

# Standards: Algebra 1

## Introduction

In high school the major works focus on learning to create, interpret, manipulate, and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations and building functions. Students will learn to define functions, describe their features and transformations. Students will understand, apply and prove congruence of similarity as defined in terms of geometric transformations. Students will also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

In Algebra I instructional time should focus on three major works: (1) solving algebraic equations (linear, exponential and quadratic); (2) understanding, comparing and representing linear, exponential and quadratic functions; (3) describing characteristics of linear, exponential and quadratic functions.

## Mathematical Practices

### *Mathematical Practice 1: Ask questions to explore mathematical ideas*

In high school, students will formulate and ask questions to highlight the difference between various function types. Students will ask targeted questions about univariate data displays to explore mathematical ideas. Students will ask questions about the underlying structure of geometric objects revealed by geometric constructions.

### *Mathematical Practice 2: Add or remove context to make sense of mathematics*

In high school, students contextualize and decontextualize functions to interpret

their properties, connecting symbolic representations to meaningful situations. Students will interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

*Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments*

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to distinguish between situations that can be modeled by various function types. They will construct arguments to justify claims such as why exponential growth eventually surpasses linear, quadratic or the growth in any other polynomial function.

*Mathematical Practice 4: Build and use models*

In high school, students build and use models in two primary ways. One of these is to use models to connect mathematical concepts to one another. Another way that students use models in high school is to model real-world phenomena.

Students create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. In high school students use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas between them.

To model real-world phenomena, students engage in *mathematical and statistical modeling* to represent, analyze, and predict real-world situations. In high school students use geometric shapes, their measures, and their properties to *model* real world objects and apply geometric methods to solve design problems.

*Mathematical Practice 5: Select and use tools appropriately and strategically*

In high school, students learn to select and leverage multiple types of tools while engaging in and applying mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential, or quadratic equations for a variable. Students select and use appropriate tools to build representations for various function types with and without technology. Students use mathematical ideas as a tool to validate the appropriateness and usefulness of a model.

### *Mathematical Practice 6: Attend to precision and reasonableness*

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students will attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

### *Mathematical Practice 7: Describe and represent structures, patterns, and relationships*

In high school, students describe the structures of linear and exponential expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define dilations and similar shapes, and then use these structures to solve problems. Students will use models to explore and describe attributes of the structure, patterns and relationships of inverses for various function types.

### *Mathematical Practice 8: Make conjectures and evaluate the results*

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to answer investigative questions. Students will make conjectures about the reasonableness and strength of the model they choose to describe the association

between two quantitative variables. Students will develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

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### Seeing Structure in Expressions (SSE)

Interpret the structure of expressions. Write expressions in equivalent forms to solve problems.

#### A1.SSE.1 - S1.A.1

*Use the structure* of linear expressions and exponential expressions with integer exponents to identify parts of the expression and interpret their meaning in context. *Interpret* complicated expressions (such as  $P(1+r)^n$ ) by viewing one or more of their parts as a single entity. (MP 7)

#### A1.SSE.2 - S2.A.1

*Use the structure* of quadratic and exponential expressions by identifying parts of the expression and interpret in *context* (ie. Use vertex form of a quadratic to identify the maximum height of an object flying through the air). *Use algebraic tools* to produce equivalent forms that reveal different properties. (MP 2, 5, 7)

### Creating Equations (CE)

Create equations that describe numbers of relationships. Limit these to linear and quadratic equations and inequalities and exponential equations. In the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.

#### A1.CE.1 - S1.A.2

*Build symbolic models* of linear equations and inequalities, and use them to solve problems *in and out of context*. (MP 3, 4)

#### A1.CE.2 - S2.A.3

*Build symbolic models* of linear and quadratic equations and inequalities, and use them to solve problems *in and out of context*. (MF 3, 4)

#### A1.CE.3 - S1.A.3

Use algebraic properties of equality *as tools* to rearrange formulas to isolate a quantity of interest. (MP 5)

#### A1.CE.4 - S1.A.6

Interpret whether a solution of a system of linear equations and inequalities is viable or non-viable *using graphic and analytic models*. (Sk 2) *Use models* of systems of linear equations and inequalities to interpret whether a solution is viable or non-viable. (MP 4)

#### A1.CE.5 - S2.A.2

Use the operations of addition, subtraction and multiplication to *build* new linear and quadratic functions. (MP 4)

### Reasoning with Equations and Inequalities (REI)

Understand solving equations as a process of reasoning and explain the reasoning. Solve equations and inequalities in one variable. Solve systems of equations. Build on student experiences graphing and solving systems of linear equations from middle school. Include cases where the two equations describe the same line - yielding infinitely many solutions - and cases where two equations describe parallel lines - yielding no solution. Represent and solve equations and inequalities

graphically. Extend the work of systems to include solving systems consisting of linear and one nonlinear equation.

#### A1.REI.1 - S1.A.4

Solve linear equations and inequalities *using algebraic tools*, starting from the assumption that a solution exists. *Communicate clear arguments* to justify if an equation does not have one unique solution. Interpret solutions *in and out of context* using both inequality and interval notation. (MP 2, 3, 5)

#### A1.REI.2 - S1.A.5

Solve systems of linear equations *using algebraic tools*. Understand and *justify* how equations are manipulated while maintaining the consistency of the system. (MP 3, 5)

#### A1.REI.3 - S1.A.7

*Build and use graphic models* to analyze and exactly or approximately solve systems of two linear equations as well as systems of one linear equation and one exponential equation in two variables and interpret their solutions. (MP 4)

#### A1.REI.4 - S1.A.8

*Build and use models graphically* to show the solutions to a linear inequality in two variables as a half-plane, and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (MP 4)

#### A1.REI.5 - S2.A.4

*Select and use algebraic and technologic tools* to strategically solve quadratic equations and inequalities in one variable. Algebraic tools include taking square roots, completing the square, the quadratic formula, and factoring. (MP 5)

#### A1.REI.6 - S2.A.5

*Select and use algebraic and technologic tools to strategically* solve a system of equations consisting of a linear equation and a quadratic equation in two variables. Algebraic tools, including factoring and the quadratic formula, should be used in

simpler cases, and technologic tools in more complicated cases. (MP 5)

## Number System (NS)

Extend the understanding of integer exponents to encompass rational exponents and explore how this relationship can be used to see patterns, reason about, and solve real world problems. Extend the number system to include all complex numbers. Build on the understanding of operations of real numbers to include imaginary numbers. Emphasize the essential role of complex numbers in solving quadratic equations by satisfying the conditions for the Fundamental Theorem of Algebra.

### A1.NS.1 - S2.NS.1

Extend *understanding of the structure* of integer exponent properties to rational exponents. (MP 8)

### A1.NS.2 - S2.NS.2

Use the properties of exponents as *tools* to rewrite expressions involving radicals and rational exponents in equivalent forms. (MP 5)

### A1.NS.3 - S2.NS.3

*Construct arguments* that verify, according to the Fundamental Theorem of Algebra, the solutions to any quadratic equation will always be numbers within the set of complex numbers and will have the form  $a + bi$  where  $a$  and  $b$  are real numbers. (MP 3)

### A1.NS.4 - S2.NS.4

*Build and use models* to solve quadratic equations with real coefficients that have complex solutions, rewriting  $x^2+4$  as  $(x + 2i)(x - 2i)$ . (MP 4)

## Interpreting Functions (IF)

Understand the concept of a linear or exponential function and use function notation. Recognize arithmetic and geometric sequences as examples of linear and exponential functions. Interpret linear or exponential functions that arise in applications in terms of a context. Analyze linear or exponential functions using different representations.

