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Draft P-12 Mathematics Standards
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Introduction

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Introduction

Utah's Core Mathematics Standards assist in opening doors of opportunity for all students. They set clear expectations and guide students through a focused and coherent progression of math concepts, considering age-level development and emphasizing proficiency of identified essential competencies. These standards establish a strong mathematical foundation in early grades that allows students to engage with complex mathematical ideas as they progress through secondary mathematics.

All students must have the opportunity to learn and meet high mathematics standards in order to understand and navigate the modern world. Engaging with mathematics helps students develop tools to analyze patterns, quantify relationships, and make informed decisions. Understanding mathematics provides students with skills that support success in college, career, and family life.
The standards set grade-specific expectations but do not dictate curriculum or teaching methods, nor do they define the strategies necessary to meet diverse learners' needs (53E-4-202). Customized supports must be provided to ensure all learners have access to the grade-level content in *Utah's Core Mathematics Standards*.

Organization of the Standards

In Utah State Code 53E-4-202, the State Board is charged with establishing core standards for Utah Public Schools that identify the basic knowledge, skills, and competencies each student is expected to acquire or master as the student advances through the public education system. These basic knowledge, skills, and competencies increase in depth and complexity from year to year.

Strands

The *Utah Core Standards* are organized into *strands*, which represent significant areas of learning within content areas. In mathematics, the strands are organized by overarching themes but do not necessarily represent isolated topics. Mathematical concepts are connected across multiple strands and grade levels. The trajectory of the strands is designed to follow a logical learning progression.

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Standards

Within each strand are *standards*. A standard articulates the knowledge and skills to be obtained by students. A standard represents the essential elements of the expected learning.

Mathematical Skills

In each grade-level core, there is a collection of *Mathematical Skills*. Each skill is a fundamental mathematical practice that students build as they engage with mathematical content. To ensure students develop capacity with these skills, they have been embedded into every standard across all strands and can be *identified by the use of italic text.*

Although specific Mathematical Skills have been intentionally identified and connected to each standard, instruction and practice of other Mathematical Skills not referenced in an individual standard may be appropriate if other skills support student learning in mathematics learning experience.

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. The list of competencies is not exhaustive of every standard; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient.

Shifts in the Standards

Changes in the standards were made to improve their consistency, clarity, and progression through the grade levels. These changes include:

- Standards were clarified, and the overall number of standards was reduced.
- Mathematical Skills were identified and are highlighted throughout the standard P-12.

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- Expectations were defined for the mathematical skills at each grade level.
- Essential Competencies were identified for each grade level specifying the success criteria associated with proficiently completing the major work of that grade level.
- Data Science standards were emphasized and incorporated into all grades to build statistical literacy essential for Utah citizens.
    - At every grade level, the Data Science standards follow a structure where students model with mathematics by:
        - Asking statistical questions,
        - Collecting and exploring data,
        - Analyzing data,
        - And interpreting results
- In Secondary Math III, specific standards that are essential for all students were highlighted, and additional standards were developed to provide students and their families agency in choosing which math course pathways will best support their post-high school goals.

Building Procedural Fluency

These standards define what students should understand and be able to do in their study of mathematics. Throughout the progression of the Utah mathematics core standards, special attention was paid to ensuring that students develop a strong foundation in conceptual understanding and procedural fluency. Procedural fluency is defined as the ability to apply procedures efficiently, flexibly, and accurately, transfer procedures to different problems and contexts, build or modify procedures from other procedures, and recognize when one strategy or procedure is more appropriate to apply than another.

Through the progression of the standards, students are provided opportunities to make sense of and build a conceptual understanding of the mathematical operations. This conceptual understanding supports children in becoming automatic in their retrieval of basic math facts. That basic fact fluency can then be

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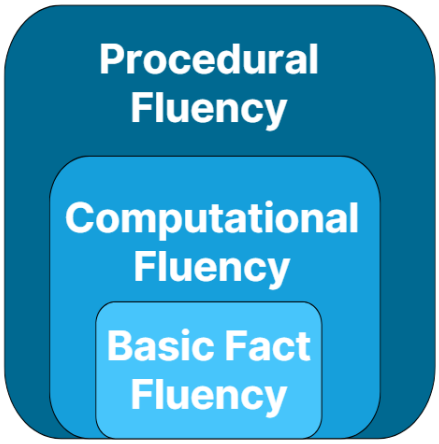
applied as students develop computational and procedural fluency using a variety of numerical strategies and efficient algorithms.



Mathematical Tools and Technology

An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to empower students to meet the learning intentions of the classroom. Software and online programs cannot replace authentic interactions between teachers and students, and the value of student-to-student discourse and collaboration. Technology and tools should be used in ways that promote mathematical reasoning and sense-making.

Mathematical Skills

The mathematical skills included in *Utah's P-12 Core Mathematics Standards* describe mathematical practices that Utah students develop throughout their educational experiences. Students become proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills. Each grade level's standards have a description in their introductory material specifying the ways students will develop these skills as they learn math concepts. Although specific Mathematical Skills have been intentionally identified and connected to each standard, instruction and practice of other Mathematical Skills not referenced in that standard are encouraged. Below, general descriptions are provided for each skill.

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Descriptions of what the skills look like in each grade band are included in the grade level standards. More details about the skills and how they are integrated throughout the standards are included in each grade level's Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Skill 1: Describe and represent structures, patterns, and relationships

Students learn the language of mathematics through exploration (making sense of a mathematical task), observation (noticing patterns and relationships in mathematics), and highlighting structure (using mathematical tools to organize those relationships). Students use their understanding to make generalizations about the structure of numbers to become more flexible and efficient in applying mathematical ideas.

Students build these skills when they have opportunities to explore new topics collaboratively and independently. As they surface patterns and relationships, they apply mathematical structures to organize ideas and work toward making mathematical generalizations.

Skill 2: Build and use models

Mathematical models use structures to represent relationships. Students build and use verbal, contextual, visual, symbolic, and physical models to represent and make sense of the relationships between quantities in real-world situations and mathematical problems.

Using models enhances student learning by providing students with multiple ways to represent and understand mathematical concepts, connect mathematical ideas to real-world situations, and develop problem-solving skills. Students explore the idea that various models can showcase mathematical thinking in valuable ways. Using multiple models helps students notice the connections between mathematical structures and promotes the development of mathematical

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generalizations that can be leveraged as tools to make sense of and persevere in solving novel problems.

Skill 3: Attend to precision and reasonableness

Students exercise mathematical precision and reasonableness in their communication, calculations, and processes. They calculate flexibly, accurately, and efficiently, using clear and concise notation to record their work. They regularly evaluate whether actions, ideas, processes, and solutions make sense and are reasonable.

Precision in communication and calculation fosters a deeper understanding of mathematical concepts. Using precise language, students articulate their thoughts clearly, enhancing their ability to explain their reasoning to others and understand others' ideas. Accurate and efficient calculations and precise notation enable students to solve problems effectively and minimize errors. The continuous evaluation of reasonableness ensures that students improve their ability to identify and learn from mathematical errors.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

Students construct arguments using the mathematical reasoning that underlies a strategy, solution, or conjecture. They justify their conjectures and clearly communicate their reasoning using concrete referents such as objects, drawings, diagrams, and actions. They justify their arguments with peers through discourse, refining their thinking to create more robust and precise arguments.

Constructing, justifying, and communicating arguments fosters a deep understanding of mathematical concepts. By defending their conjectures, students solidify their mathematical understanding and identify potential flaws in their reasoning. Clear communication enables students to articulate their thoughts effectively, enhancing their ability to explain their reasoning to others. Discourse with peers exposes students to diverse perspectives, promoting critical thinking and collaborative problem-solving.

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Skill 5: Add or remove context to make sense of mathematics

Students add or remove context to help make sense of mathematical ideas. Students add context by connecting numbers and operations to images, stories, objects, and events. They remove context from a situation by representing the mathematics using numbers, symbols, and equations. Students visualize complicated concepts as being composed of simpler parts. They identify quantities in a contextual situation, use mathematical models to represent and analyze the relationships between those quantities, and draw conclusions.

When students contextualize and decontextualize mathematical ideas, they can make sense of underlying structures by connecting representations. Working in and out of context throughout a learning progression empowers students to make sense of mathematics and persevere as they explore abstract and real-world applications. This process allows students to take their knowledge of mathematics and meaningfully apply it to their lives.

Skill 6: Ask questions to explore mathematical ideas

Students examine mathematical concepts and formulate questions based on what they notice and wonder about given situations. During mathematical discourse, students ask probing questions of one another as they examine arguments and seek to understand others' ideas.

Asking questions cultivates students' curiosity about mathematical relationships and helps them refine their understanding. As students build their capacity to ask probing questions to their peers, they engage in more meaningful discourse about mathematics. When students ask and explore rich questions, it advances the mathematical understanding of all learners.

Skill 7: Make conjectures and evaluate the results

Students make sense of mathematics when they engage in mathematical investigations. They use their understanding of structures, patterns, and relationships to speculate about the nature of a relationship and then test their conjecture to determine how to proceed. As they test their mathematical

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hypotheses and evaluate the outcomes of their investigation, they build mathematical generalizations.

Developing and assessing conjectures encourages active engagement and critical thinking. Conjecturing acts as an entry point to exploration for all students. When making conjectures, students have opportunities to continually revise their thinking. Students formulate conjectures, test them, and refine their understanding based on the outcomes. Students view mistakes as assets that help them better refine their mathematical understanding. This process fosters a deeper understanding of underlying mathematical principles. It promotes curiosity, students' ownership of their learning, and the development of positive mathematical identities.

Skill 8: Select and use tools appropriately and strategically

Students strategically select and use relevant and valuable tools for mathematical tasks with which they engage. Tools can be physical objects (e.g., compasses or manipulatives), technological tools (e.g. graphing calculators or virtual manipulatives) or mathematical constructs (e.g., diagrams, strategies, or algorithms).

Students consider the available mathematical problem-solving tools and choose the most relevant and useful ones. When students have the agency to select mathematical tools strategically, it empowers them to visualize, represent, and understand mathematical concepts effectively, fostering critical thinking, problem-solving flexibility, perseverance, and engagement.

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Preschool Age 3

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Standards: Preschool Age 3

Introduction

In Preschool Age 3 (P3) and Preschool Age 4 (P4), the major work of both grades focuses on developing an understanding of the foundational concepts of counting and cardinality and developing the ability to describe and compare measurable attributes of objects and shapes.

In P3, instructional time should focus on three major works: (1) developing concepts of counting and cardinality; (2) beginning to recognize simple patterns; and (3) beginning to describe shapes and objects. Quality learning environments focus on hands-on experiences during play and interaction with others to incorporate well-designed mathematical experiences aligned to specific learning goals.

Mathematical Skills

Link to Detailed Skills Description

Skill 1: Describe and represent structures, patterns, and relationships

In grades P-K, students explore patterns and structures in counting and with shapes. Students begin exploring the concepts of addition and subtraction with small numbers.

Skill 2: Build and use models

In grades P-2, students use a variety of models to build their understanding of numbers with increasing complexity, leading to an understanding of the base ten place value system. They model the actions and comparisons represented by the operations of addition and subtraction. Consistent exposure to these models supports fluency and helps students connect concrete experiences to foundational mathematical ideas.

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Skill 3: Attend to precision and reasonableness

In grades P-K, students use informal language as they develop and learn precise mathematical vocabulary. They begin to learn strategies for assessing their accuracy when counting, naming numbers, and describing shapes.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

In grades P3 and P4, students are learning to communicate their ideas and listen to the ideas of others. They develop strategies to agree and disagree respectfully. Students begin to explain mathematical ideas.

Skill 5: Add or remove context to make sense of mathematics

In grades P-2, students build an understanding of what the numbers, symbols, pictures, words, etc., in their work represent. Students develop flexibility in moving between concrete, pictorial, and abstract representations of mathematical ideas. They begin moving back and forth between a problem's context and its representation to use the form that best fits the situation.

Skill 6: Ask questions to explore mathematical ideas

In grades P-2, students ask questions to help them explore numbers, place value, the properties of addition and subtraction, and the properties of shapes. Students begin to ask questions about data related to the classroom and their day-to-day lives.

Skill 7: Make conjectures and evaluate the results

In grades P-2, students will make and test conjectures related to numbers, place value, the properties of addition and subtraction, and the attributes of shapes. As students explore their conjectures, they will learn to evaluate the reasonableness of their solution path and adjust as needed. Providing students opportunities to revise

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their thinking helps students develop confidence in problem solving.

Skill 8: Select and use tools appropriately and strategically

In grades P-2, students use tools when they help them make sense of mathematics. Tools can include, but are not limited to, physical tools, visual tools, and self-created tools such as counters, ten frames, place value charts, hundreds charts, open number lines, strategies, algorithms, etc. Consistent, hands-on practice with these tools builds confidence and prepares students for flexible application.

Standards: Preschool Age 3 (P3)

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Counting and Cardinality (CC)

Identify numerals by name, count to 10, use one-to-one correspondence, describe quantities of objects counted, use ordinal language, and subitize.

P3.CC.1

Attend to *structure* and *precise* language by reciting number names in sequence to 10. (Sk 1, 3)

P3.CC.2

Describe and represent mathematical structures by beginning to recognize written numbers to represent a quantity. Use a variety of *tools* to *build* and *model* numbers. (Sk 1, 2, 8)

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P3.CC.3

Demonstrate one-to-one correspondence by *precisely* counting and pointing/touching a line of 1-5 objects. (Sk 3)

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P3.CC.4

Begin to use *precise* number names to identify the number of objects in a group of up to 5 objects, recognizing that the last number in a count represents the total number of objects. (Sk 3)

P3.CC.5

With support, use mathematical language to *precisely* identify whether the number of objects in one group is the same as those in another group for sets of up to 4 objects. (Sk 3)

P3.CC.6

Identify whether an object is "first" or "second" in a sequence by noticing ordinal structure. Begin to recognize ordering words in everyday *contexts*. (Sk 5)

P3.CC.7

Use *patterns* and *structures* to subitize quantities up to 3. (Sk 1)

Operations and Algebraic Thinking (OA)

Identify and explore simple patterns.

P3.OA.1

Describe and *model* simple *patterns* in the environment and begin to duplicate and extend those patterns. (Sk 1, 2)

Measurement and Geometry (MG)

Identify and describe measurable attributes of basic shapes.

P3.MG.1

With prompting and support, identify and *describe* measurable attributes (for example, big, small, tall, short). (Sk 1)

P3.MG.2

With prompting and support, make and *evaluate conjectures* about the measurable attributes of objects and shapes. (Sk 7)

P3.MG.3

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With prompting and support, identify, *describe*, and *represent* basic shapes and their attributes. (Sk 1)

Data Science (D)

Begin asking questions that lead to collecting and analyzing data by sorting and counting objects**.**

P3.D.1

With prompting and support, *ask questions* about people or objects in the classroom. (Sk 6)

P3.D.2

With support, *select and use tools* to collect and sort data by attributes (color, shape, size). (Sk 8)

P3.D.3

With prompting and support, *attend to precision and reasonableness* when analyzing data by counting the number of objects in each category (no more than five objects per category). (Sk 3)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure

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students are proficient. Through engagement with the knowledge and skills embedded within the standards, by the end of P3, children will be able to:

- Count up to 5 objects in a set, then tell how many objects are in the set without counting again. (CC)
- Count to 10 by ones and recognize that numbers have a known sequence (for example, “1, 2, 3, 4, 5. What comes next?”). (CC)
- Quickly recognize small groups of up to 3 objects without counting. (CC)
- Describe and model simple patterns. (OA)
- Identify and describe basic shapes and their measurable attributes. (MG)
- Begin sorting and counting objects. (D)

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Preschool Age 4

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Standards: Preschool Age 4

Introduction

In Preschool Age 3 (P3) and Preschool Age 4 (P4), the major work of both grades focuses on developing an understanding of foundational concepts of counting and cardinality and developing the ability to describe and compare measurable attributes of objects and shapes.

In P4, instructional time should focus on three major works: (1) reinforcing concepts of counting and cardinality; (2) exploring concepts of addition and subtraction; and (3) exploring and describing shapes, patterns, and measurable attributes of objects. Quality learning environments focus on hands-on experiences during play and interaction with others to incorporate well-designed mathematical experiences aligned to specific learning goals.

Mathematical Skills

Link to Detailed Skills Description

Skill 1: Describe and represent structures, patterns, and relationships

In grades P-K, students explore patterns and structures in counting and with shapes. Students begin exploring the concepts of addition and subtraction with small numbers.

Skill 2: Build and use models

In grades P-2, students use a variety of models to build their understanding of numbers with increasing complexity, leading to an understanding of the base ten place value system. They model the actions and comparisons represented by the operations of addition and subtraction. Consistent exposure to these models supports fluency and helps students connect concrete experiences to foundational

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mathematical ideas.

Skill 3: Attend to precision and reasonableness

In grades P-K, students use informal language as they develop and learn precise mathematical vocabulary. They begin to learn strategies for assessing their accuracy when counting, naming numbers, and describing shapes.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

In grades P3 and P4, students are learning to communicate their ideas and listen to the ideas of others. They develop strategies to agree and disagree respectfully. Students begin to explain mathematical ideas.

Skill 5: Add or remove context to make sense of mathematics

In grades P-2, students build an understanding of what the numbers, symbols, pictures, words, etc., in their work represent. Students develop flexibility in moving between concrete, pictorial, and abstract representations of mathematical ideas. They begin moving back and forth between a problem's context and its representation to use the form that best fits the situation.

Skill 6: Ask questions to explore mathematical ideas

In grades P-2, students ask questions to help them explore numbers, place value, the properties of addition and subtraction, and the properties of shapes. Students begin to ask questions about data related to the classroom and their day-to-day lives.

Skill 7: Make conjectures and evaluate the results

In grades P-2, students will make and test conjectures related to numbers, place value, the properties of addition and subtraction, and the attributes of shapes. As students explore their conjectures, they will learn to evaluate the reasonableness of their solution path and adjust as needed. Providing students opportunities to revise

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their thinking helps students develop confidence in problem solving.

