 8: Make conjectures and evaluate the results

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to answer investigative questions. Students make conjectures about the reasonableness and strength of the model they choose to describe the association between two quantitative variables. Students develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

Standards: Secondary Math III (Data Science)

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Data Science (D)

Understand full data science cycle, focusing on processing, visualizing, and describing both categorical and quantitative data, including measures of center and variability. Build and validate statistical models (e.g., linear regression) and apply concepts like the Central Limit Theorem for statistical inference and hypothesis testing. Critically evaluate conclusions, emphasizing the role of random assignment, bias, and ethical, reproducible practices.

DS3.D.9

Represent categorical and quantitative information in organized tables suitable for analysis. *Attend to precision* in data processing by filtering through logical statements, and transforming data with a detailed set of ordered tasks that are repeatable by others or a computer. (MP 6, 7)

DS3.D.10

Describe univariate displays of large sets of data using technology. *Ask targeted questions* about the displays, including shape (skewed, symmetric), center,

variability, and unusual outcomes. (MP 1)

DS3.D.11

Describe and represent relationships between two quantitative variables. Create visual representations with or without technology, addressing strength, form, direction and unusual observations in the data. (MP 7)

DS3.D.12

Build and use models to describe relationships between two continuous variables using linear regression. Explain which variable(s) should be used for prediction. *Construct reasonable arguments* for appropriate and inappropriate use of the model for practical decision making. (MP 3, 4)

DS3.D.13

Describe the relationship between the distribution of a population, the distribution of a sample, and a sampling distribution (e.g., the distribution of sample means). *Describe* sampling distributions through summary statistics, including the mean and the standard deviation. (MP 1)

DS3.D.14

Use the central limit theorem to *represent* the distribution of sample means of large data sets as a normal distribution. *Assess conjectures* about the population means based on the sample averages through probability calculations from the normal distribution. (MP 7, 8)

DS3.D.15

Use statistical tools to validate the appropriateness and usefulness of a model, including visual and numeric assessments of model fit. Use hypothesis testing and simulation to *develop and assess conjectures* about the model. (MP 5, 8)

DS3.D.16

Evaluate results reached from data analysis. Identify ways to refine investigative questions, data collection, visualization, or analysis. *Justify and communicate clear and reasonable* conclusions. (MP 3)

DS3.D.17

Determine the appropriate scope of generalization based on the method of sampling. *Contextualize* the implications through data storytelling. (MP 2)

DS3.D.18

Understand the role of random assignment in experiments and the implications for cause-and-effect interpretations. *Justify conclusions.* (MP 3)

DS3.D.19

Construct reasonable arguments regarding the issues of data bias and confounding variables in observational studies and their implications for interpretation. (MP 3)

DS3.D.20

Use precision in practices for handling large sets of data that enhance reproducibility and ensure ethical use, including descriptions of alterations, and understand when data may contain sensitive information. (MP 6)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of Secondary III (Data Science), students will be able to:

- Summarize collected data to answer investigative questions using tables, graphs, and numerical summary statistics. Transform data. (D)
- *Describe and represent structures and patterns of* data on two categorical variables, two quantitative variables, and for univariate data. (D)
- *Build and use models* to describe relationships between two continuous variables, including linear regression. Explain which variable(s) should be used for prediction. (D)
- *Develop and use models* from sample data to estimate population means or proportions. (D)
- *Describe* sampling distributions through summary statistics, including the mean and the standard deviation. Use the central limit theorem to *represent* the distribution of sample averages as a normal distribution. (D)

- Use hypothesis testing and simulation to *develop and assess conjectures* about models. Evaluate appropriateness of models. (D)
- Evaluate results from data analysis. *Justify and communicate* conclusions, including appropriate scope of generalization, and *contextualize* the implications. (D)

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