### A1.IF.1 - S1.F.1

*Construct and communicate clear arguments* to justify whether a relation is a function. (MP 3)

### A1.IF.2 - S1.F.2

*Understand the structure* of function notation, if  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ . (MP 7)

### A1.IF.3 - S1.F.3

Evaluate functions for inputs in their domains and interpret statements that use function notation *in terms of a context*. (MP 2)

### A1.IF.4 - S1.F.4

*Build and use* explicit and recursive equations to *model* arithmetic and geometric sequences using function notation. *Describe and represent* the domain of these models as a subset of the integers. (MP 4, 7)

### A1.IF.5 - S1.F.5

Understand and interpret the features of a function represented graphically, numerically and symbolically using interval notation and inequalities by *applying and removing context*. Focus on these key features: domain, range, intervals of increase and/or decreasing, location of absolute maximum and/or absolute minimum, and intercepts. *Build graphic models* that satisfy given key features. (MP 2, 4)

### A1.IF.6 - S2.F.5

*Select and use algebraic tools* to create different but equivalent forms of functions that highlight different properties of the function. *Contextualize and decontextualize* these functions to interpret these properties. (MP 2, 5)

### A1.IF.7 - S2.F.2

Determine the average rate of change of a function over a closed interval while attending to precision and reasonableness. Connect the average rate of change

with *context* appropriately. (MP 2, 6)

### A1.IF.8 - S2.F.3

*Build graphic representations* that show key features of quadratic, linear, exponential, absolute value and piecewise functions expressed symbolically. *Select and use appropriate tools* to model with and without technology. (MP 4, 5)

### A1.IF.9 - S2.F.4

Compare and contrast the properties of quadratic, linear, exponential, absolute value and piecewise functions (represented in different ways), including zeros, extreme values, symmetry and percent rate, when appropriate. *Formulate and ask questions* to highlight the similarities and differences between function types. (MP 1)

## Building Functions (BF)

Build linear, exponential, or quadratic functions that model a relationship between two quantities. Build new functions from existing functions.

### A1.BF.1 - S1.F.8

*Build* a function that *models* linear or exponential relationships. Appropriately use explicit and recursive rules to model these relationships and translate between the two forms. (MP 4)

### A1.BF.2 - S2.F.6

*Build* a function that *models* a quadratic relationship. Use *structures and patterns* to build an equation that models explicit reasoning, or a recursive process. *Attend to the precision* of notation and labeling while building and using these models. (MP 4, 6, 7)

### A1.BF.3 - S1.F.9

*Construct arguments* that *use the structures and patterns* of relationships to connect arithmetic sequences to linear functions and geometric sequences to exponential functions. (MP 3, 7)

### A1.BF.4 - S1.F.10

*Use structures and patterns* to identify the effect of  $k$  on the graph when replacing

$f(x)$  by  $f(x) + k$ . Explore specific values of  $k$  for linear and exponential functions in and out of context. (MP 7)

### A1.BF.5 - S2.F.7

Identify the effect on the graph when replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(x+k)$  by exploring specific values of  $k$  for quadratic and absolute value functions in and out of context. *Attend to precision* of language and notation while *constructing arguments* and carefully evaluate whether the ideas and processes observed during exploration are *reasonable*. (MP 3, 6)

### A1.BF.6 - S1.F.11

*Communicate clear and reasonable arguments* to distinguish between situations that can be modeled with linear functions, exponential functions, or neither. Connect the *structure* of linear and exponential functions with their parameters, ( $y=mx+b$ ,  $y=b^x(a)$ ). *Justify* that a quantity increasing exponentially eventually exceeds a quantity increasing linearly. (MP 3, 7)

### A1.BF.7 - S2.F.8

*Communicate clear and reasonable arguments* to distinguish between situations that can be modeled with quadratic, exponential, and linear functions. Interpret the *structure* of quadratic functions written in different forms with the features of their graphs. (MP 3 & 7)

## Algebra 1 Extended Topics

### Matrices

Perform operations on matrices and use matrices in applications. Solve systems of equations.

### A1.M.1 - S1.M.1

Add, subtract, and multiply matrices of appropriate dimensions using *algebraic tools* for smaller matrices and *technology* for larger matrices. (MP 5)

## A1.M.2 - S1.M.2

Use matrices to *represent* and manipulate data, e.g., to represent payoffs or incidences relationships in a network. Multiply matrices by scalars to produce new matrices and *make sense* of this product by connecting it *with and without a context*. (MP 2, 4)

## A1.M.3 - S1.M.3

*Communicate* understanding that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. (MP 3)

## A1.M.4 - S1.M.4

*Communicate* understanding that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. (MP 3)

## A1.M.5 - S1.M.5

Use  $2 \times 2$  matrices to *model* transformations of the plane, and *interpret* the absolute value of the determinant in terms of area. (MP 4, 7)

## A1.M.6 - S1.M.6

*Model* a system of linear equations as a single-matrix equation in a vector variable. (MP 4)

## A1.M.7 - S1.M.7

Solve systems of linear equations up to three variables using matrix row reduction and technology *as tools*. (MP 5)

## A1.M.8 - S1.M.8

Find the inverse of a matrix if it exists *using technology or algebraic tools*, and use the inverse to solve systems of linear equations (using technology for matrices of dimensions  $3 \times 3$  or greater). (MP 5)

## Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of Algebra 1 students will be able to:

- Create and solve equations and inequalities. (A)
- Solve systems of equations and inequalities with pairs of linear equations and inequalities in two variables. (A)
- Create explicit and recursive equations to model arithmetic and geometric sequences. (F)
- Interpret key features of graphs that model relationships between two variables for linear, quadratic and exponential functions. (F)
- Build and interpret multiple representations of linear, quadratic and exponential functions. (F)
- Create and interpret equivalent forms of quadratic expressions to reveal different properties of interest. (A)
- Solve quadratic equations and inequalities. (A)
- Compare different representations for linear, exponential and quadratic functions. (F)

## Standards: Geometry

### Introduction

In high school the major works focus on learning to create, interpret, manipulate,

and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations and building functions. Students will learn to define functions, describe their features and transformations. Students will understand, apply and prove congruence of similarity as defined in terms of geometric transformations. Students will also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

In Geometry instructional time should focus on three major works: (1) understanding and applying congruence as defined in terms of geometric transformations; (2) proving congruence and similarity in terms of geometric transformations; and (3) analyzing and interpreting data that can be used to answer statistical questions of interest related to comparing multiple distributions or exploring associations between two quantitative variables.

## Mathematical Practices

### *Mathematical Practice 1: Ask questions to explore mathematical ideas*

In high school, students will formulate and ask questions to highlight the difference between various function types. Students will ask targeted questions about univariate data displays to explore mathematical ideas. Students will ask questions about the underlying structure of geometric objects revealed by geometric constructions.

### *Mathematical Practice 2: Add or remove context to make sense of mathematics*

In high school, students contextualize and decontextualize functions to interpret their properties, connecting symbolic representations to meaningful situations. Students will interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students

interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

*Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments*

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to distinguish between situations that can be modeled by various function types. They will construct arguments to justify claims such as why exponential growth eventually surpasses linear, quadratic or the growth in any other polynomial function.

*Mathematical Practice 4: Build and use models*

In high school, students build and use models in two primary ways. One of these is to use models to connect mathematical concepts to one another. Another way that students use models in high school is to model real-world phenomena.

Students create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. In high school students use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas between them.

To model real-world phenomena, students engage in *mathematical and statistical modeling* to represent, analyze, and predict real-world situations. In high school students use geometric shapes, their measures, and their properties to *model* real world objects and apply geometric methods to solve design problems.

*Mathematical Practice 5: Select and use tools appropriately and strategically*

In high school, students learn to select and leverage multiple types of tools while engaging in and applying mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential,

or quadratic equations for a variable. Students select and use appropriate tools to build representations for various function types with and without technology. Students use mathematical ideas as a tool to validate the appropriateness and usefulness of a model.

*Mathematical Practice 6: Attend to precision and reasonableness*

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students will attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

*Mathematical Practice 7: Describe and represent structures, patterns, and relationships*

In high school, students describe the structures of linear and exponential expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define dilations and similar shapes, and then use these structures to solve problems. Students will use models to explore and describe attributes of the structure, patterns and relationships of inverses for various function types.