Skill 8: Select and use tools appropriately and strategically

In grades P-2, students use tools when they help them make sense of mathematics. Tools can include, but are not limited to, physical tools, visual tools, and self-created tools such as counters, ten frames, place value charts, hundreds charts, open number lines, strategies, algorithms, etc. Consistent, hands-on practice with these tools builds confidence and prepares students for flexible application.

Standards: Preschool Age 4 (P4)

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Counting and Cardinality (CC)

Count to 20, recognize numerals, demonstrate one-to-one correspondence, compare quantities, use ordinal language, and subitize.

P4.CC.1

With support, attend to mathematical *structure* and *precisely* recite number names and the counting sequence to 20. (Sk 1, 3)

P4.CC.2

Describe and represent structures and patterns by associating a quantity with written numerals up to 10. Begin to use a variety of *tools* to form numerals and represent quantities. (Sk 1, 8)

P4.CC.3

Demonstrate one-to-one correspondence by *precisely* counting and pointing to or touching one object per count for an arrangement of up to 10 objects. (Sk 3)

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P4.CC.4

Use *precise* number names to identify the number of objects in a group of up to 10, recognizing that the last number in a count represents the total number of objects. (Sk 3)

P4.CC.5

Use mathematical language to *precisely* identify whether the number of objects in one group is more than, less than, or the same as the number of objects in another group for sets of up to 6 objects. (Sk 3)

P4.CC.6

Use ordering words to *precisely* identify and state whether an object is "first," "second," "third," or "last" in a sequence. Begin to apply ordering words to everyday *contexts*. (Sk 3, 5)

P4.CC.7

Use *patterns* and *structures* to subitize quantities of up to five. (Sk 1)

Operations and Algebraic Thinking (OA)

Identify and manipulate simple patterns and represent addition and subtraction problems involving numbers up to and including 5.

P4.OA.1

Build and use models to explore and represent real-world situations involving addition (adding to or putting together) and subtraction (taking from or taking apart) with numbers up to and including 5. (Sk 2)

P4.OA.2

Select and use tools to decompose numbers less than or equal to five. *Make a conjecture* about why a number or set of objects can be broken apart in multiple ways. (Sk 7, 8)

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P4.OA.3

Select and use tools to make sums of 5 using quantities from 0–5. (Sk 8)

P4.OA.4

Describe and represent simple *patterns* by duplicating, extending, and creating patterns. (Sk 1)

Measurement and Geometry (MG)

Identify, describe, build, and compare basic shapes and their measurable attributes.

P4.MG.1

Describe objects using vocabulary specific to measurable attributes (for example, length [long/short], weight [heavy/light], size [big/small], and distance [near/far]). (Sk 1)

P4.MG.2

Test and *make conjectures* to directly compare two objects using measurable attributes. (Sk 7)

P4.MG.3

Identify, *describe, and represent* two- and three-dimensional shapes regardless of size and orientation. Describe objects in the environment by using names of shapes, and identify the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to. (Sk 1)

P4.MG.4

Build and use models to create basic shapes using a variety of tools. Explore combining basic shapes to create new shapes. (Sk 2)

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Data Science (D)

Ask questions that lead to collecting, analyzing, and interpreting data by sorting and counting objects.

P4.D.1

With prompting and support, *ask questions* about a given topic to investigate situations leading to collecting and analyzing data within the classroom. (Sk 6)

P4.D.2

Select and use tools to collect and sort data by explicit attributes (color, shape, size). (Sk 8)

P4.D.3

With prompting and support, *attend to precision and reasonableness* when analyzing data by counting the number of objects in each category. (Sk 3)

P4.D.4

With prompting and support, *construct, justify, and communicate clear and reasonable arguments* when interpreting data to identify which groups contain more, less, or the same amount. (Sk 4)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge

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embedded within the *Utah Core Mathematics Standards*, by the end of P4, children will be able to:

- Count up to 10 objects in a set, then tell how many objects are in the set without counting again. (CC)
- Begin to connect number names and quantities with written numerals (CC)
- Count to 20. (CC)
- Quickly recognize groups of up to 5 objects without counting. (CC)
- Create and extend simple patterns. (OA)
- Understand and model addition and subtraction with numbers up to and including 5 with concrete objects, fingers, movement, and simple drawings. (OA)
- Identify, describe, build, and compare basic shapes using their measurable attributes. (MG)
- Use positional language to describe objects in the environment (above, below, next to, etc.) (MG)
- Begin interpreting data by sorting and counting objects (D)

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Kindergarten

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Standards: Kindergarten (K)

Introduction

In kindergarten through second grade, the major work of each grade focuses on developing an understanding of addition and subtraction and building an understanding of the base-ten place value number system. Students begin to develop an understanding of foundational concepts of linear measurement and explore the defining attributes of shapes.

In kindergarten, instructional time should focus on three major works: (1) mastering concepts of counting and cardinality; (2) representing, relating, and operating on whole numbers, initially with sets of objects; and (3) describing shapes and space. More learning time in kindergarten should be devoted to number sense than to other topics, as students build fluency through concrete experiences, visual models, and verbal expressions .

Mathematical Skills

Link to Detailed Skills Description

Skill 1: Describe and represent structures, patterns, and relationships

In grades P-K, students explore patterns and structures in counting and with shapes. Students begin exploring the concepts of addition and subtraction with small numbers.

Skill 2: Build and use models

In grades P-2, students use a variety of models to build their understanding of numbers with increasing complexity, leading to an understanding of the base ten place value system. They model the actions and comparisons represented by the operations of addition and subtraction. Consistent exposure to these models supports fluency and helps students connect concrete experiences to foundational mathematical ideas.

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Skill 3: Attend to precision and reasonableness

In grades P-K, students use informal language as they develop and learn precise mathematical vocabulary. They begin to learn strategies for assessing their accuracy when counting, naming numbers, and describing shapes.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

In grades K-2, students are learning to communicate their ideas and listen to the ideas of others. They develop strategies to agree and disagree respectfully. They begin to justify their arguments with evidence and draw connections between mathematical representations. Students also begin to compare and contrast their strategies and solutions to others'.

Skill 5: Add or remove context to make sense of mathematics

In grades P-2, students build an understanding of what the numbers, symbols, pictures, words, etc., in their work represent. Students develop flexibility in moving between concrete, pictorial, and abstract representations of mathematical ideas. They begin moving back and forth between a problem's context and its representation to use the form that best fits the situation.

Skill 6: Ask questions to explore mathematical ideas

In grades P-2, students ask questions to help them explore numbers, place value, the properties of addition and subtraction, and the properties of shapes. Students begin to ask questions about data related to the classroom and their day-to-day lives.

Skill 7: Make conjectures and evaluate the results

In grades P-2, students will make and test conjectures related to numbers, place value, the properties of addition and subtraction, and the attributes of shapes. As students explore their conjectures, they will learn to evaluate the reasonableness of

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their solution path and adjust as needed. Providing students opportunities to revise their thinking helps students develop confidence in problem solving.

Skill 8: Select and use tools appropriately and strategically

In grades P-2, students use tools when they help them make sense of mathematics. Tools can include, but are not limited to physical tools, visual tools, and self-created tools such as counters, ten frames, place value charts, hundreds charts, open number lines, strategies, algorithms, etc. Consistent, hands-on practice with these tools builds confidence and prepares students for flexible application.

Standards: Kindergarten (K)

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Counting and Cardinality (CC)

Know number names and the counting sequence. Count to tell the number of objects and represent a given quantity in a set. Identify and compare quantities of objects and numerals. Identify an object's ordinal position. Subitize quantities.

K.CC.1

Attend to structure and patterns by precisely reciting numbers in sequence from 1 to 100 by ones and tens. (Sk 1, 3)

K.CC.2

Attend to structure and patterns when counting forward from a given number between 1 and 100. (Sk 1)

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K.CC.3

Use precise language and actions to identify and *justify* the quantity of objects in a group of 0–20. (Sk 3, 4)

K.CC.4

Precisely read and write numbers 0 to 20 using base-ten numerals. Represent a quantity of 0 to 20 objects with written numerals. (Sk 3)

K.CC.5

Identify and compare quantities of objects and numerals up to 10 using the words “greater than,” “less than,” and “equal to.” Use reasoning to *justify* comparisons. (Sk 4)

K.CC.6

Use precise mathematical language to identify and *justify* an object's ordinal position. Order quantities or numerals using the ordinal words “first” through “tenth.” (Sk 3, 4)

K.CC.7

Use patterns and structures to subitize quantities up to 10. (Sk 1)

Operations and Algebraic Thinking (OA)

Understand addition as putting together and adding to, and subtraction as taking apart and taking from.

K.OA.1

Select and use tools to represent addition and subtraction with sums and minuends up to and including 10. *Attend to the precision and reasonableness* of results. (Sk 3, 8)

K.OA.2

Build and use models to solve addition and subtraction story problems with sums or minuends up to and including 10. *Add or remove story context* to solve problem type

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situations of adding to and taking from with result unknown, putting together with the total unknown, and putting together with both addends unknown. (Sk 2, 5)

K.OA.3

Build and use models to decompose numbers less than or equal to 10 into pairs in multiple ways. *Make conjectures* about why a number or set of objects can be broken apart in multiple ways. (Sk 2, 7)

K.OA.4

Attend to precision and reasonableness when using strategies to add and subtract flexibly, accurately, and efficiently using numbers less than or equal to 5. (Sk 3)

Number and Operations in Base Ten (NBT)

Build and use models to compose and decompose numbers from 11-19 into ten ones and some more ones to conceptually understand place value.

K.NBT.1

Build and use models to compose and decompose numbers from 11-19 into ten ones and some more ones. *Represent structure* from those models with math language and an equation (e.g., "Eighteen is the same as 10 and 8: 18=10+8). *Describe the pattern* that teen numbers are always composed of ten ones and some more ones. (Sk 1, 2)

Measurement and Geometry (MG)

Identify, describe, and compare measurable attributes of objects. Analyze, compare, create, and compose shapes.

K.MG.1

Attend to precision and reasonableness when describing several measurable attributes of a single object. *Make conjectures* to compare two objects with a measurable attribute to see which object has "more of"/"less of" the attribute *and evaluate the results of those conjectures*. (Sk 3, 7)

K.MG.2

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Describe structures, names, and attributes of two-dimensional and three-dimensional shapes regardless of size or orientation, and *describe* their relative positions in the environment. (Sk 1)

K.MG.3

Make conjectures and evaluate the results to analyze, compare, and sort two- and three-dimensional shapes and objects in different sizes and orientations, using informal language to describe their similarities, differences, and other attributes (for example, color, size, shape, number of sides). (Sk 7)

K.MG.4

Build and use models to create two- and three-dimensional shapes and objects. Use simple shapes to form a variety of shapes. (Sk 2)

Data Science (D)

Ask questions that lead to collecting, analyzing, and interpreting data. Sort, count, and compare objects in a data set.

K.D.1

Ask questions about a given topic to investigate situations that will lead to collecting and analyzing data within the classroom. (Sk 6)

K.D.2

Select and use tools appropriately and strategically to collect and organize data with up to three categories. (Sk 8)

K.D.3

Attend to precision and reasonableness when analyzing data by counting the number of objects in each category. (Sk 3)

K.D.4

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Construct, justify, and communicate clear and reasonable arguments when interpreting data to identify which groups contain more, less, or the same amount using category quantities. (Sk 4)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of kindergarten, children will be able to:

- Count up to 20 objects in a set to answer questions like "how many." (CC)
- Read and write numbers 0-20. (CC)
- Count to 100 by ones and tens. (CC)
- Compare quantities of objects and numerals up to 10. (CC)
- Quickly recognize groups of up to 10 objects without counting. (CC)
- Fluently add and subtract when the largest number in the equation is no greater than 5. (Basic fact fluency) (OA)
- Draw pictures or use objects to solve addition and subtraction story problems where the largest number in the story is no greater than 10. (OA)
- Break apart numbers less than or equal to 10 in multiple ways. (OA)
- Break apart numbers 11-19 into ten ones and some more ones. (NBT)
- Compare and sort objects and shapes using attributes and measurable characteristics. (MG)
- Interpret data by sorting, counting, and comparing objects in a data set. (D)

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Grade 1

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Standards: Grade 1

Introduction

In kindergarten through second grade, the major work of each grade focuses on developing an understanding of addition and subtraction and building an understanding of the base-ten place value number system. Students develop an understanding of foundational concepts of linear measurement and explore the defining attributes of shapes.

In Grade 1, instructional time should focus on four major works: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

Mathematical Skills

Link to Detailed Skills Description

Skill 1: Describe and represent structures, patterns, and relationships

In grades 1-2, students explore patterns and structures such as place value, properties of operations, and composition of shapes. They use structures and patterns to develop a deep understanding of addition and subtraction. Students use structures and patterns to see complicated things as single objects or as composed of several objects.

Skill 2: Build and use models

In grades P-2, students use a variety of models to build their understanding of numbers with increasing complexity, leading to an understanding of the base ten place value system. They model the actions and comparisons represented by the

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operations of addition and subtraction. Consistent exposure to these models supports fluency and helps students connect concrete experiences to foundational mathematical ideas.

Skill 3: Attend to precision and reasonableness

In grades 1-2, students refine informal language as they develop and learn precise mathematical vocabulary. Students use structure to accurately record, label, and share their thinking. They explore when a precise answer is necessary and when making an estimate, which can help them determine if a solution makes sense. Students use estimation strategies based on place value structure and the properties of operations to assess reasonableness throughout the problem-solving process.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

In grades K-2, students are learning to communicate their ideas and listen to the ideas of others. They develop strategies to agree and disagree respectfully. They begin to justify their arguments with evidence and draw connections between mathematical representations. Students also begin to compare and contrast their strategies and solutions to others'.

Skill 5: Add or remove context to make sense of mathematics

In grades P-2, students build an understanding of what the numbers, symbols, pictures, words, etc., in their work represent. Students develop flexibility in moving between concrete, pictorial, and abstract representations of mathematical ideas. They begin moving back and forth between a problem's context and its representation to use the form that best fits the situation.

Skill 6: Ask questions to explore mathematical ideas

In grades P-2, students ask questions to help them explore numbers, place value, the properties of addition and subtraction, and the properties of shapes. Students begin to ask questions about data related to the classroom and their day-to-day

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lives.

Skill 7: Make conjectures and evaluate the results

In grades P-2, students will make and test conjectures related to numbers, place value, the properties of addition and subtraction, and the attributes of shapes. As students explore their conjectures, they will learn to evaluate the reasonableness of their solution path and adjust as needed. Providing students opportunities to revise their thinking helps students develop confidence in problem solving.

Skill 8: Select and use tools appropriately and strategically

In grades P-2, students use tools when they help them make sense of mathematics. Tools can include, but are not limited to physical tools, visual tools, and self-created tools such as counters, ten frames, place value charts, hundreds charts, open number lines, strategies, algorithms, etc. Consistent, hands-on practice with these tools builds confidence and prepares students for flexible application.

Standards: Grade 1

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

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Operations and Algebraic Thinking (OA)

Represent and solve problems involving addition and subtraction with sums and minuends less than or equal to 20. Understand and apply properties of operations and the relationship between addition and subtraction. Understand the meaning of the equal sign.

1.OA.1

Build and use models to solve addition and subtraction story problems with sums and minuends less than or equal to 20. *Add or remove story context* to solve problem

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type situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions except start unknown. (Sk 2, 5)

1.OA.2

Build and use models to solve story problems involving adding three whole numbers with sums less than or equal to 20. *Add or remove context to make sense* of problems and solutions. (Sk 2, 5)

1.OA.3

Make and test conjectures about properties of addition. *Add context* and *compare arguments* to make sense of these properties as strategies to add and subtract. (Sk 4, 5, 7)

1.OA.4

Describe and represent subtraction as an unknown-addend problem. *Construct and justify arguments* that explain how to determine the unknown whole number in an addition or subtraction equation relating to three whole numbers. (Sk 1, 4)

1.OA.5

Describe structures and relationships to connect counting to addition and subtraction. (Sk 1)

1.OA.6

Justify strategies used to add and subtract with sums and minuends less than or equal to 20, and compare strategies with others. (Sk 4)

1.OA.7

Attend to precision and reasonableness when using strategies to add and subtract flexibly, accurately, and efficiently with sums and minuends less than or equal to 10. (Sk 3)

1.OA.8

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Build and use models to make sense of the meaning of the equal sign. *Make conjectures* regarding whether addition and subtraction equations are true or false and *evaluate the reasonableness of results*. (Sk 2, 7)

Number and Operations in Base Ten (NBT)

Count and represent numbers up to and including 120. Build and use models to conceptually understand place value. Use place value understanding to add and subtract larger numbers.

1. NBT.1

Count to 120 *with precision,* starting at any number, and represent a number of objects up to and including 120 with a written numeral. (Sk 3)

1. NBT.2

Use place value understanding to count by tens and *describe the pattern* in the ones place and tens place when counting by ten. (Sk 1)

1.NBT.3

Build and use models to represent and identify the number of tens and ones in a two-digit number. (Sk 2)

1.NBT.4

Use tools and place value understanding to compare two two-digit numbers. *Represent the relationship* between the two numbers using comparison symbols <, =, and >. (Sk 1, 8)

1.NBT.5

Build and use concrete and pictorial models to determine the sum of two two-digit numbers, a two-digit number and a one-digit number, and a two-digit number and a multiple of ten (less than or equal to 90)**.** Use the models *to represent and describe the structure* of combining tens with tens and ones with ones. *Make conjectures* about when to compose ten ones into a ten. *Ask questions to compare* concrete and pictorial models. (Sk 1, 2, 6, 7)

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1.NBT.6

Build and use concrete and pictorial models to subtract a multiple of ten from a larger multiple of ten. *Ask questions to compare* concrete and pictorial models to written numeric strategies. (Sk 1, 6)

1.NBT.7

Mentally find ten more or ten less than a given two-digit number and *justify the result* using place value language. (Sk 4)

Measurement and Geometry (MG)

Measure lengths. Tell and write time. Identify the value of U.S. coins. Reason with shapes and their attributes.