*Mathematical Practice 8: Make conjectures and evaluate the results*

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to answer investigative questions. Students will make conjectures about the reasonableness and strength of the model they choose to describe the association between two quantitative variables. Students will develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

More details about the standards, including relevant vocabulary, representations

that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

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### Congruence (CO)

Experiment with transformations in the plane. Build on student experience with rigid transformations from earlier grades. Understand congruence in terms of rigid motions. Rigid motions are at the foundation of the definition of congruence. Reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can be used to prove other theorems. Make geometric constructions. Prove Geometric theorems. Encourage multiple ways of writing proofs, such as narrative paragraphs, flow diagrams, two-column format, and diagrams without words. Focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

#### G.CO.1 - S1.G.1

*Describe and represent the underlying structure of translations, rotations and reflections, in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (MP 7)*

#### G.CO.2 - S1.G.2

*Construct and clearly communicate definitions of the rigid transformations to show that these transformations are functions that take points in the plane as inputs and*

give corresponding points as outputs. (MP 3)

### G.CO.3 - S1.G.3

*Select and use tools to strategically perform rigid transformations. Attend to precision* when identifying and describing the transformation or the sequence of transformations to determine whether or not two figures are congruent. Understand that the image created from a rigid transformation or a sequence of rigid transformations is congruent to the pre-image. (MP 5, 6)

### G.CO.4 - S1.G.4

*Construct and communicate clear and reasonable arguments* about reflection and rotational symmetries of parallelograms and regular polygons. Use reflections and rotations to informally *justify* attributes of parallelograms including rhombus, rectangles and squares. (MP 3)

### G.CO.5 - S1.G.5

Precisely construct geometric objects including equilateral triangle, rhombus, square, parallel line, perpendicular bisector, and angle bisector. *Communicate clearly* about the use of geometric construction tools and analyze the underlying structure of geometric objects as revealed by the tools. (MP 3)

### G.CO.6 - S1.G.6

*Construct, justify and communicate clear arguments* that show how the criteria for triangle congruence (ASA, SAS, SSS) can be established using the definition of congruence in terms of rigid motions. (MP 3)

### G.CO.7 - S1.G.7

*Justify* whether or not a quadrilateral has the attributes of a square, rectangle, rhombus or parallelogram. Connect the Pythagorean Theorem to the distance formula and utilize slope along with distance as a means for *justification*. (MP 3)

### G.CO.8 - S2.G.1

*Construct, justify and communicate clear and reasonable arguments* to prove geometric theorems. Support refinement of reasoning *by asking questions*. Develop

flexibility in creating logical arguments and proofs of various formats, including: narrative paragraphs, flow diagrams, two-column format, and diagrams without words. (MP 1, 3)

Proofs should focus on:

- a. theorems about lines and angles; vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
- b. theorems about triangles; measures of interior angles of a triangle sum to  $180^\circ$ ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- c. theorems about parallelograms; opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

### Similarity, Right Triangles, and Trigonometry (SRT)

Understand similarity in terms of similarity transformations. Prove theorems involving similarity. Define trigonometric ratios and solve problems involving right triangles.

#### G.SRT.1 - S2.G.2

*Describe and represent the underlying structures and properties that define dilations and similar shapes. Use the structures and properties of dilation and similarity to solve problems. (MP 7)*

#### G.SRT.2 - S2.G.3

*Build and use models that include congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. Construct, justify and communicate a clear and reasonable proof of the Pythagorean Theorem using*

triangle similarity. (MP 3, 4)

### G.SRT.3 - S2.G.4

*Describe and represent* side ratios in right triangles based upon the *underlying structure* of similarity. Connect consistency across similar right triangles to definitions of trigonometric ratios for acute angles. (MP 7)

### G.SRT.4 - S2.G.5

*Verify, justify and communicate* the relationships that exist between trigonometric ratios and trigonometric identities. Keep the focus on the relationship between sine and cosine of complementary angles, the Pythagorean Identity, and the tangent ratio in terms of sine and cosine. (MP 3)

### G.SRT.5 - S2.G.6

Flexibly, accurately, and efficiently use trigonometric ratios and the Pythagorean Theorem to solve *real-world and mathematical problems* that can be *modeled* with right triangles. (MP 2, 4)

## Circles (C)

Understand and apply theorems about circles. Find arc lengths and areas of sectors of circles. Use this as a basis for introducing the radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.

### G.C.1 - S2.G.7

*Construct, justify and communicate a clear and reasonable argument* that all circles are similar. *Connect the underlying structures* of similarity to relationships between angles, segments and arcs within circles. (MP 3, 7)

### G.C.2 - S2.G.8

*Describe and represent structures and patterns* between inscribed angles, radii, and chords. Relationships include the relationships between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (MP 7)

**G.C.3 - S2.G.9**

*Describe and represent*, using similarity, the definition of a radian as the ratio of arc length to radius. Create a distinction between arc length and arc measure and be able to *attend to precisions* while relating each of these to radian measure or the formula for the area of a sector. (MP 6, 7)

**G.C.4 - S2.G.10**

*Use structures and patterns* to work with equations of circles, find different forms of the equations and different defining features of the circle, including the center, radius, and points located on the circle. Use right triangles with the same hypotenuse length, and the Pythagorean Theorem to derive the equation of a circle. (MP 7)

**Geometric Measurement and Dimension (GMD)**

Examine volume formulas and use them to solve problems.

**G.GMD.1 - S2.G.11**

*Attend to precision and reasonableness* with respect to area and volume formulas (circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone) by applying informal arguments about how area and volume scale under similarity transformations. When scale factor  $k$  is applied to a figure, area scales by  $k^2$  and volume scales by  $k^3$ . (MP 6)

**G.GMD.2 - S3.G.1**

Use geometric shapes, their measures, and their properties to *model* real world objects and solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost). *Use these models* to ask *targeted and probing questions* with respect to area and volume *in context*. (MP 1, 2, 4)

**G.GMD.3 - S3.G.2**

Apply concepts of density based on area and volume in *modeling* situations. (MP 4)

**G.GMD.4 - S3.G.**

*Attend to precision and reasonableness* while using volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (MP 6)

## Data Science (D)

Investigate and analyze situations with large data sets. Describe, represent, and interpret data on a single quantitative variable and on two quantitative variables and evaluate results. Use linear, exponential and quadratic functions as models. Identify ways to refine investigative questions, data collection, visualization, or analysis.

### G.D.1 - S1.D.1

*Ask statistical questions* to investigate situations that can be explored using random samples from populations to make inferences about differences between two populations or associations between two quantitative variables. Explain and *justify* the distinction between correlation and causation. (MP 1, 3)

### G.D.2 - S1.D.2

*Select the appropriate statistical tools* to create a data collection plan. Collect and organize primary data for the statistical investigative question of interest or determine the validity that the data collected is useful to answer the statistical question of interest when using secondary data. (MP 5)

### G.D.3 - S1.D.3

Compare two or more distributions with *precision* by selecting graphical displays (dot plots, histograms, and modified box plots to show outliers) that highlight features of interest. Analyze data by comparing measures of center (mean and median) and measures of variability (range, inter-quartile range and standard deviation) that are appropriate for the shape (skew, symmetry, outliers, modes) of the data distribution. Recognize how standard deviation builds on the mean absolute deviation and is another measure of how data values vary in a distribution. (MP 6)

### G.D.4 - S1.D.4

Analyze data to explore the association between two quantitative variables by using a scatterplot to determine and describe patterns of association such as clustering, positive or negative association, linear or nonlinear association, and effects of outliers. Use technology to find and interpret the correlation coefficient

and use it to assess the strength of the linear relationship. *Attend to precision and reasonableness* with calculations. (MP 6)

#### G.D.5 - S1.D.5

Analyze and use statistical evidence to *evaluate results* and answer statistical investigative questions about the differences between two or more population parameters. Identify ways to refine the question, data collection, visualization, or analysis. (MP 8)

#### G.D.6 - S1.D.6

*Justify* statistical reasoning to *evaluate results* from analyses to answer the statistical investigative question about associations between two quantitative variables. For scatter plots that suggest a linear association, represent the relationship with a line of best fit and use the model of the linear equation to solve problems, including interpreting the slope and intercept in the context of the data. *Justify* statistical reasoning and results to others in a variety of formats including verbal, written, and visual. (MP 3, 8)

#### G.D.7 - S2.D.1

*Ask statistical questions* to investigate linear, exponential, and quadratic associations between two quantitative variables. (MP 1)

#### G.D.8 - S2.D.2

*Select the appropriate statistical tools* to create a data collection plan. Collect and organize primary data to investigate the association between two variables and make conjectures. For secondary data, determine if data collected are useful to answer the statistical question of interest. (MP 5)

#### G.D.9 - S2.D.3

Analyze data to explore the association between two quantitative variables. Use scatter plots to determine and explain patterns of association. Use technology to find and interpret the correlation coefficient and *evaluate the reasonableness* and strength of the model used. Determine whether a linear, exponential or quadratic function is the most *reasonable* model for the data. (MP 6, 8)

### G.D.10 - S2.D.4

Use the statistical evidence to *evaluate results* from analyses to answer questions about associations between two quantitative variables. Identify ways to refine investigative questions, data collection, visualization, or analysis. Represent the relationship with a curve of best fit and use the model of the equation to solve problems. *Justify* statistical reasoning and results to others in a variety of formats including verbal, written, and visual. (MP 3, 8)

## Geometry: Extended Topics

### Vectors

Represent and model with vector quantities. Perform operations on vectors.

#### G.V.1 - S2.V.1

Recognize vector quantities as having both magnitude and direction. *Represent* vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes. (MP 7)

#### G.V.2 - S2.V.2

Find and *represent* the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. (MP 7)

#### G.V.3 - S2.V.3

Add and subtract vectors and multiply a vector by a scalar. *Make sense* of the operations by connecting *with and without a context*. (MP 2)

#### G.V.4 - S2.V.4

Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. *Make sense* of this product by connecting it *with and without a context*. (MP 2)

G.V.5 - S2.V.5

Solve *real world and mathematical problems* involving velocity and other quantities that can be represented by vectors. (MP 2)

## Conic Sections

Translate between the geometric description and the equation for a conic section.

G.CS.1 - S2.G.12

*Derive* the equation of a parabola given a focus and directrix. (MP 3)

G.CS.2 - S2.G.13

*Derive* the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. (MP 3)

## Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of Geometry students will be able to:

- Establish triangle congruence criteria which will be used to prove other theorems. (G)
- Describe the transformation or the sequence of transformations to prove two figures are congruent. (G)
- Use the definition of congruence and triangle congruence criteria to create logical arguments for geometric theorems. (G)
- Develop properties of segments and angles related to circles (G).
- Develop trigonometric ratios for sides of right triangles. (G)

- Describe, represent, and interpret data on a single variable and evaluate results. (D)
- Describe, represent, and interpret data on two quantitative variables and evaluate results. (D)

1.16.25 Draft

# Utah's Secondary Mathematics Pathways

The Algebra 2 course has been divided into two distinct parts to better meet the needs of every student in the state than what a single course can provide.

The first set of standards listed for Algebra 2 includes the mathematics that all students benefit from engaging with (Math for All). These standards are intentionally designed to build the core mathematical knowledge—specifically in algebra, functions, and data science—that ensures all students have the critical and essential skills necessary regardless of their plans after high school. The second part of Algebra 2 provides students and families with a choice that can better prepare them for their post-secondary aspirations.

The Intermediate Algebra Pathway is a traditional Algebra 2 course that prepares students for precalculus. It is intended for those interested in the same careers as the Calculus Pathway described below, who prefer a more measured pace before tackling advanced mathematics. This course strengthens the foundational algebra skills necessary to prepare students for AP Pre-Calculus or Math 1050/1060 in the subsequent year.

The Calculus Pathway is another option, designed specifically to prepare students for careers in business, biological and physical sciences, engineering and/or mathematics. This course focuses heavily on the core algebraic and trigonometric concepts necessary to equip students with the conceptual understanding and procedural skills necessary for the study of Calculus and other advanced mathematics and science courses. It is also aligned with the courses Utah Institutions of Higher Education (IHE) offer for the above majors, including Math 1050/1060 and Calculus (which can be taken concurrently enrolled or once the student reaches University). AP precalculus and AP Calculus courses are also aligned to this pathway.

The Data Science Pathway is the next available option, specifically designed to prepare students for careers in fields such as history, nursing, psychology and the social sciences. This course concentrates on the descriptive and statistical methods essential for understanding and interpreting data. It provides the foundation necessary for students to pursue concurrent enrollment in Stat 1040 or AP

Statistics.

The final option involves the creation of a new pathway that builds on the Math for All content towards another career goal. This pathway will eventually include math courses from a variety of CTE backgrounds, alongside the current Applied and Advanced Foundational courses currently acceptable as graduation requirements when a student opts out of Algebra 2. The vision going forward is that these Board-approved versions of Algebra 2 would replace the opt out provision currently in place for the Algebra 2 course.

LEAs across the state will have flexibility in how to design their schedules around these Algebra 2 courses, running them as full year experiences or two half year classes, depending on their needs. USBE staff will work with USHE and Utah Institutions of Higher Education to find further alignment and explore the possibility of extended versions of these Algebra 2 courses that would allow students to earn additional credit or gain access to accelerated coursetaking options upon completion.

What follows are the standards our writing committee has developed for the core pathways. The USBE Mathematics team will work with LEA math communities across the state over the next few years to develop further Algebra 2 course pathways related to the third option described above.

# Standards: Algebra 2 (Math for All Students)

## Introduction

In high school the major works focus on learning to create, interpret, manipulate, and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations and building functions. Students learn to define functions, describe their features and transformations. Students understand, apply and prove congruence of similarity as defined in terms of geometric transformations. Students also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

The Algebra 2: Math for All Students standards are foundational content embedded within every Algebra 2 pathway. These standards are intentionally designed to build the core mathematical knowledge—specifically in algebra, functions, and data science—that ensures all students have the essential skills to access and succeed in any subsequent pathway.

This core foundation equips every student with the mathematical literacy necessary to be an informed and productive citizen.

In Algebra 2 instructional time should focus on four major works: (1) solving algebraic equations (polynomial, logarithmic, and radical); (2) understanding, comparing and representing functions - polynomial, exponential, logarithmic, and trig functions (sine and cosine); (3) describing characteristics of functions; (4) drawing and justifying conclusions from sample surveys, experiments, and observational studies.

## Mathematical Practices

*Mathematical Practice 1: Ask questions to explore mathematical ideas*

In high school, students formulate and ask questions to highlight the difference between various function types. Students ask targeted questions about univariate data displays to explore mathematical ideas. Students ask questions about the underlying structure of geometric objects revealed by geometric constructions.

*Mathematical Practice 2: Add or remove context to make sense of mathematics*

In high school, students contextualize and decontextualize functions to interpret their properties, connecting symbolic representations to meaningful situations. Students interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

*Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments*

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to distinguish between situations that can be modeled by various function types. They will construct arguments to justify claims such as why exponential growth eventually surpasses linear, quadratic or the growth in any other polynomial function.

*Mathematical Practice 4: Build and use models*

In high school, students build and use models in two primary ways. One of these is to use models to connect mathematical concepts to one another. Another way that students use models in high school is to model real-world phenomena.

Students create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. In high school students use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas between them.

To model real-world phenomena, students engage in mathematical and statistical modeling to represent, analyze, and predict real-world situations. In high school students use geometric shapes, their measures, and their properties to model real world objects and apply geometric methods to solve design problems.

*Mathematical Practice 5: Select and use tools appropriately and strategically*

In high school, students learn to select and leverage multiple types of tools while engaging in and applying mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential, or quadratic equations for a variable. Students select and use appropriate tools to build representations for various function types with and without technology. Students use mathematical ideas as a tool to validate the appropriateness and usefulness of a model.

*Mathematical Practice 6: Attend to precision and reasonableness*

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

*Mathematical Practice 7: Describe and represent structures, patterns, and relationships*

In high school, students describe the structures of linear and exponential expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define dilations and similar shapes, and then use these structures to solve problems. Students use models to explore and describe attributes of the structure, patterns

and relationships of inverses for various function types.

### *Mathematical Practice 8: Make conjectures and evaluate the results*

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to answer investigative questions. Students make conjectures about the reasonableness and strength of the model they choose to describe the association between two quantitative variables. Students develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

## Standards: Algebra 2 (Math for All Students)

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

### Algebra (A)

Interpret the structure of expressions. Extend to polynomial expressions. Create equations and inequalities that describe relationships, using all available types of functions to create such equations. Understand solving equations as a process of reasoning and explain the reasoning. Represent and solve equations and inequalities graphically.

#### A2.A.1 - S3.A.1

*Interpret* polynomial expressions given in factored form or standard form that represent a quantity in terms of a context. *Represent* polynomial expressions by using *technological tools* to produce equivalent forms (standard or factored) that reveal different properties, and *interpret one or more of their parts as a single entity*. (MP 2, 5, 7)

#### A2.A.2 - S3.A.2

Create equations and inequalities in one variable to *model a context* and use them to solve problems. Include equations arising from linear, quadratic, simple cubic (e.g.,  $f(x)=2x^3$ ), simple exponential, square root and cube root functions. (MP 3)

#### A2.A.3 - S3.A.3

For exponential models, express as a logarithm the solution to  $f(t)=ab^{ct} = d$  where  $a$ ,  $c$ , and  $d$  are numbers, and the base  $b$  is 2 or 10; evaluate the logarithm using *technological tools*. (MP 2)

#### A2.A.4 - S3.A.4

Solve an equation of the form  $f(x)=c$  for a simple function  $f$  that has an inverse. *Use the attributes of inverse relationships* to write inverse expressions. Include linear, quadratic, simple cubic (e.g.,  $f(x)=2x^3$ ), simple exponential, logarithmic, square root, and cube root functions. (MP7)

### Functions (F)

Describe, represent, and analyze functional relationships and their features across different representations, contextualizing and interpreting function expressions in terms of the modeled situation. Build, refine, and assess models relating two quantities. Construct and compare linear, quadratic, polynomial, and exponential functions to solve problems. Model periodic behavior with sine and cosine functions.