1.MG.1

Select and use tools appropriately and strategically to express the length of an object as a whole number. Measure lengths directly by iterating (repeating) non-standard length units. (Sk 8)

1.MG.2

Construct a reasonable argument about the order of three objects sorted by length. Compare the lengths of two objects indirectly using a third object and *justify the results.* (Sk 4)

1.MG.3

Attend to precision and reasonableness when telling and writing time in hours and half-hours using analog and digital clocks. (Sk 3)

1.MG.4

Ask questions to explore attributes of and identify common U.S. coins. *Attend to precision* when representing their comparative values and use appropriate notation (5¢). (Sk 3, 6)

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1.MG.5

Ask questions to distinguish between defining and non-defining attributes of shapes. Given defining attribute(s), build and draw shapes that possess the defining attribute(s). (Sk 6)

1.MG.6

Use structures, patterns, and relationships to create a composite shape formed by two- and three-dimensional shapes and compose new shapes from the composite shape. (Sk 1)

1.MG.7

Build and use models to partition circles and rectangles into two or four equal shares. *Justify, compare, and evaluate* how decomposing a shape into more equal shares creates smaller shares. (Sk 2, 4)

Data Science (D)

Ask questions that lead to collecting, analyzing, and interpreting data. Develop strategies for organizing data. Interpret visual representations of data.

1.D.1

Ask questions to investigate situations that will lead to collecting and analyzing data within the context of the classroom. (Sk 6)

1.D.2

Select and use tools appropriately and strategically to collect and organize data. Consider and organize a given data set about a classroom context. (Sk 8)

1.D.3

Attend to precision and reasonableness when analyzing data sets with up to three categories by creating visual representations to organize data. (Sk 3)

1.D.4

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Construct, justify, and communicate clear and reasonable arguments while interpreting data to answer investigative questions. (Sk 4)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of first grade, children will be able to:

- Fluently add and subtract when the largest number in the equation is no greater than 10. (Basic fact fluency) (OA)
- Draw pictures, use objects or write equations to solve addition and subtraction story problems where the largest number in the story is no greater than 20. (OA)
- Use a variety of strategies to add and subtract. (OA)
- Count, write, and recognize numbers to 120 (NBT)
- Make sense of place value and the meaning of tens and ones in two-digit numbers. (NBT)
- Add two-digit numbers using objects and pictures. (NBT)
- Measure length using non-standard measurement units. (MG)
- Build two- and three-dimensional shapes based on their defining attributes. (MG)
- Partition circles and rectangles equally. (MG)
- Develop strategies for organizing data and interpret visual representations of data. (D)

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Grade 2

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Standards: Grade 2

Introduction

In kindergarten through second grade, the major work of each grade focuses on developing an understanding of addition and subtraction and building an understanding of the base-ten place value number system. Students develop an understanding of foundational concepts of linear measurement and explore the defining attributes of shapes.

In Grade 2, instructional time should focus on four major works: (1) extending understanding of place value, including grouping in hundreds, tens and ones; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

Mathematical Skills

Link to Detailed Skills Description

Skill 1: Describe and represent structures, patterns, and relationships

In grades 1-2, students explore patterns and structures such as place value, properties of operations, and composition of shapes. They use structures and patterns to develop a deep understanding of addition and subtraction. Students use structures and patterns to see complicated things as single objects or as composed of several objects.

Skill 2: Build and use models

In grades P-2, students use a variety of models to build their understanding of numbers with increasing complexity, leading to an understanding of the base ten place value system. They model the actions and comparisons represented by the operations of addition and subtraction. Consistent exposure to these models supports fluency and helps students connect concrete experiences to foundational mathematical ideas.

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Skill 3: Attend to precision and reasonableness

In grades 1-2, students refine informal language as they develop and learn precise mathematical vocabulary. Students use structure to accurately record, label, and share their thinking. They explore when a precise answer is necessary and when making an estimate, which can help them determine if a solution makes sense. Students use estimation strategies based on place value structure and the properties of operations to assess reasonableness throughout the problem-solving process.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

In grades K-2, students are learning to communicate their ideas and listen to the ideas of others. They develop strategies to agree and disagree respectfully. They begin to justify their arguments with evidence and draw connections between mathematical representations. Students also begin to compare and contrast their strategies and solutions to others'.

Skill 5: Add or remove context to make sense of mathematics

In grades P-2, students build an understanding of what the numbers, symbols, pictures, words, etc., in their work represent. Students develop flexibility in moving between concrete, pictorial, and abstract representations of mathematical ideas. They begin moving back and forth between a problem's context and its representation to use the form that best fits the situation.

Skill 6: Ask questions to explore mathematical ideas

In grades P-2, students ask questions to help them explore numbers, place value, the properties of addition and subtraction, and the properties of shapes. Students begin to ask questions about data related to the classroom and their day-to-day lives.

Skill 7: Make conjectures and evaluate the results

In grades P-2, students will make and test conjectures related to numbers, place

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value, the properties of addition and subtraction, and the attributes of shapes. As students explore their conjectures, they will learn to evaluate the reasonableness of their solution path and adjust as needed. Providing students opportunities to revise their thinking helps students develop confidence in problem solving.

Skill 8: Select and use tools appropriately and strategically

In grades P-2, students use tools when they help them make sense of mathematics. Tools can include, but are not limited to physical tools, visual tools, and self-created tools such as counters, ten frames, place value charts, hundreds charts, open number lines, strategies, algorithms, etc. Consistent, hands-on practice with these tools builds confidence and prepares students for flexible application.

Standards: Grade 2

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Operations and Algebraic Thinking (OA)

Represent and solve problems involving addition and subtraction. Add and subtract with sums and minuends less than or equal to 20 and work with equal groups of objects to gain foundations for multiplication.

2.OA.1

Build and use models to solve one-step addition and subtraction story problems with sums and minuends less than or equal to 100, and two-step addition and subtraction story problems with sums and minuends less than or equal to 20. *Add or remove story context* to solve problem type situations with unknowns in all positions while attending to the meaning of the equal sign. (Sk 2, 5)

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2.OA.2

Attend to precision and reasonableness when using strategies and properties to add and subtract flexibly, accurately, and efficiently with sums and minuends less than or equal to 20. Know from memory all sums of two one-digit numbers. (Sk 3)

2.OA.3

Describe and represent structures and patterns to determine if a group of up to 20 objects has an odd or even number of items by pairing them or counting by twos. *Construct arguments* about why numbers are even or odd, and write equations to *justify and clearly communicate the argument*. (Sk 1, 4)

2.OA.4

Build and use models involving addition to determine the total number of objects in rectangular arrays with up to 5 rows and up to 5 columns. *Describe and represent the structures and patterns* of the arrays. *Make and evaluate conjectures* about whether they are the same to add by row or column. (Sk 1, 2, 7)

Number and Operations in Base Ten (NBT)

Build and use models to conceptually understand place value. Use place value understanding to add and subtract.

2.NBT.1

Show place value understanding by bundling ten 10s to make a "hundred." Use a variety of *models to represent* the amount of hundreds, tens, and ones in a three-digit number. Identify the number of hundreds, tens, and ones in a three-digit number and describe the value of each digit. (Sk 2)

2.NBT.2

Count within 1,000, *making conjectures* about counting patterns; Skip-count by twos, fives, tens, and hundreds. (Sk 7)

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2.NBT.3

Read three-digit whole numbers and *represent structures in* those numbers using numerals, base 10 word form, and expanded form. (Sk 1)

2.NBT.4

Use tools and place value understanding to compare two three-digit numbers. *Represent the relationship* between the two numbers using comparison symbols <, =, and >. (Sk 1, 8)

2.NBT.5

With attention to precision and reasonableness, use numeric strategies to determine the sum of up to four two-digit numbers, and the difference of two two-digit numbers flexibly, accurately, and efficiently. *Justify* the process using the relationship between addition and subtraction, properties of addition, and/or place value language. (Sk 3, 4)

2.NBT.6

Build and use concrete and pictorial models to determine the sums and differences of three-digit numbers with sums and minuends up to and including 999. Use the models to *represent and describe the structure of* combining hundreds with hundreds, tens with tens, and ones with ones. *Make conjectures about* when it is necessary to compose or decompose hundreds, tens, and ones. *Ask questions to compare* concrete and pictorial models with written numeric strategies. (Sk 1, 2, 6, 7)

2.NBT.7

Mentally add and subtract multiples of 10 or 100 from a three-digit number and *justify the result.* (Sk 4)

Measurement and Geometry (MG)

Measure and estimate lengths in standard units and relate addition and subtraction to length. Work with time and money. Analyze and partition shapes to build a foundation for multiplication, area, and fractions.

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2.MG.1

Select and use appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes to measure the length of objects to the nearest whole unit. (Sk 8)

2.MG.2

Describe and represent the length of an object using two different units of measure. *Describe the relationship* of the two measurements and relate them to the units chosen. (Sk 1)

2.MG.3

Construct, justify, and communicate clear and reasonable arguments when estimating lengths using inches, feet, centimeters, and meters. (Sk 4)

2.MG.4

Attend to precision and reasonableness when measuring to determine how much longer one object is than another and express the length difference in terms of a standard length unit. (Sk 3)

2.MG.5

Build and use models to represent whole numbers as lengths from zero on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2,.... *Strategically use the number line as a tool* to represent whole-number sums and differences within 100 and in story problems. (Sk 2, 8)

2.MG.6

Attend to precision and reasonableness when telling and writing time to the nearest five minutes. *Ask questions* to determine when to use a.m. and p.m. (Sk 3, 6)

2.MG.7

Add or remove context when solving problems involving dollar bills, quarters, dimes, nickels, and pennies. (Sk 5)

2.MG.8

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Ask questions to identify and describe shapes having specific attributes. Build and draw shapes that possess specific attributes. (Sk 6)

2.MG.9

Build and use models to partition a rectangle into rows and columns of same-size squares and count to find the total number of squares. (Sk 2)

2.MG.10

Describe and represent structures, patterns and relationships when partitioning two-dimensional shapes into two, three, or four equal shares; describe the shares using the words halves, thirds, a fourth of, quarters, etc. Recognize that equal parts of identical wholes need not have the same shape. (Sk 1)

Data Science (D)

Ask questions that lead to collecting, analyzing, and interpreting data. Organize and build visual representations of data using features such as titles, labels, and legends. Consider the impact that missing data may have on a data set.

2.D.1

Ask questions to investigate situations that will lead to collecting and analyzing data within the classroom and grade. (Sk 6)

2.D.2

Select and use tools appropriately and strategically to collect and organize data. *Ask questions* to consider a given data set about a classroom or grade level context, and organize that data using more than one categorical variable. (Sk 6, 8)

2.D.3

Attend to precision and reasonableness when analyzing data sets by creating a single-unit scale visualization to organize data using features such as titles, labels, and legends to make sense of the data. (Sk 3)

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2.D.4

Construct, justify, and communicate clear and reasonable arguments while interpreting data to answer investigative questions. Identify missing or incomplete data and the impact missing data may have on interpretation. (Sk 4)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of second grade, children will be able to:

- Fluently add and subtract when the largest number in the equation is no greater than 20. (Basic fact fluency) (OA)
- Connect numerical strategies to concrete and pictorial models to solve one- and two-step addition and subtraction story problems. (OA)
- Identify numbers as odd or even. (OA)
- Fluently add and subtract when the largest number in the equation is no greater than 100. (Computational fluency) (NBT)
- Add and subtract three-digit numbers using objects, pictures, and numerical strategies. (NBT)
- Make sense of place value and the meaning of hundreds, tens, and ones in three-digit numbers. (NBT)
- Use tools to measure and compare the lengths of objects. (MG)
- Analyze and partition shapes and begin using fractional language, for example halves, thirds and fourths. (MG)

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- Organize and build visual representations of data using features such as titles, labels, and legends. (D)

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Grade 3

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Standards: Grade 3

Introduction

In third through fifth grade, the major work of each grade focuses on representing and understanding multiplication and division and developing an understanding of fractions. Children generalize and expand their place value understanding. Measurement concepts are expanded to include two- and three-dimensional measurements.

In Grade 3, instructional time should focus on four major works: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

Mathematical Skills

Link to Detailed Skills Description

Skill 1: Describe and represent structures, patterns, and relationships

In grades 3-5, students explore patterns and structures such as place value, properties of operations, and shapes' attributes. They use structures and patterns to develop a deep understanding of the operations of multiplication and division. They extend their understanding of patterns, structures, and relationships through work with fractions, decimals, and larger whole numbers. Students use structures and patterns to see complicated things as single objects or composed of several objects.

Skill 2: Build and use models

In grades 3-5, students use a variety of models to extend their understanding of the base ten place value system, including decimals. They use models to build their understanding of fractions. They model the actions and comparisons represented

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by all four operations. In the upper grades, students focus on models that represent multiplication and division, including area models, number lines, and other multiplicative comparisons.

Skill 3: Attend to precision and reasonableness

In grades 3-5, students refine informal language as they develop and learn precise mathematical vocabulary. Students use structure to accurately record, label, and share their thinking. They determine when a precise answer is necessary and when making an estimate can help them determine if a solution makes sense. Students use estimation strategies such as rounding, utilizing benchmark fractions, and properties of operations to assess reasonableness through the problem solving process.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

In grades 3-5, students use mathematical vocabulary to communicate their ideas and listen to others' ideas. They utilize strategies to agree and disagree respectfully. They back their argument up with evidence and draw connections between mathematical representations, comparing and contrasting their strategy and solution to others.

Skill 5: Add or remove context to make sense of mathematics

In grades 3-5, students understand what the numbers, symbols, pictures, words, etc. in their work represent when using the four operations. Students flexibly move between concrete, pictorial, and abstract representations of mathematical ideas. They move back and forth between a problem's context and its representation to use the form that best fits the situation.

Skill 6: Ask questions to explore mathematical ideas

In grades 3-5, students ask questions to help them explore multi digit numbers, fractions, decimals, the properties of all four operations, and the classification of shapes. Students ask questions to help them make sense of data and different data

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representations.

Skill 7: Make conjectures and evaluate the results

In grades 3-5, students make and test conjectures related to multi digit numbers, fractions, decimals, the properties of all four operations, and the properties of shapes. As students explore their conjectures, they evaluate the reasonableness of their solution path and adjust as needed.

Skill 8: Select and use tools appropriately and strategically

In grades 3-5, students use tools when they help them understand mathematics. Tools can include, but are not limited to, physical tools, visual tools, and self-created tools such as base ten blocks, fraction tiles, area models, open number lines, strategies, algorithms, etc.

Standards: Grade 3

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Operations and Algebraic Thinking (OA)

Represent and solve problems involving multiplication and division with whole number products and quotients. Demonstrate understanding of the properties of multiplication and the relationship between multiplication and division. Use the four operations to solve multistep problems and to explain patterns in arithmetic.

3.OA.1

Build and use models to interpret the products of whole numbers. (Sk 2)

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3.OA.2

Build and use models to interpret whole-number quotients. (Sk 2)

3.OA.3

Build and use models to solve multiplication and division story problems with products or dividends less than or equal to 100 and whole number quotients. *Add or remove context* to solve situations involving equal groups, arrays, and measurement quantities. (Sk 2, 5)

3.OA.4

Evaluate the unknown whole number in a multiplication or division equation. *Represent* division as an unknown-factor problem, and *describe the relationship* between multiplication and division. (Sk 1)

3.OA.5

Make and assess conjectures about the properties of multiplication. *Add context* and *compare arguments* to make sense of these properties as strategies to multiply and divide with products and dividends less than or equal to 100. (Sk 5, 7)

3.OA.6

Attend to precision and reasonableness when flexibly, accurately, and efficiently multiplying and dividing with products, dividends, and divisors up to and including 100, and quotients up to and including 10. Know from memory all products of two one-digit numbers. (Sk 3)

3.OA.7

Build and use models to solve two-step story problems involving whole numbers and having whole-number answers using the four operations, and attend to *the reasonableness* of answers. *Remove context* to represent situations as equations with a variable standing in for the unknown quantity. (Sk 2, 3, 5)

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3.OA.8

Describe and represent arithmetic patterns using properties of operations. *Evaluate, compare, and justify patterns* involving even and odd numbers and describe how patterns occur through repeated addition or multiplication. (Sk 1, 4)

Number and Operations in Base Ten (NBT)

Use numeric strategies (founded in place value thinking and properties of operations) to perform multi-digit addition, subtraction, and multiplication.

3.NBT.1

Describe and represent place value structure by bundling ten 100s to make a thousand. *Represent* the amount of thousands, hundreds, tens, and ones in a 4-digit number, and *describe* the value of each. (Sk 1)

3.NBT.2

Use models to round 2- or 3-digit whole numbers to the nearest 10 or 100 *in and out of context.* Determine and *justify* situations where rounding is appropriate. (Sk 2, 4, 5)

3.NBT.3

Determine sums and differences of three-digit whole numbers using numeric approaches and *justify* the approach with place value language. (Sk 4)

3.NBT.4

Find the product of a one-digit whole number and a multiple of 10 (10-90). *Make and test conjectures* to generalize patterns that occur when multiplying by a multiple of ten. (Sk 7)

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Number and Operations – Fractions (NF)

Develop an understanding of fractions including whole numbers. Represent fractions, find equivalent fractions, and make sense of fractions in context. Denominators are limited to 2, 3, 4, 6, and 8 in third grade.

3.NF.1

Describe and represent a fraction, *a/b*, as a numerator and a non-zero denominator, including unit fractions and fractions equal to whole numbers. *Describe and represent patterns* to compose and decompose a fraction into its unit fractions. Identify the meaning of the whole for a given context. (Sk 1)

3.NF.2

Build and use number line models to represent fractions, using unit fractions as intervals. Connect number line models with area models to make sense of fractions. (Sk 2)

3.NF.3

Build and use models to make sense of and generate equivalent fractions including whole numbers. (Sk 2)

3.NF.4

Compare fractions with like numerators or like denominators, and *justify* the comparisons *using models* or by reasoning about their size. (Sk 2, 4)

3.NF.5

Model and represent fractions in *real-world contexts. Evaluate the reasonableness* of the fraction within the context**.** (Sk 2, 3, 5)

Measurement and Geometry (MG)

Solve problems involving estimated and precise measurements. Develop an understanding of area and perimeter. Relate area to multiplication and division. Reason with shapes and their attributes.