#### A2.F.1 - S3.F.1

Understand and interpret the features of polynomial, radical or logarithmic functions represented graphically using interval notation and inequalities by *applying and removing context*. Focus on these key features: domain, range, intervals of increase and/or decrease, location of absolute maximum and/or absolute minimum, intercepts and end behavior. *Build graphic models* that satisfy given key features. (MP 2, 4)

#### A2.F.2 - S3.F.2

*Build and use models* of periodic phenomena with sine and cosine functions that connect real world contexts with amplitude, frequency, and midline. *Use structures*

*and patterns* to support understanding of a domain extended beyond 0 to 90 degrees, or 0 to  $\frac{\pi}{2}$  radians. (MP 4, 7)

### A2.F.3 - S3.F.3

Understand and interpret key features (period, midline and amplitude) of sine and cosine functions represented graphically. *Build graphic models* that satisfy the given key features. (MP 4)

### A2.F.4 - S3.F.4

*Build graphical representations* that show key features of polynomial, square root, cube root, exponential, logarithmic, and trigonometric (sine and cosine) functions expressed symbolically. *Select and use appropriate tools* to graph, by hand for simple cases, and using technology for more complicated cases. (MP 4, 5)

### A2.F.5 - S3.F.5

Identify the effect on the graph when replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(x+k)$  by exploring specific values of  $k$  for polynomial, square root, cube root, exponential, logarithmic, and trigonometric functions in and out of context. *Attend to precision* of language and notation while *constructing arguments* and carefully evaluate whether the ideas and processes observed during exploration are *reasonable*. (MP 3, 6)

### A2.F.6 - S3.F.6

*Use the attributes of inverse relationships to build models* (tables, graphs, verbal descriptions and equations) for the inverses of linear, exponential, and quadratic functions. (MP 4, 7)

### A2.F.7 - S3.F.7

Determine which function type (polynomial, square root, cubic root, exponential, or trigonometric) best models a given real world context. (MP 4)

### A2.F.8 - S3.F.8

*Justify* that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically, or (more generally) as a polynomial function. (MP 3)

## Data Science (D)

Use probability models to interpret data and compute probabilities of compound and conditional events. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages using technology. Summarize, represent, and interpret data on a single categorical or quantitative variable. Draw and justify conclusions from sample surveys, experiments, and observational studies. Employ simulation to compute p-values and determine statistical significance.

### A2.D.1 - S3.D.1

*Describe and represent structures and patterns of data on two categorical variables. Attend to precision and reasonableness when summarizing categorical data in two-way frequency models, and construct, justify, and communicate clear and reasonable arguments about possible associations and trends in the data. (MP 3, 6, 7)*

### A2.D.2. - S3.D.2

*Build and use probability models including organized lists, venn-diagrams, tree diagrams, and two-way tables to find or estimate probabilities of compound or conditional events. (MP 4)*

### A2.D.3 - S3.D.3

*Use multiple representations as tools to describe and represent structure to build understanding of the addition rule,  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ . Understand the conditional probability of A given B as  $P(A \cap B)/P(B)$ . Interpret independence of A and B as saying that the conditional probability of B given A is the same as the probability of B. (MP 7)*

### A2.D.4 - S3.D.4

Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. *Select and use physical and/or tech tools to estimate areas under the normal curve. (MP 5)*

### A2.D.5 - S3.D.5

*Formulate statistical investigative questions* that are either summary-based or comparative for surveys, observational studies, or experiments using primary and/or secondary data. Select *appropriate statistical tools* to develop a plan for data collection and analysis to address these questions. (MP 1, 5)

#### A2.D.6 - S3.D.6

*Select appropriate statistical tools* to implement a data collection plan for primary data related to an investigative question. When using secondary data, assess its validity and relevance to answering the question. *Construct an argument* justifying the chosen data collection method (survey, observational study or experiment). Understand and apply best practices for designing sample surveys, experiments, and observational studies. (MP 3, 5)

#### A.2.D.7 - S3.D.7

*Attend to precision and reasonableness* when summarizing and organizing collected data to answer an investigative question using tables, graphs, and numerical summary statistics. *Use models* from sample data to estimate population means or proportions. Employ simulation models for random sampling to determine approximate sampling distributions and compute p-values from those distributions. (MP 4, 6)

#### A2.D. 8 - S3.D.8

Use statistical evidence from the analyses to *evaluate results* and answer questions. Identify ways to refine investigative questions, data collection, visualization, or analysis. *Justify* outcomes or estimates of population characteristics, considering their plausibility compared to chance variation, in a variety of formats (verbal, written, visual). Communicate the interpretation of simulated p-values and determine statistical significance. (MP 3, 8)

## Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide

interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of Algebra 2 (Math for All), students will be able to:

- Solve simple equations (square root, cube root and polynomial). (A)
- Understand, distinguish between and represent functions including polynomial, trigonometric, logarithmic, and inverse functions. (F)
- Interpret and analyze key features of linear, quadratic, exponential, polynomial, logarithmic and trigonometric functions (sine and cosine). (F)
- Use congruency, similarity and right triangle ratios to extend the domain of trigonometric functions (sine and cosine). (F)
- Compute probabilities for compound and conditional events and use them to interpret data. (D)
- Draw and justify conclusions from sample surveys, experiments, and observational studies. (D)

# Standards: Algebra 2 (Intermediate Algebra Pathway)

## Introduction

In high school the major works focus on learning to create, interpret, manipulate, and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations and building functions. Students learn to define functions, describe their features and transformations. Students understand, apply and prove congruence of similarity as defined in terms of geometric transformations. Students also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

The Algebra 2 Intermediate Algebra Pathway is designed for students who plan to pursue college degrees in STEM and business fields (including engineering, physical and biological sciences, and mathematics) but prefer a more deliberate pace before advancing to Calculus.

This course concentrates on the essential foundational algebra concepts needed to build a strong mathematical base. It positions students for success in AP Pre-Calculus or College Algebra/Trigonometry the following academic year.

The Intermediate Algebra Pathway shares but does not encompass all standards from the Calculus Pathway course. As a result, some of the Intermediate Algebra Pathway standards are nonsequential. Standards with the prefix *A2* represent standards from the *Algebra 2 Math for All Students* content. Standards with the prefix *CA2* are specific to the Calculus Pathway course.

In the Algebra 2 Intermediate Algebra Pathway, instructional time should focus on four major works: (1) understanding, comparing, and representing polynomial and rational functions; (2) understanding, comparing, and representing exponential and

logarithmic functions; (3) understanding, and representing trigonometric functions; and (4) creating, interpreting, manipulating, and solving algebraic equations.

## Mathematical Practices

### *Mathematical Practice 1: Ask questions to explore mathematical ideas*

In high school, students formulate and ask questions to highlight the difference between various function types. Students ask targeted questions about univariate data displays to explore mathematical ideas. Students ask questions about the underlying structure of geometric objects revealed by geometric constructions.

### *Mathematical Practice 2: Add or remove context to make sense of mathematics*

In high school, students contextualize and decontextualize functions to interpret their properties, connecting symbolic representations to meaningful situations. Students interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

### *Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments*

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to distinguish between situations that can be modeled by various function types. They will construct arguments to justify claims such as why exponential growth eventually surpasses linear, quadratic or the growth in any other polynomial function.

### *Mathematical Practice 4: Build and use models*

In high school, students build and use models in two primary ways. One of these is

to use models to connect mathematical concepts to one another. Another way that students use models in high school is to model real-world phenomena.

Students create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. In high school students use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas between them.

To model real-world phenomena, students engage in mathematical and statistical modeling to represent, analyze, and predict real-world situations. In high school students use geometric shapes, their measures, and their properties to model real world objects and apply geometric methods to solve design problems.

### *Mathematical Practice 5: Select and use tools appropriately and strategically*

In high school, students learn to select and leverage multiple types of tools while engaging in and applying mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential, or quadratic equations for a variable. Students select and use appropriate tools to build representations for various function types with and without technology. Students use mathematical ideas as a tool to validate the appropriateness and usefulness of a model.