3.MG.1

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Select and use tools appropriately and strategically to make reasonable estimates and find measurements involving length, weight, and liquid volume. (Sk 8)

3.MG.2

Attend to precision and reasonableness when telling and writing time to the nearest minute. (Sk 3)

3.MG.3

Add or remove context to solve one-step story problems involving length, liquid volumes, masses of objects, and time intervals in minutes. (Sk 5)

3.MG.4

Recognize area and perimeter as attributes of plane figures and *build and use models* to understand the concept of unit squares and area. *Select and use tools strategically and appropriately* to distinguish between linear (perimeter) and area measures. Use tiling to show that the area of a rectangle is the same as would be found by multiplying side lengths. (Sk 2, 8)

3.MG.5

Build and use area models to represent the distributive property and relate the representations to the operations of addition and multiplication. (Sk 2)

3.MG.6

Add and remove context to solve real-world and mathematical problems involving perimeters of simple polygons and rectangular areas. *Make and evaluate conjectures* about rectangles with the same perimeter and different areas or with the same area and different perimeters. Describe how these measures are related. (Sk 5, 7)

3.MG.7

Describe and represent relationships between quadrilaterals and *build models* of quadrilaterals that do not belong to a given subcategory. (Sk 1, 2)

3.MG.8

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Attend to precision and reasonableness when partitioning shapes into parts with equal areas and expressing the area of each part as a unit fraction of the whole. (Sk 3)

Data Science (D)

Identify whether questions will result in quantitative or categorical data collection. Represent data sets with more than three categories using scaled visualizations and determine the benefits and drawbacks of different visual representations and scales. Identify patterns, trends, and outliers in a data set.

3.D.1

Ask questions to investigate situations that will lead to collecting and analyzing data within the classroom, school, or community. Identify whether the question asked will result in quantitative or categorical data. (Sk 6)

3.D.2

Select and use tools appropriately and strategically to generate and collect data. *Ask questions* to consider a given data set about classroom, school, or community contexts. Identify data as quantitative or categorical. (Sk 6, 8)

3.D.3

Attend to precision and reasonableness when analyzing data by creating scaled visualizations representing a data set with more than three categories. *Make and evaluate conjectures* about benefits and drawbacks of different visual representations and scales used to analyze a set of data. (Sk 3, 7)

3.D.4

Construct, justify and communicate clear and reasonable arguments while interpreting data, including scaled visualizations, to answer investigative questions. Begin identifying patterns, trends, and outliers in the data. (Sk 4)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in

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future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of third grade, children will be able to:

- Use pictures, objects, and equations to solve multiplication and division story problems when the largest number in the story is no greater than 100. (OA)
- Fluently multiply two one-digit numbers. (Basic fact fluency) (OA)
- Fluently divide numbers, with no remainders, when the largest number in the equation is no greater than 100. (Basic fact fluency) (OA)
- Describe and represent a fraction as a numerator and denominator, a fraction as a part of a whole, and as a location on a number line. (NF)
- Build models to compare fractions and to generate equivalent fractions. (NF)
- Use models to determine the area and perimeter of rectangles. (MG)
- Represent data sets with more than three categories using scaled visualizations and identify patterns, trends, and outliers in a data set. (D)

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Grade 4

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Standards: Grade 4

Introduction

In third through fifth grade, the major work of each grade focuses on representing and understanding multiplication and division and developing an understanding of fractions. Children generalize and expand their place value understanding. Measurement concepts are expanded to include two- and three-dimensional measurements.

In Grade 4, instructional time should focus on four major works: (1) generalizing understanding of place value to 1,000,000, understanding the multiplicative relationship between the value of each place; (2) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (3) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (4) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

Mathematical Skills

Link to Detailed Skills Description

Skill 1: Describe and represent structures, patterns, and relationships

In grades 3-5, students explore patterns and structures such as place value, properties of operations, and shapes' attributes. They use structures and patterns to develop a deep understanding of the operations of multiplication and division. They extend their understanding of patterns, structures, and relationships through work with fractions, decimals, and larger whole numbers. Students use structures and patterns to see complicated things as single objects or composed of several objects.

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Skill 2: Build and use models

In grades 3-5, students use a variety of models to extend their understanding of the base ten place value system, including decimals. They use models to build their understanding of fractions. They model the actions and comparisons represented by all four operations. In the upper grades, students focus on models that represent multiplication and division, including area models, number lines, and other multiplicative comparisons.

Skill 3: Attend to precision and reasonableness

In grades 3-5, students refine informal language as they develop and learn precise mathematical vocabulary. Students use structure to accurately record, label, and share their thinking. They determine when a precise answer is necessary and when making an estimate can help them determine if a solution makes sense. Students use estimation strategies such as rounding, utilizing benchmark fractions, and properties of operations to assess reasonableness through the problem solving process.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

In grades 3-5, students use mathematical vocabulary to communicate their ideas and listen to others' ideas. They utilize strategies to agree and disagree respectfully. They back their argument up with evidence and draw connections between mathematical representations, comparing and contrasting their strategy and solution to others.

Skill 5: Add or remove context to make sense of mathematics

In grades 3-5, students understand what the numbers, symbols, pictures, words, etc. in their work represent when using the four operations. Students flexibly move between concrete, pictorial, and abstract representations of mathematical ideas. They move back and forth between a problem's context and its representation to

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use the form that best fits the situation.

Skill 6: Ask questions to explore mathematical ideas

In grades 3-5, students ask questions to help them explore multi digit numbers, fractions, decimals, the properties of all four operations, and the classification of shapes. Students ask questions to help them make sense of data and different data representations.

Skill 7: Make conjectures and evaluate the results

In grades 3-5, students make and test conjectures related to multi digit numbers, fractions, decimals, the properties of all four operations, and the properties of shapes. As students explore their conjectures, they evaluate the reasonableness of their solution path and adjust as needed.

Skill 8: Select and use tools appropriately and strategically

In grades 3-5, students use tools when they help them understand mathematics. Tools can include, but are not limited to, physical tools, visual tools, and self-created tools such as base ten blocks, fraction tiles, area models, open number lines, strategies, algorithms, etc.

Standards: Grade 4

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Operation and Algebraic Thinking (OA)

Understand multiplication as a comparison. Use the four operations to solve multistep problems in which remainders must be interpreted. Explore the relationship between factors and multiples. Generate and interpret multiplicative patterns.

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4.OA.1

Build and use models to visualize a multiplication equation as a comparison. *Represent* verbal statements of multiplicative comparisons as multiplication equations. (Sk 1, 2)

4.OA.2

Build and use models (including drawings and equations with a symbol for the unknown number) to solve story problems involving multiplicative comparison. *Make and evaluate conjectures* about the difference between multiplicative and additive comparison. (Sk 2, 7)

4.OA.3

Use and name properties of operations to *justify* strategies used to solve problems involving the four operations. (Sk 4)

4.OA.4

Build and use models to solve multi-step story problems involving whole numbers using the four operations, including problems in which remainders must be interpreted, and *attend to the reasonableness* of answers. *Remove context* to represent situations as equations with a variable standing in for the unknown quantity. (Sk 2, 3, 5)

4.OA.5

Ask questions to explore the mathematical idea that a whole number is a multiple of its factors. *Make and evaluate conjectures* that all whole numbers are divisible by their factors. (Sk 6, 7)

4.OA.6

Generate a number or shape pattern based on a given rule. *Describe and represent* the features of the pattern that are not explicitly stated in the rule itself. *Make conjectures* as to how the pattern will continue. (Sk 1, 7)

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Number and Operations in Base Ten (NBT)

Use place value understanding to make multiplicative comparisons. Write, compare, and round whole numbers less than or equal to 1,000,000. Use numeric strategies, place value understanding, and properties of operations to perform multi-digit addition, subtraction, multiplication, and division.

4.NBT.1

Describe patterns in place value highlighting how the value of one place represents ten times more than the place to its right. *Justify and represent those patterns* with a statement of multiplicative comparison. (Sk 1, 4)

4.NBT.2

Read multi-digit whole numbers up to and including 1,000,000 and *represent* multi-digit whole numbers using numerals and expanded form. (Sk 1)

4.NBT.3

Compare two whole numbers with up to 6 digits and *represent the relationship* between two numbers using comparison symbols <, =, and >. (Sk 1)

4.NBT.4

Apply knowledge of place value structure to round whole numbers up to and including six digits *in and out of context*. Determine and *justify* situations where rounding is appropriate. (Sk 4, 5)

4.NBT.5

Attend to precision when using efficient algorithms to determine sums and differences of multi-digit whole numbers in real-world problems and equations flexibly, accurately, and efficiently. (Sk 3)

4.NBT.6

Find the product of up to four digits by a one-digit whole number, and two, two-digit numbers in real-life situations or equations. *Develop visual and numeric models* that demonstrate place value and/or the properties of multiplication. (Sk 2)

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4.NBT.7

Find the quotient of up to a three-digit dividend and one-digit divisor. *Develop visual and numeric models* that demonstrate place value and the relationship between multiplication and division**;** *justify and communicate* the meaning of the remainder. (Sk 2, 4)

Number and Operations–Fractions (NF)

Extend understanding of equivalence and ordering fractions. Build on previous understanding of fractions to begin operating with fractions and explore the relationship between fractions and decimals. Understand decimal notation to the hundredths. Denominators for fourth grade are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.

4.NF.1

Build and use models to compose and decompose fractions greater than one and mixed numbers. Recognize that a value can be expressed as a fraction greater than one and an equivalent mixed number. *Justify* the equivalence of the representations. (New Standard) (Sk 2, 4)

4.NF.2

Make and assess conjectures about the identity property of multiplication as a strategy to recognize and generate equivalent fractions. Connect numeric strategies to visual fraction models. (Sk 7)

4.NF.3

Construct, justify, and communicate clear and reasonable arguments to compare and order fractions with different numerators and denominators. (Sk 4)

4.NF.4

Build and use models to add and subtract fractions with like denominators, including mixed numbers. *Add or remove context* to solve problems involving addition and subtraction of fractions. (Sk 2, 5)

4.NF.5

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Build and use models to multiply a fraction by a whole number. Apply and extend previous understanding of multiplication *structures, patterns, and relationships* when using numeric strategies to multiply a whole number and a fraction. *Add or remove context* to solve problems involving multiplying a fraction by a whole number. (Sk 1, 2, 5)

4.NF.6

Build and use models to express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. (Sk 2)

4.NF.7

Construct, justify, and communicate clear and reasonable arguments to connect decimal notation to fractions with denominators 10 or 100. (Sk 4)

4.NF.8

Compare two decimals to hundredths by reasoning about their size, and *justify* the comparison. (Sk 4)

Measurement and Geometry (MG)

Solve problems involving measurement and conversion of measurements. Solve real-world area and perimeter problems. Understand various concepts of angles and angle measurement. Draw and identify lines and angles, as well as classify shapes by properties of their lines and angles.

4.MG.1

Add and remove context to solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit within the same system given a conversion key. Record the measurement equivalents in a two-column table. (Sk 5)

4.MG.2

Attend to precision and reasonableness when using the four operations involving distances, time intervals, liquid volumes, masses of objects, and money. Include one-step problems involving simple fractions and decimals. (Sk 3)

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4.MG.3

Add and remove context to solve real-world and mathematical problems involving the area and perimeter of rectangles and rectilinear shapes. *Build and use models* to make sense of the area and perimeter formulas for rectangles. (Sk 2, 5)

4.MG.4

Draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. *Describe these structures and their relationships* within two-dimensional figures. (Sk 1)

4.MG.5

Describe and represent the structure and relationship of angles as geometric figures that are formed when two rays share a common endpoint. *Describe the relationship* of the measure of angles in reference to a circle. (Sk 1)

4.MG.6

Attend to precision and reasonableness when measuring or sketching angles in whole-number degrees using a protractor. (Sk 3)

4.MG.7

Understand and demonstrate the additive nature of angle measures. Solve addition and subtraction problems to find unknown angles on a diagram in *real-world and mathematical problems*. (Sk 5)

4.MG.8

Compare and classify two-dimensional figures including quadrilaterals and triangles based on the presence or absence of: parallel lines, perpendicular lines, and angles of specified sizes. *Justify and evaluate* the classification. (Sk 4)

4.MG.9

Build and use models to represent a line of symmetry for a two-dimensional figure. (Sk 2)

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Data Science (D)

Investigate and analyze situations with large data sets. Identify and discuss potential sources of bias in data collection, representation, and interpretation, and evaluate how data bias may affect how people draw conclusions from data.

4.D.1

Ask questions to investigate situations that will lead to collecting and analyzing data within the classroom, school, community, or state. (Sk 6)

4.D.2

Select and use tools appropriately and strategically to generate and collect data. *Ask questions* to consider a given data set about the classroom, school, community, or state contexts. *Make and evaluate conjectures* about the impact of the sampling process and sample size on results. (Sk 6, 7, 8)

4.D.3

Attend to precision and reasonableness when analyzing data by creating visual representations. *Make and evaluate conjectures* about how different visualizations might influence interpretation of data. (Sk 3, 7)

4.D.4

Construct, justify and communicate clear and reasonable arguments while interpreting data to answer investigative questions. Identify and discuss potential sources of bias in data collection, representation, and interpretation, and evaluate how data bias may affect how people draw conclusions from data. (Sk 4)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge

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embedded within the *Utah Core Mathematics Standards*, by the end of fourth grade, children will be able to:

- Understand multiplication as a comparison. (OA)
- Solve whole number, multip-step word problems involving addition, subtraction, multiplication, and division. (OA)
- Generalize place value understanding for multi-digit whole numbers (less than 1,000,000) by analyzing patterns, representing whole numbers in a variety of ways, and making comparisons. (NBT)
- Use efficient algorithms to fluently add and subtract. (Computational and procedural fluency) (NBT)
- Use models to represent and make sense of two-digit by two-digit multiplication and division with one-digit divisors (ex, 125 ÷ 5 = 25 where 5 is the divisor). (NBT)
- Make connections between numeric strategies and visual models to compare, order, and determine if fractions have equal values. (NF)
- Build and use models to add and subtract fractions with like denominators and to multiply fractions by a whole number. (NF)
- Build and use models to connect decimal values to fractions with denominators of tenths and hundredths. (NF)
- Classify shapes based on the presence or absence of specific types of lines and angles. (MG)
- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit given a conversion key. (MG)
- Apply knowledge of area and perimeter to solve real-world and mathematical problems. (MG)
- Identify and discuss potential sources of bias in data collection, representation, and interpretation, and evaluate how data bias may affect how people draw conclusions from data. (D)

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Grade 5

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Standards: Grade 5

Introduction

In third through fifth grade, the major work of each grade focuses on representing and understanding multiplication and division and developing an understanding of fractions. Children generalize and expand their place value understanding. Measurement concepts are expanded to include two- and three-dimensional measurements.

In Grade 5, instructional time should focus on three major works: (1) extend understanding of addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) solidify place value understanding to develop understanding of operations with decimals to hundredths and with whole number and decimal operations; and (3) develop an understanding of volume.

Mathematical Skills

Link to Detailed Skills Description

Skill 1: Describe and represent structures, patterns, and relationships

In grades 3-5, students explore patterns and structures such as place value, properties of operations, and shapes' attributes. They use structures and patterns to develop a deep understanding of the operations of multiplication and division. They extend their understanding of patterns, structures, and relationships through work with fractions, decimals, and larger whole numbers. Students use structures and patterns to see complicated things as single objects or composed of several objects.

Skill 2: Build and use models

In grades 3-5, students use a variety of models to extend their understanding of the

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base ten place value system, including decimals. They use models to build their understanding of fractions. They model the actions and comparisons represented by all four operations. In the upper grades, students focus on models that represent multiplication and division, including area models, number lines, and other multiplicative comparisons.

Skill 3: Attend to precision and reasonableness

In grades 3-5, students refine informal language as they develop and learn precise mathematical vocabulary. Students use structure to accurately record, label, and share their thinking. They determine when a precise answer is necessary and when making an estimate can help them determine if a solution makes sense. Students use estimation strategies such as rounding, utilizing benchmark fractions, and properties of operations to assess reasonableness through the problem solving process.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

In grades 3-5, students use mathematical vocabulary to communicate their ideas and listen to others' ideas. They utilize strategies to agree and disagree respectfully. They back their argument up with evidence and draw connections between mathematical representations, comparing and contrasting their strategy and solution to others.

Skill 5: Add or remove context to make sense of mathematics

In grades 3-5, students understand what the numbers, symbols, pictures, words, etc. in their work represent when using the four operations. Students flexibly move between concrete, pictorial, and abstract representations of mathematical ideas. They move back and forth between a problem's context and its representation to use the form that best fits the situation.

Skill 6: Ask questions to explore mathematical ideas

In grades 3-5, students ask questions to help them explore multi digit numbers,

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fractions, decimals, the properties of all four operations, and the classification of shapes. Students ask questions to help them make sense of data and different data representations.

Skill 7: Make conjectures and evaluate the results

In grades 3-5, students make and test conjectures related to multi digit numbers, fractions, decimals, the properties of all four operations, and the properties of shapes. As students explore their conjectures, they evaluate the reasonableness of their solution path and adjust as needed.

Skill 8: Select and use tools appropriately and strategically

In grades 3-5, students use tools when they help them understand mathematics. Tools can include, but are not limited to, physical tools, visual tools, and self-created tools such as base ten blocks, fraction tiles, area models, open number lines, strategies, algorithms, etc.

Standards: Grade 5

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Operations and Algebraic Thinking (OA)

Explore prime and composite numbers. Write and interpret numerical expressions. Justify and generalize understanding of properties of operations. Analyze patterns and relationships.

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5.OA.1

Ask questions to explore the mathematical ideas of prime and composite numbers. *Describe and represent patterns* to *justify* whether a given whole number between 1 and 100 is prime or composite. (Sk 1, 4, 6)

5.OA.2

Add context to make sense of multi-step problems using multiple operations that involve parentheses, brackets, or braces when given, and *remove context* to write and/or evaluate numerical expressions. Interpret expressions without evaluating to make sense of the expressions. (Sk 5)

5.OA.3

Make conjectures about solving problems involving addition, subtraction, multiplication, and division by connecting properties of operations. *Justify* why the strategy will always work. (Sk 4, 7)

5.OA.4

Generate two numerical patterns using two given rules. *Describe and represent relationships* between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. (Sk 1)

Number and Operations in Base Ten (NBT)

Understand structures and patterns in the place value system including decimals and powers of ten. Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.1

Describe the structure of place value by explaining that the value of one place represents ten times as much as the place to its right and 1/10 of the place to its left. *Justify and represent those patterns* with a statement of multiplicative comparison. (Sk 1, 4)

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5.NBT.2

Represent powers of 10 with whole number exponents**.** *Make conjectures about* and *justify* a decimal's placement when multiplying by powers of 10. (Sk 1, 4, 7)

5.NBT.3

Describe and represent decimals to the thousandths place using models, base-ten numerals, number names, and expanded forms. (Sk 1)

5.NBT.4

Compare two decimal numbers and *represent the relationship* between the two numbers using comparison symbols <, =, and >. (Sk 1)

5.NBT.5

Apply knowledge of place value structure to round decimal numbers to the nearest whole, tenth, or hundredth *in and out of context*. Determine and *justify* situations where rounding is appropriate. (Sk 4, 5)

5.NBT.6

Attend to precision and reasonableness when finding the product of up to and including four digits and a one-digit whole number, or to multiply two, two-digit numbers flexibly, accurately, and efficiently. Select numeric strategies that demonstrate properties of multiplication and/or place value. (Sk 3)

5.NBT.7

Add or remove context to find quotients with remainders with up to a four-digit dividend and one-digit divisor. Select numeric strategies that demonstrate properties of multiplication and/or place value. *Justify* and communicate the meaning of the remainder. (Sk 4, 5)

5.NBT.8

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Build and use models to find sums and differences of decimals to hundredths. *Describe the process* of adding tenths with tenths and hundredths with hundredths. *Add or remove context* to solve addition and subtraction problems involving decimals. (Sk 1, 2, 5)

5.NBT.9

Build and use models to find products of decimals to thousandths. *Add or remove context* to solve multiplication problems involving decimals. (Sk 2, 5)

5.NBT.10

Build and use models to find quotients of decimals limited to a whole number dividend with a decimal divisor of tenths between 0 and 1, or a decimal dividend with a single-digit whole number divisor. *Add or remove context* to solve division problems involving decimals. (Sk 2, 5)

Number and Operations–Fractions (NF)

Use equivalent fractions as a strategy to add and subtract fractions. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.1

Attend to precision and reasonableness when using numerical strategies to add and subtract fractions with unlike denominators. *Add or remove context* to solve problems involving addition and subtraction with unlike denominators. (Sk 3, 5)

5.NF.2

Interpret a fraction as division (a/b is the same as $a \div b$). *Add or remove context* to solve problems involving division, including situations where the whole represents a set of multiple items. (Sk 5)

5.NF.3

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Build and use models, including area models, to multiply fractions, whole numbers, and mixed numbers. *Add and remove context* to solve problems involving multiplication of fractions and mixed numbers. (Sk 2, 5)

5.NF.4

Interpret multiplication as scaling. *Construct, justify, and communicate clear and reasonable arguments* about the size of the resulting product when multiplying a given number by a fraction greater than one, a fraction less than one, a fraction equal to one. *Make conjectures* about the size of the product based on the size of the factors without computing. (Sk 4, 7)

5.NF.5

Build and use models, to divide unit fractions by whole numbers and whole numbers by unit fractions. *Add and remove context* to solve problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions. *Make conjectures* about the relationship between multiplication and division when operating with fractions. (Sk 2, 5, 7)

Measurement and Geometry (MG)

Convert like measurement units. Develop an understanding of volume. Develop an understanding of quadrant one in the coordinate plane. Classify two-dimensional figures based on their attributes.

5.MG.1

Describe and represent the relationship between different-sized standard measurement units and convert among the measurement units given a specific measurement system and conversion key. *Add or remove context* to solve real-world, one and two-step problems using conversions. (Sk 1, 5)

5.MG.2

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Recognize volume as an attribute of solid figures. *Build and use models* to understand the concept of volume. Demonstrate that unit cubes can be packed into rectangular prisms. Determine volume by counting unit cubes and expressing the volume in cubic units. (Sk 2)

5.MG.3

Build and use models to make sense of and apply the formulas for finding the volume of right rectangular prisms. *Add or remove context to solve real-world problems* involving the volume of right rectangular prisms with whole-number side lengths. (Sk 2, 5)

5.MG.4

Build and use a model of the coordinate plane to represent real-world problems. Graph points in the first quadrant of the coordinate plane and interpret coordinate values. (Sk 2)

5.MG.5

Describe and represent structure, patterns, and relationships when classifying two-dimensional figures in a hierarchy based on attributes. (Sk 1)

Data Science (D)

Investigate and analyze situations with large data sets. Interpret data to answer investigative questions, consider the impact of bias, representation, and sampling approaches. Explore features of distribution and probability.

5.D.1

Ask questions to investigate situations that will lead to collecting and analyzing data within the classroom, school, community, state, or country. (Sk 6)

5.D.2

Select and use tools appropriately and strategically to generate and collect data. *Ask questions* to consider a given data set about classroom, school, community, state, or

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country contexts. Compare two potential sampling approaches, and *make and evaluate conjectures* about the impact on results. (Sk 6, 7, 8)

5.D.3

Attend to precision and reasonableness when analyzing data by creating multiple visual representations. *Make and evaluate conjectures* about features of distributions such as center and range. (Sk 3, 7)

5.D.4

Construct, justify, and communicate clear and reasonable arguments while interpreting data to answer investigative questions, considering the impact of bias, representation, and sample size. Express confidence levels in statements based on the data available, such as “likely,” “unlikely,” “possible,” or “strong evidence” and “weak evidence.” (Sk 4)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge

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embedded within the *Utah Core Mathematics Standards*, by the end of fifth grade, children will be able to:

- Generalize place value understanding for decimals to the thousandths place by analyzing patterns, representing numbers in a variety of ways, and making comparisons. (NBT)
- Fluently multiply multi-digit whole numbers by a one-digit whole number and two two-digit whole numbers. (Computational and procedural fluency) (NBT)
- Use numeric strategies to solve division problems. Communicate the meaning of the remainder. (NBT)
- Build and use models to solve addition, subtraction, multiplication and division problems involving decimals. (NBT)
- Use numeric strategies to add and subtract fractions with unlike denominators. (NF)
- Build and use models to multiply fractions, whole numbers, and mixed numbers. (NF)
- Recognize volume as an attribute of a solid shape and explain how multiplication and addition relate to volume. (MG)
- Classify two-dimensional figures in a hierarchy. (MG)
- Interpret data to answer investigative questions, consider the impact of bias, representation, and sampling approaches. (D)

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Grade 6

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Standards: Grade 6

Introduction

In sixth through eighth grade, the major work of each grade focuses on applying and using operations with rational numbers, understanding ratio concepts and applying proportional reasoning, and simplifying expressions and solving equations.

In sixth grade, instructional time should focus on four major works: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) interpreting data to answer investigative questions.

Mathematical Skills

Link to Detailed Skills Description

Skill 1: Describe and represent structures, patterns, and relationships

In grades 6-8, students describe and represent their mathematical understanding of structures, pattern and relationships in circumstances including, but not limited to: extending their understanding of multiplication and division in order to divide fractions by fractions; to develop and apply properties of integers of exponents to generate equivalent numerical expressions and to use the structure of similar triangles to explain why the slope of a non-vertical line is constant between any two points on the coordinate plane.

Skill 2: Build and use models

In grades 6-8, students are asked to model under various circumstances, including building visual models of fractions to extend their understanding of multiplication and division, to identify the constant of proportionality and solve multi-step problems involving proportional relationships, and to analyze and solve systems of

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two linear equations.

Skill 3: Attend to precision and reasonableness

In grades 6-8, students can apply this skill when adding, multiplying, and dividing multi-digit whole numbers and decimals, attend to the reasonableness of solutions when solving multi-step, real world problems using positive and negative rational numbers, and when finding rational approximations of irrational numbers.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

In grades 6-8, students construct, justify, and communicate arguments in various mathematical contexts such as: solving problems involving ratios and rates and explaining their thinking about proportional relationships, to justify and explain the logical steps needed to solve multi-step equations and inequalities, and to communicate informal arguments to establish angle relationships for parallel lines cut by a transversal and angle relationships for triangles.

Skill 5: Add or remove context to make sense of mathematics

In grades 6-8, students will add and remove context when working with problems involving the division of fractions by fractions to make sense of the situations and interpret the resulting quotients; to make sense of the addition and subtraction of integer numbers, and use context to determine when it is appropriate to approximate the value of expressions containing irrational numbers.

Skill 6: Ask questions to explore mathematical ideas

In grades 6-8, students will formulate questions to explore mathematical ideas involving real-world problems that use rate, ratio, and percents; in the context of data analysis, students will interpret quantitative bivariate data to answer investigative questions, asking questions about patterns, associations, and potential outliers within the data.

Skill 7: Make conjectures and evaluate the results

In grades 6-8, students will develop and assess conjectures related to proportional

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relationships and to apply the concept of unit rate and percent; to develop and assess conjectures about the properties of irrational numbers by comparing them to the set of rational numbers.

Skill 8: Select and use tools appropriately and strategically

In grades 6-8, students use algebraic tools to generate equivalent expressions and to solve multi-step equations using positive and negative rational numbers, and students will use reasoning and operational tools to perform operations with scientific notation.

Standards: Grade 6

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Algebra (A)

Build on previous learning in Operations and Algebraic Thinking to reason with and solve one-variable equations and inequalities. Students will also represent and analyze quantitative relationships between dependent and independent variables.

6.A.1

Describe and represent patterns and structures of expressions to write, read, and evaluate algebraic expressions involving whole-number exponents and variables. (Sk 1)

6.A.2

Strategically use tools to generate equivalent expressions. Use properties of operations within a *context* to identify when two expressions are equivalent. (Sk 5, 8)

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6.A.3

Use of substitution to reason about whether a given number in a specified set makes an equation or inequality true. Provide *justification* to support that reasoning. (Sk 4)

6.A.4

Describe, represent and solve real-world and mathematical problems using variables within the *structure* of one-step equations. (Sk 1)

6.A.5

Describe, represent and solve real-world and mathematical problems using variables within the *structure* of one-step inequalities. (Sk 1)

6.A.6

Create equations to *model* relationships between two quantities *in and out of context.* (Sk 2, 5)

6.A.7

Build and use models including tables, graphs, and equations to represent and understand the relationship between independent and dependent variables. (Sk 2)

Number Systems (NS)

Build on previous work involving operations to compute fluently with multi-digit numbers, decimal and fractions. Understand that there is a set of numbers called integers and create models to show and describe integers and their relationships.

6.NS.1

Describe and represent structures and patterns to extend understanding of multiplication and division in order to divide fractions by fractions. *Build and use visual fraction models* to make sense of and solve problems of dividing fraction by

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fractions. *Add and remove context with* problems involving division of fractions by fractions and interpret the quotients. (Sk 1, 2, 5)

6.NS.2

Describe and represent structures and patterns to apply an algorithm to divide multi-digit decimals flexibly, accurately, and efficiently. Limit to a whole number dividend with a decimal divisor or a decimal dividend with a whole number divisor. Determine and *justify* situations where estimation is appropriate. (Sk 1, 4)

6.NS.3

Attend to precision and reasonableness when adding, subtracting, multiplying, and dividing multi-digit whole numbers and decimals flexibly, accurately, and efficiently. (Sk 3)

6.NS.4

Find the greatest common factor between two whole numbers 1 to 100 and the least common multiples of two whole numbers 1 to 12 by making use of *structures and patterns*. Relate greatest common factors to the distributive property. For example, re-express 36 + 8 as 4(9 + 2). (Sk 1)

6.NS.5

Use positive and negative numbers together to represent quantities in *real world contexts*, explain the meaning of zero in *context*. *Communicate* understanding that

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positive and negative numbers are used together to describe quantities having opposite directions or values. (Sk 4,5)

6.NS.6

Build and use models to show positive and negative numbers on a number line and coordinate plane. Use these models to order positive and negative numbers. (Sk 2)

6.NS.7

Build and use models to develop the definition of absolute value. Work with absolute value *in and out of context*. (Sk 2, 5)

Ratio and Proportional Relationships (RP)

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1

Clearly communicate and justify understanding of ratio relationships including unit rates. *Build and use models* to represent real-world ratio relationships. (Sk 2, 4)

6.RP.2

Construct, justify and communicate clear and reasonable arguments using ratio reasoning to solve problems involving ratios and rates. *Ask questions* about real world contexts involving rate, ratio and percent. (Sk 4, 6)

Geometry (G)

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.1

Describe and represent structures, patterns, and relationships when finding the area of right triangles, other triangles, special quadrilaterals, and polygons by composing and decomposing into rectangles, triangles and/or other shapes. (Sk 1)

6.G.2

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Describe and represent structures, patterns, and relationships when representing the nets of 3-dimensional figures composed of rectangles and triangles. *Build and use* nets to make sense of surface area. (Sk 1, 2)

6.G.3

Select and use tools appropriately to solve real-world and mathematical problems by *contextualizing and decontextualizing* the surface area and volume of right rectangular prisms with fractional edge lengths.
(Sk 5, 8)

6.G.4

Contextualize and decontextualize polygons on the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same *x* coordinate or the same *y* coordinate. (Sk 5)

Data Science (D)

Describe and represent structures and patterns of statistical variability of data. Interpret data to answer investigative questions about distributions, considering the interpretation of the following features of distributions: center, variability, overall shape, and deviations.

6.D.1

Ask questions that anticipate variability in data collection to investigate situations that lead to analyzing data. (Sk 6)

6.D.2

Select and use tools appropriately and strategically to generate and collect data. *Ask questions* about how the data was collected and whether the data is useful to answer the statistical question of interest. (Sk 6, 8)

6.D.3

Attend to precision and reasonableness when exploring data by creating and analyzing multiple visual representations, including plots on a number line, dot plots, histograms and box plots. *Justify* which graph or plot would best represent the data. Analyze data by interpreting numerical summaries for measures of center

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(mean and median) and measures of variability (inter-quartile range and/or mean absolute deviation) and recognize how these numbers measure how the data's values vary with a single number. (Sk 3, 4)

6.D.4

Construct, justify, and communicate clear and reasonable arguments while interpreting data to answer investigative questions about distributions, considering the Interpretation of the following features of distributions: center (mean and median), variability (inter-quartile range and/or mean absolute deviation), and overall shape (symmetric or non-symmetric), as well as describing any striking deviations (outliers). (Sk 4)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge

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embedded within the *Utah Core Mathematics Standards*, by the end of sixth grade, students will be able to:

- Apply previous understanding of arithmetic to algebraic expressions including variables and exponents. (A)
- Reason with and solve one-variable equations and analyze the relationship between independent and dependent variables in a real-world context. (A)
- Fluently add, subtract, multiply, and divide with multi-digit numbers, fractions and decimals. (Computational and procedural fluency) (NS)
- Find the greatest common factor and common multiples of two whole numbers. (NS)
- Define, identify, and explain the meaning of integers on number lines, coordinate grids, and in real-world context. (NS)
- Define the absolute value of an integer and apply the concept to solve real world problems. (NS)
- Build and use models to represent real-world ratio relationships and solve ratio/rate problems. (RP)
- Solve real-world and mathematical problems involving area, surface area, and volume, with and without context. (G)
- Answer investigative questions about distributions, considering the interpretation of the following features: center, variability, overall shape, and deviations. (D)

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Grade 7

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Standards: Grade 7

Introduction

In sixth through eighth grade, the major work of each grade focuses on applying and using operations with rational numbers, understanding ratio concepts and applying proportional reasoning, and simplifying expressions and solving equations.

In seventh grade, instructional time should focus on four major works: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

Mathematical Skills

Link to Detailed Skills Description

Skill 1: Describe and represent structures, patterns, and relationships

In grades 6-8, students describe and represent their mathematical understanding of structures, pattern and relationships in circumstances including, but not limited to: extending their understanding of multiplication and division in order to divide fractions by fractions; to develop and apply properties of integers of exponents to generate equivalent numerical expressions and to use the structure of similar triangles to explain why the slope of a non-vertical line is constant between any two points on the coordinate plane.

Skill 2: Build and use models

In grades 6-8, students are asked to model under various circumstances, including building visual models of fractions to extend their understanding of multiplication and division, to identify the constant of proportionality and solve multi-step

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problems involving proportional relationships, and to analyze and solve systems of two linear equations.

Skill 3: Attend to precision and reasonableness

In grades 6-8, students can apply this skill when adding, multiplying, and dividing multi-digit whole numbers and decimals, attend to the reasonableness of solutions when solving multi-step, real world problems using positive and negative rational numbers, and when finding rational approximations of irrational numbers.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

In grades 6-8, students construct, justify, and communicate arguments in various mathematical contexts such as: solving problems involving ratios and rates and explaining their thinking about proportional relationships, to justify and explain the logical steps needed to solve multi-step equations and inequalities, and to communicate informal arguments to establish angle relationships for parallel lines cut by a transversal and angle relationships for triangles.

Skill 5: Add or remove context to make sense of mathematics

In grades 6-8, students will add and remove context when working with problems involving the division of fractions by fractions to make sense of the situations and interpret the resulting quotients; to make sense of the addition and subtraction of integer numbers, and use context to determine when it is appropriate to approximate the value of expressions containing irrational numbers.

Skill 6: Ask questions to explore mathematical ideas

In grades 6-8, students will formulate questions to explore mathematical ideas involving real-world problems that use rate, ratio, and percents; in the context of data analysis, students will interpret quantitative bivariate data to answer investigative questions, asking questions about patterns, associations, and

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potential outliers within the data.

Skill 7: Make conjectures and evaluate the results

In grades 6-8, students will develop and assess conjectures related to proportional relationships and to apply the concept of unit rate and percent; to develop and assess conjectures about the properties of irrational numbers by comparing them to the set of rational numbers.