### *Mathematical Practice 6: Attend to precision and reasonableness*

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

### *Mathematical Practice 7: Describe and represent structures, patterns, and*

### *relationships*

In high school, students describe the structures of linear and exponential expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define dilations and similar shapes, and then use these structures to solve problems. Students use models to explore and describe attributes of the structure, patterns and relationships of inverses for various function types.

### *Mathematical Practice 8: Make conjectures and evaluate the results*

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to answer investigative questions. Students make conjectures about the reasonableness and strength of the model they choose to describe the association between two quantitative variables. Students develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

## **Standards: Algebra 2 (Intermediate Algebra Pathway)**

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

### **Algebra (A)**

Create equations that describe numbers or relationships, using all types of functions to create such equations. Interpret the structure of polynomial and rational expressions. Write expressions in equivalent forms to solve problems. Write expressions in equivalent forms to solve problems. Perform arithmetic operations on polynomials and understand the relationship between zeros and factors of polynomials. Rewrite rational expressions.

**CA2.A.5 - CS3.A.5 (Extension of S3.A.2)**

Create equations and inequalities in one variable to *model a context* and use them to solve problems. Include equations arising from polynomial, exponential, rational and radical functions. (MP 3)

#### CA2.A.6 - CS3.A.6 (Extension of S3.A.3)

For exponential models, express as a logarithm the solution to  $f(t)=ab^{ct} = d$  where  $a$ ,  $b$ ,  $c$ , and  $d$  are numbers,  $b>0$ ; evaluate the logarithm using *technological tools*. *Justify* and use the properties of logarithms to generate equivalent expressions, connecting them to the properties of exponents. (MP 2, 3)

#### CA2.A.7 - CS3.A.7

Understand the inverse relationship between exponents and logarithms. Use this *relationship* to solve problems involving exponents and logarithms. (MP 7)

#### CA2.A.10 - CS3.A.10

Add, subtract, multiply and divide polynomial and rational functions. *Justify* whether these operations result in new polynomial or rational functions. (MP 3)

#### CA2.A.11 - CS3.A.11

Identify zeros of polynomial functions *using algebraic tools*, including factoring or the Remainder Theorem. (MP 5)

#### CA2.A.12 - CS3.A.12

Expand and simplify polynomial sums, products, and powers using *structures and patterns*. (MP 7)

#### CA2.A.5 - CSS3.A.13

Solve compound inequalities in one variable, including absolute value inequalities *using algebraic or technological tools*. (MP 5)

### Number System (NS)

Perform operations with complex numbers and represent them on the complex plane. Apply complex numbers to polynomial identities and equations, emphasizing their essential role in satisfying the Fundamental Theorem of Algebra.

### CA2.NS.2 - CS3.NS.2

Using the Fundamental Theorem of Algebra, *construct arguments* that verify the solutions to polynomial equations will always be numbers within the set of complex numbers and will have the form  $a + bi$  where  $a$  and  $b$  are real numbers. (MP 3)

### CA2.NS.3 - CS3.NS.3

Find solutions for polynomials that are limited to integer coefficients *using algebraic or technological tools*. (MP 5)

## Functions (F)

Analyze functional relationships and their features using different representations, contextualizing and interpreting function expressions in terms of the modeled situation. Build, refine, and compare polynomial and exponential models to solve problems. Build new functions from existing functions. Extend the domain of trigonometric functions using the unit circle. Model periodic phenomena with trigonometric functions.

### CA2.F.9 - CS3.F.9 (Extension of S3.F.1)

Understand and interpret the features of polynomial, radical, rational or logarithmic functions represented graphically using interval notation and inequalities *by applying and removing context*. Focus on these key features: domain, range, intervals of increase and/or decreasing, location of absolute maximum and/or absolute minimum, intercepts, asymptotes, and end behavior. *Build graphic models* that satisfy given key features. (MP 2, 4)

### CA2.F.10 - CS3.F.10 (Extension of S3.F.2)

*Build and use models* of periodic phenomena with trigonometric functions that connect real world contexts with amplitude, frequency, and midline. *Use structures and patterns* to support understanding of a domain extended beyond 0 to 90 degrees, or 0 to  $\frac{\pi}{2}$  radians. (MP 4, 7)

### CA2.F.11 - CS3.F.11 (Extension of S3.F.3)

Understand and interpret key features (period, midline and amplitude/asymptote)

of trigonometric functions represented graphically. *Build graphic models* that satisfy the given key features. (MP 4)

#### CA2.F.12 - CS3.F.12 (Extension of S3.F.4)

*Build graphical representations* that show key features of polynomial, radical, exponential, logarithmic, rational and trigonometric functions expressed symbolically. *Select and use appropriate tools* to graph, by hand for simple cases, and use technology for more complicated cases. (MP 4, 5)

#### CA2.F.13 - CS3.F.13 (Extension of S3.F.5)

Identify the effect on the graph when replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ ,  $f(x+k)$  by exploring specific values of  $k$  for polynomial, radical, exponential, logarithmic, rational, and trigonometric functions *in and out of context*. *Attend to precision* of language and notation while constructing arguments and carefully evaluate whether the ideas and processes observed during exploration are *reasonable*. (MP 3, 6)

#### CA2.F.16 - CS3.F.16

*Build* a new function from a composition of functions that *models a relationship* between two quantities. (MP 4)

#### CA2.F.17 - CS3.F.17

Verify by composition that one function is the inverse of another function. *Build an invertible function* from a non-invertible function by restricting the domain. (MP 4)

### Geometry/Trigonometry (G)

Apply geometric reasoning to real world modeling situations. Apply prior understanding of trigonometric ratios to all types of triangles using the Law of Sines and Cosines. Build and use the unit circle to extend the domain of trigonometric functions.

#### CA2.G.1 - CS3.G.4

Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles *with and without context*. (MP 2)

**CA2.G.2 - CS3.G.5**

*Use special triangles as tools to construct the unit circle with degrees and radians. Describe and represent patterns in the unit circle and use them as tools to evaluate the trigonometric functions at multiples of  $\frac{\pi}{4}$  and  $\frac{\pi}{6}$ . (MP 5, 7)*

## Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of Algebra 2 (Intermediate Algebra), students will be able to:

- Solve equations for all families of functions (A)
- Describe, represent and build polynomials, using zeros and factors. (A)
- Solve equations (rational and radical) as well as inequalities including compound inequalities. (A)
- Understand, distinguish between, and represent functions including polynomial, rational, trigonometric, logarithmic, and inverse functions. (F)
- Interpret and analyze key features of linear, quadratic, exponential, polynomial, rational, logarithmic and trigonometric functions. (F)
- Use congruence, similarity and right triangle ratios to work with the unit circle, and trigonometric identities. (F)

# Standards: Algebra 2 (Calculus Pathway)

## Introduction

In high school the major works focus on learning to create, interpret, manipulate, and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations and building functions. Students learn to define functions, describe their features and transformations. Students understand, apply and prove congruence of similarity as defined in terms of geometric transformations. Students also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

The Algebra 2 Calculus Pathway is designed to establish the essential mathematical foundation for students pursuing college degrees in STEM and business fields, including engineering, physical and biological sciences, and mathematics.

The standards focus heavily on the core algebraic concepts necessary to equip students with the conceptual understanding and procedural skills necessary for the study of calculus and other advanced mathematics and science courses.

Completion of this pathway directly prepares students for introductory concurrent enrollment Calculus, AP Pre-Calculus, AP Calculus, or IB Math courses.

Within each domain, the standard numbering begins where the *Algebra 2 Math for All Students* standards conclude. Standards with the prefix *S3* represent standards from the *Math for All Students* content. Standards with the prefix *CS3* are specific to the Calculus Pathway course.

In the Algebra 2 Calculus Pathway, instructional time should focus on five major works: (1) understanding, comparing, and representing polynomial and rational functions; (2) understanding, comparing, and representing exponential and logarithmic functions; (3) understanding and representing trigonometric functions; (4) creating, interpreting, manipulating, and solving algebraic equations; and (5)

understanding and representing parametric and polar functions.

## Mathematical Practices

### *Mathematical Practice 1: Ask questions to explore mathematical ideas*

In high school, students formulate and ask questions to highlight the difference between various function types. Students ask targeted questions about univariate data displays to explore mathematical ideas. Students ask questions about the underlying structure of geometric objects revealed by geometric constructions.

### *Mathematical Practice 2: Add or remove context to make sense of mathematics*

In high school, students contextualize and decontextualize functions to interpret their properties, connecting symbolic representations to meaningful situations. Students interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

### *Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments*

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to distinguish between situations that can be modeled by various function types. They construct arguments to justify claims such as why exponential growth eventually surpasses linear, quadratic or the growth in any other polynomial function.

### *Mathematical Practice 4: Build and use models*

In high school, students build and use models in two primary ways. One of these is to use models to connect mathematical concepts to one another. Another way that students use models in high school is to model real-world phenomena.

Students create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. In high school students use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas between them.

To model real-world phenomena, students engage in mathematical and statistical modeling to represent, analyze, and predict real-world situations. In high school students use geometric shapes, their measures, and their properties to model real world objects and apply geometric methods to solve design problems.

*Mathematical Practice 5: Select and use tools appropriately and strategically*

In high school, students learn to select and leverage multiple types of tools while engaging in and applying mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential, or quadratic equations for a variable. Students select and use appropriate tools to build representations for various function types with and without technology. Students use mathematical ideas as a tool to validate the appropriateness and usefulness of a model.

*Mathematical Practice 6: Attend to precision and reasonableness*

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

*Mathematical Practice 7: Describe and represent structures, patterns, and relationships*

In high school, students describe the structures of linear and exponential

expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define dilations and similar shapes, and then use these structures to solve problems. Students use models to explore and describe attributes of the structure, patterns and relationships of inverses for various function types.

### *Mathematical Practice 8: Make conjectures and evaluate the results*

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to answer investigative questions. Students make conjectures about the reasonableness and strength of the model they choose to describe the association between two quantitative variables. Students develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

## **Standards: Algebra 2 (Calculus Pathway)**

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

### **Algebra (A)**

Analyze and interpret the structure and patterns of polynomial, rational and radical expressions, and rewrite them in equivalent form to simplify and solve problems. Construct and represent series, rational expressions, and higher-degree polynomials using zeros and factors. Use a variety of algebraic and technologic tools to solve rational, radical, and compound inequalities.

#### **CA2.A.5 - CS3.A.5 (Extension of S3.A.2)**

Create equations and inequalities in one variable to *model a context* and use them to solve problems. Include equations arising from polynomial, exponential, rational and radical functions. (MP 3)

### CA2.A.6 - CS3.A.6 (Extension of S3.A.3)

For exponential models, express as a logarithm the solution to  $f(t)=ab^{ct} = d$  where  $a$ ,  $b$ ,  $c$ , and  $d$  are numbers,  $b>0$ ; evaluate the logarithm using *technological tools*. *Justify* and use the properties of logarithms to generate equivalent expressions, connecting them to the properties of exponents. (MP 2, 3)

### CA2.A.7 - CS3.A.7

Understand the inverse relationship between exponents and logarithms. Use this *relationship* to solve problems involving exponents and logarithms. (MP 7)

### CA2.A.8 - CS3.A.8

*Model* arithmetic and geometric series, including infinite geometric series algebraically, graphically, and numerically. (MP 4)

### CA2.A.9 - CS3.A.9

*Build and use expressions* including sigma notation to represent the sum of a finite arithmetic or geometric series. (MP 4)

### CA2.A.10 - CS3.A.10

Add, subtract, multiply and divide polynomial and rational functions. *Justify* whether these operations result in new polynomial or rational functions. (MP 3)

### CA2.A.11 - CS3.A.11

Identify zeros of polynomial functions *using algebraic tools*, including factoring or the Remainder Theorem. (MP 5)

### CA2.A.12 - CS3.A.12

Expand and simplify polynomial sums, products, and powers using *structures and patterns*. (MP 7)

### CA2.A.13 - CS3.A.13

Solve compound inequalities in one variable, including absolute value inequalities *using algebraic or technological tools*. (MP 5)

## Number System (NS)

Build on understanding of real numbers and quadratic equations to include complex solutions of polynomial equations and satisfy the conditions for the Fundamental Theorem of Algebra. Use and perform operations with imaginary and complex numbers and represent them on the complex plane.

### CA2.NS.1 - CS3.NS.1

*Represent* addition, subtraction, and multiplication geometrically on the complex plane. Perform algebraic operations with complex numbers on the complex plane. (MP 7)

### CA2.NS.2 - CS3.NS.2

Using the Fundamental Theorem of Algebra, *construct arguments* that verify the solutions to polynomial equations will always be numbers within the set of complex numbers and will have the form  $a + bi$  where  $a$  and  $b$  are real numbers. (MP 3)

### CA2.NS.3 - CS3.NS.3

Find solutions for polynomials that are limited to integer coefficients *using algebraic or technological tools*. (MP 5)

## Functions (F)

Analyze functional relationships and their features using different representations, contextualizing and interpreting function expressions in terms of the modeled situation. Build, refine, and compare polynomial and exponential models to solve problems. Construct and use parametric and graphic models. Build new functions from existing functions. Extend the domain of trigonometric functions using the unit circle. Model periodic phenomena with trigonometric functions. Use inverse trigonometric functions to solve equations and inequalities.

### CA2.F.9 - CS3.F.9 (Extension of S3.F.1)

Understand and interpret the features of polynomial, radical, rational or logarithmic functions represented graphically using interval notation and

inequalities by *applying and removing context*. Focus on these key features: domain, range, intervals of increase and/or decreasing, location of absolute maximum and/or absolute minimum, intercepts, asymptotes, and end behavior. *Build graphic models* that satisfy given key features. (MP 2, 4)

#### CA2.F.10 - CS3.F.10 (Extension of S3.F.2)

*Build and use models* of periodic phenomena with trigonometric functions that connect real world contexts with amplitude, frequency, and midline. *Use structures and patterns* to support understanding of a domain extended beyond 0 to 90 degrees, or 0 to  $\frac{\pi}{2}$  radians. (MP 4, 7)

#### CA2.F.11 - CS3.F.11 (Extension of S3.F.3)

Understand and interpret key features (period, midline and amplitude/asymptote) of trigonometric functions represented graphically. *Build graphic models* that satisfy the given key features. (MP 4)

#### CA2.F.12 - CS3.F.12 (Extension of S3.F.4)

*Build graphical representations* that show key features of polynomial, radical, exponential, logarithmic, rational and trigonometric functions expressed symbolically. *Select and use appropriate tools* to graph, by hand for simple cases, and using technology for more complicated cases. (MP 4, 5)

#### CA2.F.13 - CS3.F.13 (Extension of S3.F.5)

Identify the effect on the graph when replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ ,  $f(x+k)$  by exploring specific values of  $k$  for polynomial, radical, exponential, logarithmic, rational, and trigonometric functions in and out of context. *Attend to precision* of language and notation while *constructing arguments* and carefully evaluate whether the ideas and processes observed during exploration are *reasonable*. (MP 3, 6)

#### CA2.F.14 - CS3.F.14

Define a curve parametrically, and *create a graphical model*. (MP 4)

#### CA2.F.15 - CS3.F.15

*Build* a new function from a composition of functions that *models* a relationship

between two quantities. (MP 4)

### CA2.F.16 - CS3.F.16

*Verify* by composition that one function is the inverse of another function. *Build* an invertible function from a non-invertible function by restricting the domain. (MP 3, 4)

### CA2.F.17 - CS3.F.17

Use inverse trigonometric functions to solve equations and inequalities *with and without a context*. Determine the *reasonableness* of solutions using *context*. (MP 2, 6)

## Geometry/Trigonometry (G)

Apply geometric reasoning to real world modeling situations. Apply prior understanding of trigonometric ratios to all types of triangles using the Law of Sines and Cosines. Build and use the unit circle to extend the domain of trigonometric functions.

### CA2.G.1 - CS3.G.4

Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles *with and without context*. (MP 2)

### CA2.G.2 - CS3.G.5

Use special triangles as *tools* to construct the unit circle with degrees and radians. *Describe and represent patterns* in the unit circle and use them as *tools* to evaluate the trigonometric functions at multiples of  $\frac{\pi}{4}$  and  $\frac{\pi}{6}$ . (MP 5, 7)

### CA2.G.3 - CS3.G.6

*Prove* the addition and subtraction formulas for sine, cosine and tangent and use them *as tools* to solve problems. (MP 5)

## Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in

future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of Algebra 2 (Calculus), students will be able to:

- Understand, distinguish between, and represent functions including polynomial, rational, trigonometric, logarithmic, and inverse functions. (F)
- Interpret and analyze key features of linear, quadratic, exponential, polynomial, rational, logarithmic, and trigonometric functions. (F)
- Use congruence, similarity, and right triangle ratios to work with the unit circle and trigonometric identities. (F)
- Describe, represent, and build polynomials using zeros and factors. (A)
- Solve equations (rational and radical) as well as inequalities including compound inequalities. (A)
- Perform operations with complex numbers. (NS)
- Apply trigonometric identities and inverse trigonometric functions in solving equations and inequalities. (F)

# Standards: Algebra 2 (Data Science Pathway)

## Introduction

In high school, the major works will focus on learning to create, interpret, manipulate, and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations, and building functions. Students learn to define functions and describe their features and transformations. Students understand, apply and prove congruence of similarity as defined in terms of geometric transformations. Students also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

The Algebra 2 Data Science Pathway prepares students for a diverse range of college majors and careers that rely on understanding, interpreting, and communicating data. This includes fields like nursing, psychology, history, and the social sciences.