Skill 8: Select and use tools appropriately and strategically

In grades 6-8, students use algebraic tools to generate equivalent expressions and to solve multi-step equations using positive and negative rational numbers, and students will use reasoning and operational tools to perform operations with scientific notation.

Standards: Grade 7

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Algebra (A)

Use properties of operations to generate equivalent expressions. Represent and solve mathematical problems using algebraic expressions and equations.

7.A.1

Use tools strategically to apply properties of operations (add, subtract, factor and expand) linear expressions with rational coefficients in different forms. (Sk 8)

7.A.2

Use tools strategically to solve multi-step, real-world problems using positive and negative rational numbers. *Attend to the reasonableness* of the solutions. (Sk 3, 8)

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7.A.3

Construct and justify multi-step equations and inequalities using variables to solve mathematical problems, with and without context, flexibly, accurately and efficiently. (Sk 4)

The Number System (NS)

Build upon understanding of fractions and decimals to define the set of rational numbers. Apply understanding of rational numbers and operations to solve problems.

7.NS.1

Build and use models including a horizontal or vertical number line diagram to extend previous understanding of addition and subtraction to add and subtract rational numbers *in and out of context* including signed decimals and fractions. (Sk 2, 5)

7.NS.2

Build and use models to extend previous understanding of multiplication and division to multiply and divide rational numbers *in and out of context*. (SK 5)

7.NS.3

Attend to precision and reasonableness to add, subtract, multiply and divide rational numbers including signed decimals and fractions flexibly, accurately, and efficiently *in and out of context*. (Sk 3, 8)

7.NS.4

Describe and represent structures, patterns and relationships to make connections between fraction, decimal and percent representations of a rational number. *Attend*

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to the precision and reasonableness in flexibly using the various representations of rational numbers. (Sk 1, 3)

Ratio and Proportional Relationships (RP)

Use ratio and rate understanding to develop a clear understanding of proportional relationships. Recognize proportional relationships in the world and use patterns in proportional relationships to reason about and solve real-world problems.

7.RP.1

Make and evaluate conjectures about proportional relationships to construct and apply the concept of unit rate and percent. *Describe and represent structures and patterns* that connect percent to ratio. (Sk 1, 7)

7.RP.2

Ask questions about and analyze real world contexts to determine whether two quantities have a proportional relationship and use multiple representations to *construct, justify, and communicate clear and reasonable arguments* to support that reasoning. (Sk 4, 6)

7.RP.3

Contextualize and decontextualize problems involving proportional relationships to find solutions. (Sk 5)

7.RP.4

Build and use models (tables, graphs, equations, diagrams and verbal descriptions) to identify the constant of proportionality and solve multi-step problems involving

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proportional relationships. *Describe the structure and relationship* between the points (0, 0) and (1, r) where r is the unit rate. (Sk 1, 2)

Geometry (G)

Draw, construct, and describe geometrical figures from given criteria. Use and connect 2D and 3D geometrical figures to real-life situations and use them to solve real-world and mathematical problems.

7.G.1

Select and use tools appropriately and strategically (i.e. a compass, straightedge, dynamic geometry software) to accurately draw, construct, and describe scaled geometrical figures. *Utilize the underlying structure* of scale drawings to understand scale factors. (Sk 1, 8)

7.G.2

Select and use tools appropriately and strategically when creating triangles from given conditions and determine whether a unique triangle, more than one triangle, or no triangle can be formed. (Sk 8)

7.G.3

Flexibly, accurately and efficiently solve real-world and mathematical problems by *contextualizing and decontextualizing* situations involving perimeter and area of

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circles, triangles, quadrilaterals and polygons as well as surface area, and volume for right-prisms and right-rectangular pyramids. (Sk 5)

7.G.4

Describe and represent structures, patterns, and relationships related to supplementary, complementary, vertical, and adjacent angles to solve for an unknown angle measures in a figure. (Sk 1)

Data Science (D)

Make inferences about populations based on samples. Draw informal comparative inferences about two populations. Investigate chance processes and develop, use, and evaluate probability models.

7.D.1

Ask questions that anticipate variability in data collection to investigate situations that lead to analyzing data produced from random sampling to make inferences about a population. (Sk 6)

7.D.2

Select and use tools appropriately and strategically to generate and collect randomly sampled data that is representative of the population. *Construct and communicate clear and reasonable arguments* to explain why random sampling is more likely to produce representative samples and support valid inferences. (Sk 4, 8)

7.D.3

Attend to precision and reasonableness when analyzing data produced by random sampling by creating multiple visual representations, including plots on a number lines, dot plots, histograms and box plots. Analyze data by interpreting numerical summaries for measures of center (mean and median) and measures of variability (inter-quartile range and/or mean absolute deviation). (Sk 3)

7.D.4

Construct, justify, and communicate clear and reasonable arguments while interpreting data to answer investigative questions about a population or comparisons between two different populations. Use the comparison of features such as center (mean

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and median), variability (spread/range) and overall shape (symmetric or nonsymmetric). (Sk 4)

7.D.5

Communicate clear and reasonable arguments that show understanding that the probability of a chance event is the likelihood of an event occurring and is expressed as a number between 0 and 1. (Sk 4)

7.D.6

Build and use models of sampling distributions to investigate chance processes. Estimate the probability of a chance event by observing its long-run relative frequency**.** *Evaluate the reasonableness of results.* (Sk 2, 7)

7.D.7

Build and use probability models such as organized lists, tables, tree diagrams, and simulation to find or estimate probabilities of simple and compound events. (Sk 2)

7.D.8

Build and develop a uniform probability *model* by assigning equal probability to all outcomes. *Use the model* to determine probabilities of events. (Sk 2)

7.D.9

Build a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. (Sk 2)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge

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embedded within the *Utah Core Mathematics Standards*, by the end of seventh grade, students will be able to:

- Use properties of operations to generate equivalent expressions and solve real-world problems using algebraic equations and inequalities. (A)
- Add, subtract, multiply, and divide rational numbers including negative and positive numbers to solve real world problems, with and without context. (NS)
- Flexibly use various representations of fractions, percents, and decimals. (NS)
- Recognize proportional relationships with and without context. Use patterns in proportional relationships to reason about and solve problems. (RP)
- Find the perimeter and area of triangles, quadrilaterals, and polygons. Find the surface area and volume of right rectangular prisms. (G)
- Find the area and circumference of a circle. (G)
- Draw, construct, and describe geometrical figures from a given criteria. (G)
- Make inferences about populations based on samples, supported by measures of center and measures of variability. (D)
- Draw informal comparative inferences about two populations. (D)
- Investigate chance processes and develop, use, and evaluate probability models. (D)

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Grade 8

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Standards: Grade 8

Introduction

In sixth through eighth grade, the major work of each grade focuses on applying and using operations with rational numbers, understanding ratio concepts and applying proportional reasoning, and simplifying expressions and solving equations.

In eighth grade, instructional time should focus on three major works: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

Mathematical Skills

Link to Detailed Skills Description

Skill 1: Describe and represent structures, patterns, and relationships

In grades 6-8, students describe and represent their mathematical understanding of structures, pattern and relationships in circumstances including, but not limited to: extending their understanding of multiplication and division in order to divide fractions by fractions; to develop and apply properties of integers of exponents to generate equivalent numerical expressions and to use the structure of similar triangles to explain why the slope of a non-vertical line is constant between any two points on the coordinate plane.

Skill 2: Build and use models

In grades 6-8, students are asked to model under various circumstances, including building visual models of fractions to extend their understanding of multiplication and division, to identify the constant of proportionality and solve multi-step problems involving proportional relationships, and to analyze and solve systems of

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two linear equations.

Skill 3: Attend to precision and reasonableness

In grades 6-8, students can apply this skill when adding, multiplying, and dividing multi-digit whole numbers and decimals, attend to the reasonableness of solutions when solving multi-step, real world problems using positive and negative rational numbers, and when finding rational approximations of irrational numbers.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

In grades 6-8, students construct, justify, and communicate arguments in various mathematical contexts such as: solving problems involving ratios and rates and explaining their thinking about proportional relationships, to justify and explain the logical steps needed to solve multi-step equations and inequalities, and to communicate informal arguments to establish angle relationships for parallel lines cut by a transversal and angle relationships for triangles.

Skill 5: Add or remove context to make sense of mathematics

In grades 6-8, students will add and remove context when working with problems involving the division of fractions by fractions to make sense of the situations and interpret the resulting quotients; to make sense of the addition and subtraction of integer numbers, and use context to determine when it is appropriate to approximate the value of expressions containing irrational numbers.

Skill 6: Ask questions to explore mathematical ideas

In grades 6-8, students will formulate questions to explore mathematical ideas involving real-world problems that use rate, ratio, and percents; in the context of data analysis, students will interpret quantitative bivariate data to answer investigative questions, asking questions about patterns, associations, and potential outliers within the data.

Skill 7: Make conjectures and evaluate the results

In grades 6-8, students will develop and assess conjectures related to proportional

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relationships and to apply the concept of unit rate and percent; to develop and assess conjectures about the properties of irrational numbers by comparing them to the set of rational numbers.

Skill 8: Select and use tools appropriately and strategically

In grades 6-8, students use algebraic tools to generate equivalent expressions and to solve multi-step equations using positive and negative rational numbers, and students will use reasoning and operational tools to perform operations with scientific notation.

Standards: Grade 8

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Algebra (A)

Build on prior understanding of expressions and equations to include work with radical and integer exponents. Through the use of multiple representations, make connections between linear relationships and solve single-variable equations and inequalities.

8.A.1

Use structure and patterns to develop and use the properties of integer exponents to generate equivalent numerical expressions. (Sk 1)

8.A.2

Use tools strategically to simplify and evaluate square and cube roots of positive rational numbers. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. (Sk 8)

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8.A.3

Represent the structure and patterns of numbers using scientific notation. *Use reasoning and operational tools strategically* to perform operations with scientific notation. (Sk 1, 8)

8.A.4

Represent patterns by graphing proportional relationships and interpret the unit rate as the slope of the graph. *Use the structure* of similar triangles to explain why the slope is the same between any two distinct points on a non-vertical line in the coordinate plane. (Sk 1)

8.A.5

Represent and compare linear relationships using graphs, tables, and equations *with or without context*. (Sk 5)

8.A.6

Use structures, patterns and properties strategically to solve single-variable equations and inequalities with rational number coefficients, and absolute value equations. (Sk 1)

8.A.7

Build and use graphic models to analyze and solve systems of two linear equations in two variables and interpret their solutions. Identify systems that have zero, one, and infinite solutions *with or without context*. (Sk 2, 5)

The Number System (NS)

Understand that there is a set of numbers which are not rational, called irrational numbers. Create models to show and describe irrational numbers and their relationships to rational numbers.

8.NS.1

Describe and represent structures and patterns comparing the set of irrational numbers to the set of rational numbers, in order to determine properties of irrational numbers. (Sk 1)

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8.NS.2

Find rational approximations of irrational numbers, *attending to precision and reasonableness*. Use a number line diagram as a *model* to compare values of irrational numbers. (Sk 2, 3)

8.NS.3

Estimate the value of expressions containing irrational numbers, *with and without a context*. (SK 5)

8.NS.4

Justify why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational. *Build and use models* to connect to physical situations. (Sk 2, 4)

Functions (F)

Describe functions and their features. Use functions to model relationships between two variables.

8.F.1

Construct and communicate clear arguments as to whether a relation is a function. (Sk 4)

8.F.2

Build and use algebraic and graphic representations of linear functions to model relationships between two quantities. *Contextualize and decontextualize* the rate of change and initial value as found in y=mx+b. (Sk 2, 5)

8.F.3

Describe and represent features of relationships between two quantities graphically, including intervals where the function is increasing or decreasing, the location of its maximum and minimum values. *Build graphic models* that satisfy given qualitative features. (Sk 1, 2)

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8.F.4

Use multiple representations (tables, graphs, equations, diagrams and verbal descriptions) to compare the properties of linear and nonlinear functions with and without a context. *Construct and communicate clear and reasonable arguments to justify* whether a function is linear or non-linear. (Sk 4)

Geometry (G)

Understand and describe both congruence and similarity in terms of transformations. Explore and verify angle relationships connected to parallel lines cut by a transversal. Understand and use the Pythagorean Theorem.

8.G.1

Clearly communicate about the general properties of dilations, rotations, reflections, and translations with and without coordinates. Use rigid transformations to establish a definition of congruent figures. *Justify* that two shapes are congruent through the use of rigid transformations. (Sk 4)

8.G.2

Justify and clearly communicate the difference between congruence and similarity in terms of transformations. *Ask targeted and probing questions* that clarify the distinction between congruent and similar figures. (Sk 4, 6)

8.G.3

Construct, justify and communicate informal arguments to establish angle relationships for parallel lines cut by a transversal and angle relationships for triangles, including the angle sum and exterior angle. (Sk 4)

8.G.4

Use geometric structures and numeric patterns to explain proofs of the Pythagorean Theorem, use this theorem when solving problems with right triangles and finding distance between two coordinate points. (Sk 1)

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8.G.5

Flexibly, accurately, and efficiently solve real-world and mathematical problems by *contextualizing and decontextualizing* situations involving right triangles, distance, and volume of cones, cylinders, and spheres. (Sk 5)

Data Science (D)

Investigate patterns of association in bivariate data.

8.D.1

Ask questions about bivariate data sets (quantitative and categorical) to investigate patterns of associations between two quantities. (Sk 6)

8.D.2

Select and use tools appropriately and strategically to identify existing data sets or generate and collect data to investigate associations between two quantities. (Sk 8)

8.D.3

Attend to precision and reasonableness when exploring, describing and comparing variability in bivariate data for quantitative variables by creating scatter plots. For scatter plots that suggest a linear association, *represent the relationship* with a linear equation, and informally assess the line of fit by judging the closeness of the data points to the line. (Sk 1, 3)

8.D.4

Describe, represent and explain patterns of association such as clustering, positive or negative association, linear or nonlinear association and effects of outliers. (Sk 1)

8.D.5

Construct, justify, and communicate clear and reasonable arguments while interpreting data to answer investigative questions about quantitative bivariate data. *Use the equation of a linear model* to solve problems in the context of bivariate measurement data, including interpreting the slope and intercept. (Sk 2, 4)

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8.D.6

Attend to precision and reasonableness when exploring, describing and comparing variability in bivariate data for categorical variables by constructing two-way tables. Calculate and interpret relative frequencies for rows or columns in a two-way table. (Sk 3)

8.D.7

Construct, analyze, and communicate clear and reasonable arguments to answer investigative questions about the association between categorical bivariate data. Use the relative frequencies calculated for rows or columns as evidence to justify and describe possible associations between the two variables. (Sk 4)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of eighth grade, students will be able to:

- Work with radicals and integer exponents, understand the connections between proportional relationships, lines, and linear equations. (A)
- Analyze and solve linear equations and inequalities. (A)
- Solve single-variable equations with rational number coefficients, inequalities with rational number coefficients, and absolute value equations. (A)
- Analyze graphs to solve systems of two linear equations in two variables and interpret their solutions. (A)
- Explain the difference between irrational and rational numbers. (NS)

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- Extend understanding of the number line to include rational and an approximation of irrational numbers. (NS)
- Use functions to model relationships between two variables. (F)
- Describe and represent features of relationships between two quantities graphically. (F)
- Use multiple representations to compare the properties of linear and nonlinear functions. (F)
- Understand and describe both congruence and similarity in terms of transformations. (G)
- Explore and verify angle relationships connected to parallel lines cut by a transversal. (G)
- Understand and use the Pythagorean Theorem. (G)
- Interpret data to answer investigative questions about quantitative bivariate data. (D)
- Solve problems in the context of bivariate measurement data, including interpreting the slope and intercept. (D)

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Secondary I

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Standards: Secondary Math I

Introduction

In secondary math 1 through secondary math 3 the major works of each grade will focus on learning to create, interpret, manipulate, and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations and building functions. Students will learn to define functions, describe their features and transformations. Students will understand, apply and prove congruence of similarity as defined in terms of geometric transformations. Students will also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

In secondary mathematics I instructional time should focus on four major works: (1) solve algebraic equations (linear and exponential); (2) understand, compare and represent linear and exponential functions; (3) describe characteristics of linear and exponential functions; (4) understand and apply congruence as defined in terms of geometric transformations; and (5) analyze and interpret data that can be used to answer statistical questions of interest related to comparing multiple distributions or exploring associations between two quantitative variables.

Mathematical Skills

Link to Detailed Skills Description

Skill 1: Describe and represent structures, patterns, and relationships

In high school, students describe the structures of linear and exponential expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define dilations and similar shapes, and then use these structures to solve problems. Students will use models to explore and describe attributes of the structure,

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patterns and relationships of inverses for various function types.

Skill 2: Build and use models

In high school, students will build and use models in two primary ways. Students use models to connect mathematical concepts and to model real-world phenomena. Students will create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. In high school students will use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas such between them.

To model real-world phenomena, students will engage in *mathematical and statistical modeling* to represent, analyze, and predict real-world situations. In high school students will use geometric shapes, their measures, and their properties to *model* real world objects and apply geometric methods to solve design problems.

Skill 3: Attend to precision and reasonableness

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students will attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to distinguish between situations that can be modeled by various function types. They

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will construct arguments to justify claims such as why exponential growth eventually surpasses linear, quadratic or the growth in any other polynomial function.

Skill 5: Add or remove context to make sense of mathematics

In high school, students contextualize and decontextualize functions to interpret their properties, connecting symbolic representations to meaningful situations. Students will interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

Skill 6: Ask questions to explore mathematical ideas

In high school, students will formulate and ask questions to highlight the difference between various function types. Students will ask targeted questions about univariate data displays to explore mathematical ideas. Students will ask questions about the underlying structure of geometric objects revealed by geometric constructions.

Skill 7: Make conjectures and evaluate the results

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to answer investigative questions. Students will make conjectures about the reasonableness and strength of the model they choose to describe the association between two quantitative variables. Students will develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

Skill 8: Select and use tools appropriately and strategically

In high school, students will learn to select and leverage multiple types of tools while exploring mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential, or quadratic equations for a variable. Students will select and use appropriate tools to build representations for various function types with and without technology.

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Students will use mathematical ideas as a tool to validate the appropriateness and usefulness of a model.

Standards: Secondary I

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Algebra (A)

Interpret the structure and patterns of linear and exponential expressions. Create inequalities to describe linear and exponential relationships. Understand solving equations/inequalities or systems of equations as a process of reasoning and having a knowledge of the tools that can be used to create arguments or justifications for solutions.

S1.A.1

Describe and represent the structures of linear and exponential expressions by identifying whole expressions as being made of parts, like-terms and factors. (Sk 1)

S1.A.2

Build and use models to *describe and represent patterns and relationships* of linear equations in two or more variables, inequalities, and simple exponential functions. (Sk 1, 2)

S1.A.3

Select and use algebraic tools appropriately and strategically to rearrange formulas to isolate a quantity of interest. (Sk 8)

S1.A.4

Construct and communicate clear and reasonable arguments using properties of equality to justify each step in solving a linear equation, while starting from the

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assumption that the original equation has a solution. *Strategically select and use the appropriate tools* to solve one variable equations and inequalities. (Sk 4, 8)

S1.A.5

Given a system of equations, *use context* to make sense of the ways in which the equation(s) can be algebraically adjusted while conserving the relationship. *Apply these strategies as tools* to help solve systems of linear equations exactly and approximately. (Sk 5, 8)

S1.A.6

Build and use graphic models to analyze and solve systems of two linear equations as well as systems of one linear equation and one exponential equation in two variables and interpret their solutions. (Sk 2)

S1.A.7

Build and use models graphically to show the solutions to a linear inequality in two variables as a half-plane, and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (Sk 2)

Functions (F)

Describe and represent arithmetic and geometric sequences with linear and exponential functions using function notation. Analyze functions using different representations, and contextualize functions and their features when appropriate. Engage in the process of developing and assessing conjectures to support the work of building functions that model a relationship between two quantities and develop an understanding of how existing models can be combined to build new functions. Construct and compare linear and exponential models and solve problems.

S1.F.1

Construct and communicate clear arguments to justify whether a relation is a function. (Sk 4)

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S1.F.2

Use representations as a *tool* to identify the domain of a function and describe the quantitative relationship. (Sk 8)

S1.F.3

Attend to precision using function notation to evaluate functions for inputs in their domains and interpret statements that use function notation *in terms of a context.* (Sk 3, 5)

S1.F.4

Build and use explicit and recursive equations to *model* arithmetic and geometric sequences using function notation. *Describe and represent* the domain of these models as a subset of the integers. (Sk 1, 2)

S1.F.5

Understand and interpret the features of a function represented graphically, numerically and symbolically by *applying and removing context*. Focus on these key features: domain, range, intervals of increase and/or decreasing, location of absolute maximum and/or absolute minimum, and intercepts. *Build* graphic *models* that satisfy given key features. (Sk 2, 5)

S1.F.6

Build representations that show key features of linear and exponential functions expressed symbolically. *Select and use appropriate tools to* model with and without technology. (Sk 2, 8)

S1.F.7

Compare and contrast the properties of linear and exponential functions (represented in different ways). *Formulate and ask questions* to highlight the similarities and differences between the functions. (Sk 6)

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S1.F.8

Build a function that *models* linear or exponential relationships. Appropriately use explicit and recursive rules to model these relationships and translate between the two forms. (Sk 2)

S1.F.9

Construct arguments that *use the structures and patterns* of relationships to connect arithmetic sequences to linear functions and geometric sequences to exponential functions. (Sk 1, 4)

S1.F.10

Identify the effect on the graph when replacing f(x) by f(x) + k by exploring specific values of k for linear and exponential functions in and out of context**.** *Attend to precision* of language and notation while *constructing arguments* and carefully evaluate whether the ideas and processes observed during exploration are *reasonable*. (Sk 3, 4)

S1.F.11

Communicate clear and reasonable arguments to distinguish between situations that can be modeled with linear functions, exponential functions, or neither. Connect the *structure* of linear and exponential functions with their parameters, (y=mx+b, $y=b^x(a)$). *Justify* that a quantity increasing exponentially eventually exceeds a quantity increasing linearly. (Sk 1, 4)

Geometry (G)

Build on student experience with rigid motions from earlier grades to establish an understanding of congruence in terms of rigid motions. The underlying structure of rigid motions established in this course is foundational for future work with geometrical proof. Rigid motions and their properties are to be used to establish triangle congruence criteria which will be used to prove other theorems. Geometric constructions are to be understood as a means for gaining deeper understanding of the underlying structures of rigid transformations as well as the logical reasoning that connects rigid motions and justifications of triangle congruence criteria. Reasoning about and justifying attributes of geometric figures placed on the

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coordinate grid allows students to connect numeric and algebraic processes with geometric justification.

S1.G.1

Describe and represent the underlying structure of translations, rotations and reflections, in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (Sk 1)

S1.G.2

Construct, justify and clearly communicate definitions of the rigid transformations to show that these transformations are functions that take points in the plane as inputs and give corresponding points as outputs. (Sk 4)

S1.G.3

Select and use tools appropriately and strategically to perform rigid transformations. *Attend to precision* when identifying and describing the transformation or the sequence of transformations to determine whether or not two figures are congruent. Understand that the image created from a rigid transformation or a sequence of rigid transformations is congruent to the pre-image. (Sk 3, 8)

S1.G.4

Construct, justify and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Use reflections and rotations to informally *justify* attributes of parallelograms including rhombus, rectangles and squares. (Sk 4)

S1.G.5

Construct, justify and communicate clearly as to how geometric construction tools can be used to precisely construct geometric objects including equilateral triangle, rhombus, square, parallel line, perpendicular bisector, and angle bisector. Analyze the underlying structure of geometric objects as revealed by the tools for geometric construction. (Sk 4)

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S1.G.6

Construct, justify and communicate clear and reasonable arguments that show how the criteria for triangle congruence (ASA, SAS, SSS) can be established using the definition of congruence in terms of rigid motions. (Sk 4)

S1.G.7

Ask targeted and probing questions and construct and communicate clear and reasonable arguments about quadrilaterals located on the coordinate grid to justify whether or not a quadrilateral has the attributes of a square, rectangle, rhombus or parallelogram. Connect the Pythagorean Theorem to the distance formula and utilize slope along with distance as a means for justification. (Sk 4, 6)

Data Science (D)

Describe, represent**,** and interpret data on a single variable and on two quantitative variables and evaluate results.

S1.D.1

Ask statistical questions to investigate situations that can be explored using random samples from populations to make inferences about differences between two populations or associations between two quantitative variables. Explain and *justify* the distinction between correlation and causation. (Sk 4, 6)

S1.D.2

Select the appropriate statistical tools to apply a data collection plan when collecting primary data for the statistical investigative question of interest or determine the validity that the data collected is useful to answer the statistical question of interest when using secondary data. (Sk 8)

S1.D.3

Attend to precision and reasonableness when analyzing data to compare two or more distributions by selecting graphical displays (dot plots, histograms, and modified box plots to show outliers) that highlight features of interest. Analyze data by comparing measures of center (mean and median) and measures of variability (range, inter-quartile range and standard deviation) that are appropriate for the

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shape (Skew, symmetry, outliers, modes) of the data distribution. Recognize how standard deviation builds on the mean absolute deviation and is another measure of how data values vary in a distribution. (Sk 3)

S1.D.4

Attend to precision and reasonableness when analyzing data to explore the association between two quantitative variables by using a scatterplot to determine and describe patterns of association such as clustering, positive or negative association, linear or nonlinear association and effects of outliers. Use technology to find and interpret the correlation coefficient and use it to assess the strength of the linear relationship. (Sk 3)

S1.D.5

Analyze and use statistical evidence to *evaluate results* and answer statistical investigative questions about the differences between two or more population parameters. Then *justify* statistical reasoning and results to others in a variety of formats including verbal, written, and visual. (Sk 4, 7)

S1.D.6

Justify statistical reasoning to *evaluate results* from analyses to answer the statistical investigative question about associations between two quantitative variables. For scatter plots that suggest a linear association, represent the relationship with a line of best fit and use the model of the linear equation to solve problems, including interpreting the slope and intercept in the context of the data. *Justify* statistical reasoning and results to others in a variety of formats including verbal, written, and visual. (Sk 4, 7)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge

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embedded within the *Utah Core Mathematics Standards*, by the end of Secondary 1

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students will be able to:

- Interpret parts of a linear and/or exponential expression in terms of a context. (A)
- Create equations and inequalities in one, two or more variables and use them to solve problems. (A)
- Fluently solve systems of equations exactly and approximately (numerically, algebraically, and graphically) with pairs of linear equations in two variables. (A)
- Fluently solve linear equations and inequalities and represent solutions graphically. (A)
- Fluently produce explicit and recursive equations to model arithmetic and geometric sequences. (F)
- Interpret key features of graphs that model relationships between two variables for linear and exponential functions. (F)
- Build multiple representations of linear and exponential functions. (F)
- Compare and contrast the properties of linear and exponential functions. (F)
- Distinguish between situations that can be modeled with linear functions and exponential functions. (F)
- Use function notation to represent linear and exponential functions, including arithmetic and geometric sequences. (F)
- Build on student experience with rigid motions from earlier grades to establish an understanding of congruence in terms of rigid motions. (G)
- Establish triangle congruence criteria which will be used to prove other theorems.
- Describe the transformation or the sequence of transformations to prove two figures are congruent. (G)

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- Describe, represent**,** and interpret data on a single variable and on two quantitative variables and evaluate results. (D)

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Secondary II

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Standards: Secondary Math II

Introduction

In secondary math I through secondary math III the major works of each grade will focus on learning to create, interpret, manipulate, and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations and building functions. Students will learn to define functions, describe their features and transformations. Students will understand, apply and prove congruence of similarity as defined in terms of geometric transformations. Students will also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

In secondary mathematics II instructional time should focus on four major works: (1) solve algebraic equations (linear, exponential and quadratic); (2) understand, compare and represent functions; (3) describe characteristics functions; (4) prove congruence and similarity in terms of geometric transformations; and (5) analyze and interpret data that can be used to answer statistical questions of interest related associations between two quantitative variables.

Mathematical Skills

Link to Detailed Skills Description

Skill 1: Describe and represent structures, patterns, and relationships

In high school, students describe the structures of linear and exponential expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define dilations and similar shapes, and then use these structures to solve problems. Students will use models to explore and describe attributes of the structure, patterns and relationships of inverses for various function types.

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Skill 2: Build and use models

In high school, students will build and use models in two primary ways. Students use models to connect mathematical concepts and to model real-world phenomena. Students will create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. In high school students will use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas such between them.

To model real-world phenomena, students will engage in *mathematical and statistical modeling* to represent, analyze, and predict real-world situations. In high school students will use geometric shapes, their measures, and their properties to *model* real world objects and apply geometric methods to solve design problems.

Skill 3: Attend to precision and reasonableness

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students will attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to distinguish between situations that can be modeled by various function types. They will construct arguments to justify claims such as why exponential growth

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eventually surpasses linear, quadratic or the growth in any other polynomial function.

Skill 5: Add or remove context to make sense of mathematics

In high school, students contextualize and decontextualize functions to interpret their properties, connecting symbolic representations to meaningful situations. Students will interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

Skill 6: Ask questions to explore mathematical ideas

In high school, students will formulate and ask questions to highlight the difference between various function types. Students will ask targeted questions about univariate data displays to explore mathematical ideas. Students will ask questions about the underlying structure of geometric objects revealed by geometric constructions.

Skill 7: Make conjectures and evaluate the results

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to answer investigative questions. Students will make conjectures about the reasonableness and strength of the model they choose to describe the association between two quantitative variables. Students will develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

Skill 8: Select and use tools appropriately and strategically

In high school, students will learn to select and leverage multiple types of tools while exploring mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential, or quadratic equations for a variable. Students will select and use appropriate tools to build representations for various function types with and without technology. Students will use mathematical ideas as a tool to validate the appropriateness and

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usefulness of a model.

Standards: Secondary II

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Algebra (A)

Interpret the structure and patterns of expressions. Extend to quadratic expressions and use structure to highlight different aspects of quadratic functions. Build linear and quadratic polynomials using addition, subtraction, and multiplication. Create inequalities to describe linear, exponential, and quadratic relationships. Understand solving equations/inequalities or systems of equations as a process of reasoning and having a knowledge of the tools that can be used to create arguments or justifications for solutions.

S2.A.1

Describe and represent the structures of quadratic and exponential expressions by identifying whole expressions as being made of parts, like terms and factors. *Interpret* these structures in a *context* and use them to produce equivalent forms that reveal different properties. (Sk 1, 5)

S2.A.2

Build models of linear and quadratic functions. Use the operations of addition, subtraction and multiplication to create new linear and quadratic functions. (Sk 2)

S2.A.3

Describe and represent patterns in relationships to *build and use models of* linear and quadratic inequalities to represent possible solutions. (Sk 1, 2)

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S2.A.4

Select and use tools strategically to solve quadratic equations and inequalities in one variable. (Sk 8)

S2.A.5

Select and use tools strategically to solve a system of equations consisting of a linear equation and a quadratic equation in two variables. (Sk 8)

Number System (NS)

Extend the understanding of integer exponents to encompass rational exponents and explore how this relationship can be used to see patterns, reason about, and solve real world problems. Extend the number system to include all complex numbers. Build on the understanding of operations of real numbers to include imaginary numbers. Emphasize the essential role of complex numbers in solving quadratic equations by satisfying the conditions for the Fundamental Theorem of Algebra.

S2.NS.1

Develop, assess, and evaluate conjectures about properties of exponents for rational exponents. *Describe and represent patterns* that emerge from the extension of properties of integer exponents to rational exponents including equivalency of radical expressions. (Sk 1, 7)

S2.NS.2

Construct, justify, and communicate clear and reasonable arguments that verify, according to the Fundamental Theorem of Algebra, the solutions to any quadratic equation will always be numbers within the set of complex numbers and will have the form a + bi where a and b are real numbers. (Sk 4)

S2.NS.3

Build and use models to solve quadratic equations with real coefficients that have

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complex solutions, rewriting x^2+4 as (x + 2i)(x – 2i). (Sk 2)

Functions (F)

Describe and represent functional relationships and their features. Analyze functions using different representations and contextualize functions and their features when appropriate. Building a function that models a quadratic relationship between two quantities. Develop an understanding of how existing models can be combined to build new functions. Construct and compare linear, quadratic, and exponential models.

S2.F.1

Understand and interpret the features of a function represented graphically, numerically and symbolically by *applying and removing context*. Focus on these key features: domain, range, intervals of increase and/or decreasing, location of

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absolute maximum and/or absolute minimum, and intercepts. *Build* graphic *models* that satisfy given key features. (Sk 2, 5)

S2.F.2

Attend to precision and reasonableness when determining the average rate of change of a function over a closed interval. Connect the average rate of change with *context* appropriately. (Sk 3, 5)

S2.F.3

Build representations that show key features of quadratic, linear, exponential, absolute value and piecewise functions expressed symbolically. *Select and use appropriate tools to* model with and without technology. (Sk 2, 8)

S2.F.4

Compare and contrast the properties of quadratic, linear, exponential, absolute value and piecewise functions (represented in different ways). *Formulate and ask questions* to highlight the similarities and differences between function types. (Sk 6)

S2.F.5

Select and use algebraic tools to create different but equivalent forms of functions that highlight different properties of the function. *Contextualize and decontextualize* these functions to interpret these properties. (Sk 5, 8)

S2.F.6

Build a function that *models* a quadratic relationship. Use *structures and patterns* to build an equation that models explicit reasoning, or a recursive process**.** *Attend to the precision* of notation and labeling while building and using these models. (Sk 1, 2, 3)

S2.F.7

Identify the effect on the graph when replacing f(x) by f(x) + k, kf(x), f(kx), f(x+k) by exploring specific values of k for quadratic and absolute value functions in and out of context**.** *Attend to precision* of language and notation while *constructing*

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arguments and carefully evaluate whether the ideas and processes observed during exploration are *reasonable*. (Sk 3, 4)

S2.F.8

Communicate clear and reasonable arguments to distinguish between situations that can be modeled with quadratic, exponential, and linear functions. Interpret the *structure* of quadratic functions written in different forms with the features of their graphs. (Sk 1, 4)

Geometry (G)

Build upon the work of previous courses by using rigid motions as well as triangle congruence criteria to prove geometric theorems. Develop an understanding of dilations and similarity. Build upon this understanding of dilation and similarity to support work with right triangle ratios. Use this knowledge and skills to solve real world problems that can be modeled by right triangles. Understand and explain why all circles are similar. Use this principle to support reasoning about angles, lines and segments connected with circles. Develop an understanding of radian measure for angles in terms of the ratio of arc length and radius. Describe circles on a coordinate grid with algebraic equations. Build connections to the Pythagorean Theorem and transformations. Explain area and volume formulas as they scale with similarity transformations.