This course focuses on descriptive and statistical methods—the essential tools used to analyze, model, and draw insights from data. Completion of this pathway provides students with the foundational skills necessary to successfully enroll in introductory university statistics courses or AP Statistics.

Within each domain the standard numbering begins where the Algebra 2 *Math for All Students* standards conclude. Standards with the prefix *S3* represent standards from the *Math for All Students* content. Standards with the prefix *DS3* are specific to the Data Science Pathway course.

In the Algebra 2 Data Science Pathway instructional time should focus on five major works: (1) data acquisition and preparation; (2) exploratory data analysis; (3) computational thinking; (4) data driven decision making; and (5) model evaluation and validation.

These five pillars are the foundations of the mathematics students explore in the Data Science Pathway.

- Data Acquisition and Preparation:
  - Create usable datasets by collecting, filtering, cleaning, and scaling data, while prioritizing data privacy.
- Exploratory Data Analysis:
  - Describe patterns and variations within data by analyzing visualizations and identifying trends and features. Provide contextual interpretations of visual and numeric findings. Explore relationships between variables and formulate questions to guide further analysis.
- Computational Thinking:
  - Frame problems for computational solutions. Apply algorithmic thinking to decompose complex problems into manageable parts, enabling reuse of components across applications.
- Data-Driven Decision Making:
  - Build and apply models relevant to specific contexts for real-world applications. Critically evaluate the ethical implications of generalizations. Justify conclusions based on rigorous data analysis.
- Model Evaluation and Validation:
  - Assess the validity and potential bias of models. Consider the role of randomness in inference and computation. Evaluate the representativeness of data in relation to model validity. Distinguish between statistical significance and practical importance. Conduct hypothesis testing to validate findings.

## Mathematical Practices

*Mathematical Practice 1: Ask questions to explore mathematical ideas*

In high school, students formulate and ask questions to highlight the difference

between various function types. Students ask targeted questions about univariate data displays to explore mathematical ideas. Students ask questions about the underlying structure of geometric objects revealed by geometric constructions.

*Mathematical Practice 2: Add or remove context to make sense of mathematics*

In high school, students contextualize and decontextualize functions to interpret their properties, connecting symbolic representations to meaningful situations. Students interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

*Mathematical Practice 3: Construct, justify, and communicate clear and reasonable arguments*

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to distinguish between situations that can be modeled by various function types. They construct arguments to justify claims such as why exponential growth eventually surpasses linear, quadratic or the growth in any other polynomial function.

*Mathematical Practice 4: Build and use models*

In high school, students build and use models in two primary ways. One of these is to use models to connect mathematical concepts to one another. Another way that students use models in high school is to model real-world phenomena.

Students create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. In high school students use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas between them.

To model real-world phenomena, students engage in mathematical and statistical

modeling to represent, analyze, and predict real-world situations. In high school students use geometric shapes, their measures, and their properties to model real world objects and apply geometric methods to solve design problems.

*Mathematical Practice 5: Select and use tools appropriately and strategically*

In high school, students learn to select and leverage multiple types of tools while engaging in and applying mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential, or quadratic equations for a variable. Students select and use appropriate tools to build representations for various function types with and without technology. Students use mathematical ideas as a tool to validate the appropriateness and usefulness of a model.

*Mathematical Practice 6: Attend to precision and reasonableness*

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

*Mathematical Practice 7: Describe and represent structures, patterns, and relationships*

In high school, students describe the structures of linear and exponential expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define dilations and similar shapes, and then use these structures to solve problems. Students use models to explore and describe attributes of the structure, patterns and relationships of inverses for various function types.

### *Mathematical Practice 8: Make conjectures and evaluate the results*

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to answer investigative questions. Students make conjectures about the reasonableness and strength of the model they choose to describe the association between two quantitative variables. Students develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

## **Standards: Algebra 2 (Data Science)**

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the [Utah Mathematics Core Guides](#).

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

### **Data Science (D)**

Understand full data science cycle, focusing on processing, visualizing, and describing both categorical and quantitative data, including measures of center and variability. Build and validate statistical models (e.g., linear regression) and apply concepts like the Central Limit Theorem for statistical inference and hypothesis testing. Critically evaluate conclusions, emphasizing the role of random assignment, bias, and ethical, reproducible practices.

#### **DA2.D.9 - DS3.D.9**

*Represent* categorical and quantitative information in organized tables suitable for analysis. *Attend to precision* in data processing by filtering through logical statements, and transforming data with a detailed set of ordered tasks that are repeatable by others or a computer. (MP 6, 7)

#### **DA2.D.10 - DS3.D.10**

Describe univariate displays of large sets of data using technology. *Ask targeted questions* about the displays, including shape (skewed, symmetric), center, variability, and unusual outcomes. (MP 1)

### DA2.11 - DS3.D.11

*Describe and represent relationships* between two quantitative variables. Create visual representations with or without technology, addressing strength, form, direction and unusual observations in the data. (MP 7)

### DA2.12 - DS3.D.12

*Build and use models* to describe relationships between two continuous variables using linear regression. Explain which variable(s) should be used for prediction. *Construct reasonable arguments* for appropriate and inappropriate use of the model for practical decision making. (MP 3, 4)

### D.A2.13 - DS3.D.13

*Describe the relationship* between the distribution of a population, the distribution of a sample, and a sampling distribution (e.g., the distribution of sample means). *Describe* sampling distributions through summary statistics, including the mean and the standard deviation. (MP 1)

### D.A2.14 - DS3. D14

Use the central limit theorem to *represent* the distribution of sample means of large data sets as a normal distribution. *Assess conjectures* about the population means based on the sample averages through probability calculations from the normal distribution. (MP 7, 8)

### DA2.15 - DS3.D.15

*Use statistical tools* to validate the appropriateness and usefulness of a model, including visual and numeric assessments of model fit. Use hypothesis testing and simulation to *develop and assess conjectures* about the model. (MP 5, 8)

### DA2.16 - DS3.D.16

*Evaluate results* reached from data analysis. Identify ways to refine investigative questions, data collection, visualization, or analysis. *Justify and communicate clear and reasonable* conclusions. (MP 3)

### DA2.17 - DS3.D.17

Determine the appropriate scope of generalization based on the method of sampling. *Contextualize* the implications through data storytelling. (MP 2)

### DA2.D.18 - DS3.D.18

Understand the role of random assignment in experiments and the implications for cause-and-effect interpretations. *Justify conclusions.* (MP 3)

### DA2.D.19 - DS3.D.19

*Construct reasonable arguments* regarding the issues of data bias and confounding variables in observational studies and their implications for interpretation. (MP 3)

### DA2.D.20 - DS3.D.20

*Use precision in practices* for handling large sets of data that enhance reproducibility and ensure ethical use, including descriptions of alterations, and understand when data may contain sensitive information. (MP 6)

## Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of Algebra 2 (Data Science), students will be able to:

- Summarize collected data to answer investigative questions using tables, graphs, and numerical summary statistics. Transform data. (D)
- *Describe and represent structures and patterns of* data on two categorical variables, two quantitative variables, and for univariate data. (D)
- *Build and use models* to describe relationships between two continuous variables, including linear regression. Explain which variable(s) should be used for prediction. (D)
- *Develop and use models* from sample data to estimate population means or proportions. (D)
- *Describe* sampling distributions through summary statistics, including the mean and the standard deviation. Use the central limit theorem to *represent*

the distribution of sample averages as a normal distribution. (D)

- Use hypothesis testing and simulation to *develop and assess conjectures* about models. Evaluate appropriateness of models. (D)
- Evaluate results from data analysis. *Justify and communicate* conclusions, including appropriate scope of generalization, and *contextualize* the implications. (D)

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