S2.G.1

Construct, justify and communicate clear and reasonable arguments to prove geometric theorems. Support refinement of reasoning *by asking questions.* Develop flexibility in creating arguments and proofs of various formats, including: narrative paragraphs, flow diagrams, two-column format, and diagrams without words. Proofs should focus on theorems about lines and angles, triangles, and

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parallelograms and allow students to utilize previous work with transformations and triangle congruence. (Sk 4, 6)

S2.G.2

*Describe and represent the underlying structure***s** and properties that define dilations and similar shapes. Use the structures and properties of dilation and similarity to solve problems. (Sk 1)

S2.G.3

Build and use models that include congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. *Construct, justify and communicate* a clear and reasonable proof of the Pythagorean Theorem using triangle similarity. (Sk 2, 4)

S2.G.4

Describe and represent side ratios in right triangles based upon the *underlying structure* of similarity. Connect consistency across similar right triangles to definitions of trigonometric ratios for acute angles. (Sk 1)

S2.G.5

Verify, justify and communicate the relationships that exist between trigonometric ratios and trigonometric identities. Keep the focus on the relationship between sine

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and cosine of complementary angles, the Pythagorean Identity, and the tangent ratio in terms of sine and cosine. (Sk 4)

S2.G.6

Flexibly, accurately, and efficiently use trigonometric ratios and the Pythagorean Theorem to solve *real-world and mathematical problems* that can be *modeled* with right triangles. (Sk 2, 5)

S2.G.7

Construct, justify and communicate a clear and reasonable argument that all circles are similar. *Connect the underlying structures* of similarity to relationships between angles, segments and arcs within circles. (Sk 1, 4)

S2.G.8

Describe and represent structures and patterns between inscribed angles, radii, and chords. Relationships include the relationships between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (Sk 1)

S2.G.9

Describe and represent, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define radian measure of an angle as the constant of proportionality. Create a distinction between arc length and arc measure and be able to *attend to precisions* while relating each of these to radian measure or the formula for the area of a sector. (Sk 1, 3)

S2.G.10

Use structures and patterns to work with equations of circles, find different forms of the equations and different defining features of the circle, including the center, radius, and points located on the circle. Use right triangles with the same

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hypotenuse length, and the Pythagorean Theorem to derive the equation of a circle. (Sk 1)

S2.G.11

Attend to precision and reasonableness with respect to area and volume formulas by applying informal arguments about how area and volume scale under similarity transformations. When scale factor k is applied to a figure, area scales by k^2 and volume scales by k^3. (Sk 3)

Data Science (D)

Describe, represent**,** and interpret data on two quantitative variables and evaluate results. Use linear, exponential and quadratic functions as models.

S2.D.1

Ask statistical questions to investigate linear, exponential, and quadratic associations between two quantitative variables. (Sk 6)

S2.D.2

Select and use tools appropriately and strategically apply a data collection plan when collecting primary data to investigate for the purpose of making inferences about the association between two variables, or determine the validity that the data collected is useful to answer the statistical question of interest when using secondary data. (Sk 8)

S2.D.3

Attend to precision and reasonableness when analyzing data to explore the association between two quantitative variables by using scatter plots to determine and explain patterns of association. Use technology to find and interpret the correlation coefficient and *evaluate the reasonableness* and strength of the linear,

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exponential, or quadratic model used. Determine whether a linear, exponential or quadratic function is the most appropriate to model the data. (Sk 3, 7)

S2.D.4

Use the statistical evidence to *evaluate results* from analyses to answer an investigative question about associations between two quantitative variables. Represent the relationship with a curve of best fit and use the model of the equation to solve problems. *Justify* statistical reasoning and results to others in a variety of formats including verbal, written, and visual. (Sk 4, 7)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge

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embedded within the *Utah Core Mathematics Standards*, by the end of Secondary II, students will be able to:

- Produce equivalent forms of quadratic expressions to reveal different properties of interest. (A)
- Fluently solve quadratic equations and inequalities in one variable. (A)
- Extend the number system to include complex numbers when real solutions do not exist for a quadratic equation. (NS)
- Interpret different forms of quadratic functions. (F)
- Interpret and compare different representations for linear, exponential and quadratic functions. (F)
- Build multiple representations of linear and exponential and quadratic functions. (F)
- Compare key characteristics of quadratic functions to those of linear and exponential functions. (F)
- Interpret and analyze key features of quadratic functions. (F)
- Prove congruence and similarity in terms of geometric transformations. (G)
- Prove geometric theorems related to congruence and similarity. (G)
- Develop trigonometric ratios for sides of right triangles. (G)
- Describe, represent**,** and interpret data on two quantitative variables and evaluate results. (D)

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Secondary Math Pathways

Utah's Secondary Mathematics Pathways

The Secondary III course has been divided into two distinct parts to better meet the needs of every student in the state than what a single course can provide. The first part of Secondary III includes the mathematics that all students would benefit from engaging with (Math for All). This content is tailored to be crucial for all students regardless of their plans after high school. The second part of Secondary III provides students and families with a choice that can better prepare them for their post-secondary aspirations.

The first option is the calculus pathways, which is designed to be preparatory for careers in business, biological and physical sciences, engineering and/or mathematics. This course focuses on the foundations of algebra needed to pursue the study of limits and later on, calculus. It is also aligned with the courses Utah Institutions of Higher Education (IHE) offer for the above majors, including Math 1050/1060 and Calculus (which can be taken concurrently enrolled or once the student reaches University). AP precalc and AP Calc courses are also aligned to this pathway.

The second option is the data science pathway, which is designed to be preparatory for careers in history, nursing, psychology and the social sciences. This course focuses on the descriptive and statistical methods used to understand data. It provides the foundations for students to pursue Math 1040 concurrently enrolled or if they decide to at University, while also setting students up for success in AP statistics.

The third option is the creation of a pathway that builds on the Math for All content towards another career goal. This option will eventually include math courses from a variety of CTE backgrounds, alongside courses that have been adapted from our previously leveraged Applied and Advanced Foundational courses currently acceptable as graduation requirements when a student opts out of Secondary III. The vision going forward is that these Board-approved versions of Secondary III would replace the opt out provision currently in place for the Secondary III course.

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LEAs across the state will have flexibility in how to design their schedules around these Secondary III courses, running them as full year experiences or two half year classes, depending on their needs. USBE staff will work with USHE and Utah Institutions of Higher Education to find further alignment and explore the possibility of extended versions of these Secondary III courses that would allow students to earn additional credit or gain access to accelerated coursetaking options upon completion of.

What follows are the standards our writing committee has developed for the core pathways. The USBE Mathematics team will work with LEA math communities across the state over the next few years to develop further Secondary III course pathways related to the third option described above.

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Secondary III (Math for All Students)

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Standards: Secondary Math III (All Students)

Introduction

In secondary math I through secondary math III, the major works of each grade will focus on learning to create, interpret, manipulate, and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations and building functions. Students will learn to define functions, describe their features and connect them to transformations. Students will understand, apply and prove congruence or similarity as defined by geometric transformations. Students will also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

In secondary mathematics III instructional time should focus on four major works: (1) solve algebraic equations (polynomial, logarithmic, radical, rational and trigonometric); (2) understand, compare and represent functions - polynomial, rational, logarithmic and trig functions (sine and cosine); (3) describe characteristics of functions; (4) extend congruence and similarity; and (4) draw and justify conclusions from sample surveys, experiments, and observational studies.

Mathematical Skills

Link to Detailed Skills Description

Skill 1: Describe and represent structures, patterns, and relationships

In high school, students describe the structures of linear and exponential expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define dilations and similar shapes, and then use these structures to solve problems. Students will use models to explore and describe attributes of the structure,

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patterns and relationships of inverses for various function types.

Skill 2: Build and use models

In high school, students will build and use models in two primary ways. Students use models to connect mathematical concepts and to model real-world phenomena. Students will create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. In high school students will use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas such between them.

To model real-world phenomena, students will engage in *mathematical and statistical modeling* to represent, analyze, and predict real-world situations. In high school students will use geometric shapes, their measures, and their properties to *model* real world objects and apply geometric methods to solve design problems.

Skill 3: Attend to precision and reasonableness

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students will attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to distinguish between situations that can be modeled by various function types. They

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will construct arguments to justify claims such as why exponential growth eventually surpasses linear, quadratic or the growth in any other polynomial function.

Skill 5: Add or remove context to make sense of mathematics

In high school, students contextualize and decontextualize functions to interpret their properties, connecting symbolic representations to meaningful situations. Students will interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

Skill 6: Ask questions to explore mathematical ideas

In high school, students will formulate and ask questions to highlight the difference between various function types. Students will ask targeted questions about univariate data displays to explore mathematical ideas. Students will ask questions about the underlying structure of geometric objects revealed by geometric constructions.

Skill 7: Make conjectures and evaluate the results

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to answer investigative questions. Students will make conjectures about the reasonableness and strength of the model they choose to describe the association between two quantitative variables. Students will develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

Skill 8: Select and use tools appropriately and strategically

In high school, students will learn to select and leverage multiple types of tools while exploring mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential, or quadratic equations for a variable. Students will select and use appropriate tools to build representations for various function types with and without technology.

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Students will use mathematical ideas as a tool to validate the appropriateness and usefulness of a model.

Standards: Secondary III (Math for All Students)

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Algebra (A)

Interpret the structure and patterns of polynomial and rational expressions.

S3.A.1

Describe, represent, and interpret the structures and patterns of polynomial and rational expressions by identifying whole expressions as being made of parts, like terms and factors. Interpret these structures in a *context* and use them to produce equivalent forms that reveal different properties. (Sk 1, 5)

Functions (F)

Describe and represent functional relationships and their features. Analyze functions using different representations, and contextualize functions and their features when appropriate. Develop and assess conjectures to support the work of building functions that model a relationship between two quantities. Develop an understanding of how models can be refined to model more complex or sophisticated situations. Construct and compare linear, quadratic, and exponential models and solve problems. Interpret expressions for functions in terms of the situation it models. Extend the definition of trigonometric ratios beyond 0 to 90 degrees and use trigonometric functions to model periodic behavior.

S3.F.1

Understand and interpret the features of a function represented graphically, numerically and symbolically by *applying and removing context*. Focus on these key

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features: domain, range, intervals of increase and/or decreasing, location of absolute maximum and/or absolute minimum, intercepts and end behavior. *Build* graphic *models* that satisfy given key features. (Sk 2, 5)

S3.F.2

Build representations that show key features of polynomial, rational, square root, cube root, exponential, logarithmic, and trigonometric functions expressed symbolically. *Select and use appropriate tools to model* with and without technology. (Sk 2, 8)

S3.F.3

Compare and contrast the properties of these functions polynomial, rational, square root, cube root, exponential, logarithmic, and trigonometric functions. *Formulate and ask questions* to highlight the similarities and differences between function types. (Sk 6)

S3.F.4

Identify the effect on the graph when replacing f(x) by f(x) + k, kf(x), f(kx), f(x+k) by exploring specific values of k for polynomial, rational, square root, cube root, exponential, logarithmic, and trigonometric functions in and out of context**.** *Attend to precision* of language and notation while *constructing arguments* and carefully evaluate whether the ideas and processes observed during exploration are *reasonable*. (Sk 3, 4)

S3.F.5

Use models to explore and *describe* attributes of the *structure, patterns and relationships* of inverses for linear, exponential and quadratic functions**.** *Strategically use tools* to build a model for the inverse of a linear, exponential, or quadratic function. (Sk 1, 2, 8)

S3.F.6

Communicate clear and reasonable arguments to distinguish between situations that can be *modeled* with polynomial, rational, square root, cube root, exponential, logarithmic, and trigonometric functions. *Justify* that a quantity increasing

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exponentially eventually exceeds a quantity increasing linearly or quadratically. (Sk

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2, 4)

S3.F.7

Describe structures and patterns to extend the domain of trigonometric functions beyond 0 to 90 degrees to *build and use models* of periodic phenomena with sine and cosine functions with specified amplitude, frequency, and midline. (Sk 1, 2)

Geometry (G)

Apply geometric reasoning to real world modeling situations.

S3.G.1

Use geometric shapes, their measures, and their properties to *model* real world objects and apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost). *Use these models* to *ask targeted and probing questions* with respect to area and volume *in context*. (Sk 2, 5, 6)

S3.G.2

Apply concepts of density based on area and volume in *modeling* situations. (Sk 2)

Data Science (D)

Understand independence and conditional probability and use them to interpret data and compute probabilities of compound and conditional events in a uniform probability model or using a normal distribution when appropriate. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use technology to estimate areas under the normal curve. Summarize, represent, and interpret data on a single count or measurement variable. Understand and evaluate random processes underlying statistical experiments. Draw and justify conclusions from sample surveys, experiments, and observational studies. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.

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S3.D.1

Describe and represent structures and patterns of data on two categorical variables. *Attend to precision and reasonableness* when summarizing categorical data in two-way frequency models, and *construct, justify, and communicate clear and reasonable arguments* about possible associations and trends in the data. (Sk 1, 3, 4)

S3.D.2

Build and use probability models including organized lists, venn-diagrams, tree diagrams, and two-way tables to find or estimate probabilities of compound or conditional events. (Sk 2)

S3.D.3

Describe and represent structure to build understanding of the addition rule P(A or B) = P(A) + P(B) – P(A and B) and understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of B given A is the same as the probability of B. (Sk 1)

S3.D.4

Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. *Select and use physical and/or tech tools* to estimate areas under the normal curve. (Sk 8)

S3.D.5

Formulate statistical investigative questions that are either summary-based or comparative**,** for surveys, observational studies, or experiments using primary and/or secondary data. Select *appropriate statistical tools* to develop a plan for data collection and analysis to address these questions. (Sk 6, 8)

S3.D.6

Select appropriate statistical tools to implement a data collection plan for primary data related an investigative question. When using secondary data, assess its validity and relevance to answering the question. *Construct an argument* justifying the chosen data collection method (survey, observational study or experiment).

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Understand and apply best practices for designing sample surveys, experiments, and observational studies. (Sk 4, 8)

S3.D.7

Attend to precision and reasonableness when summarizing collected data to answer an investigative question using tables, graphs, and numerical summary statistics. *Develop and use models* from sample data to estimate population means or proportions, including constructing prediction intervals. Employ simulation models for random sampling to develop a margin of error, determine approximate sampling distributions, and compute p-values from those distributions. (Sk 2, 3)

S3.D.8

Use statistical evidence from the analyses to *evaluate results* and answer statistical investigative questions. *Justify* outcomes or estimates of population characteristics, considering their plausibility compared to chance variation, in a variety of formats (verbal, written, visual). Interpret the margin of error associated with estimates and appropriately communicate the interpretation of simulated p-values. (Sk 4, 7)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge

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embedded within the *Utah Core Mathematics Standards*, by the end of Secondary III (Math for All), students will be able to:

- Understand, compare and represent functions including polynomial, rational, trigonometric, logarithmic, and inverse functions. (F)
- Build on prior knowledge of key features of transformations of linear, quadratic and exponential models and extend to polynomial, rational, logarithmic and trig functions (sine and cosine). (F)
- Build off prior knowledge of congruency and similarity and right triangle ratios to extend the domain of trigonometric functions (sine and cosine). (F)
- Distinguish between situations that can be modeled with polynomial, rational, square root, cube root, exponential, logarithmic, and trigonometric functions. (F)
- Compute probabilities for compound and conditional events and use them to interpret data. (D)
- Draw and justify conclusions from sample surveys, experiments, and observational studies. (D)

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Secondary III (Calculus)

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Standards: Secondary Math III (Calculus Pathway)

Introduction

In secondary math I through secondary math III the major works of each grade will focus on learning to create, interpret, manipulate, and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations and building functions. Students will learn to define functions, describe their features and transformations. Students will understand, apply and prove congruence of similarity as defined in terms of geometric transformations. Students will also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

In secondary mathematics III instructional time should focus on four major works: (1) Solve algebraic equations (polynomial, logarithmic, radical, rational and trigonometric), (2) Understand, compare and represent functions (polynomial and inverse), (3) describe characteristics of functions (4) extend congruence and similarity and (4) Draw and justify conclusions from sample surveys, experiments, and observational studies.

Mathematical Skills

Link to Detailed Skills Description

Skill 1: Describe and represent structures, patterns, and relationships

In high school, students describe the structures of linear and exponential expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define dilations and similar shapes, and then use these structures to solve problems.

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Students will use models to explore and describe attributes of the structure, patterns and relationships of inverses for various function types.

Skill 2: Build and use models

In high school, students will build and use models in two primary ways. Students use models to connect mathematical concepts and to model real-world phenomena. Students will create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. In high school students will use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas such between them.

To model real-world phenomena, students will engage in *mathematical and statistical modeling* to represent, analyze, and predict real-world situations. In high school students will use geometric shapes, their measures, and their properties to *model* real world objects and apply geometric methods to solve design problems.

Skill 3: Attend to precision and reasonableness

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students will attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to

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distinguish between situations that can be modeled by various function types. They will construct arguments to justify claims such as why exponential growth eventually surpasses linear, quadratic or the growth in any other polynomial function.

Skill 5: Add or remove context to make sense of mathematics

In high school, students contextualize and decontextualize functions to interpret their properties, connecting symbolic representations to meaningful situations. Students will interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

Skill 6: Ask questions to explore mathematical ideas

In high school, students will formulate and ask questions to highlight the difference between various function types. Students will ask targeted questions about univariate data displays to explore mathematical ideas. Students will ask questions about the underlying structure of geometric objects revealed by geometric constructions.

Skill 7: Make conjectures and evaluate the results

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to answer investigative questions. Students will make conjectures about the reasonableness and strength of the model they choose to describe the association between two quantitative variables. Students will develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

Skill 8: Select and use tools appropriately and strategically

In high school, students will learn to select and leverage multiple types of tools while exploring mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential, or quadratic equations for a variable. Students will select and use appropriate tools to

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build representations for various function types with and without technology. Students will use mathematical ideas as a tool to validate the appropriateness and usefulness of a model.

Standards (Secondary III [Calculus Pathway])

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Algebra (A)

Interpret the structure and patterns of polynomial and rational expressions. Write expressions in equivalent forms to solve problems. Perform arithmetic operations on polynomials. Describe and represent structures and patterns for series. Describe, represent and build polynomials beyond quadratic, using zeros and factors. Describe, represent and build rational expressions. Select and use tools appropriately and strategically to solve rational and radical equations and inequalities and compound inequalities.

Standards with the prefix *S3* represent standards from the *Math for All Students* content. Standards with the prefix *CS3* are specific to the Calculus Pathway.

S3.A.1

Describe, represent, and interpret the structures and patterns of polynomial and rational expressions by identifying whole expressions as being made of parts, like terms and factors. Interpret these structures in a *context* and use them to produce equivalent forms that reveal different properties. (Sk 1, 5)

CS3.A.2

Describe and represent structures and patterns for arithmetic and geometric series, including infinite geometric series. (Sk 1)

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CS3.A.3

Communicate clear and reasonable arguments that polynomials and rationals are closed under addition, subtraction, and multiplication (and non-zero division for rationals). *Attend to precision* while using these operations with polynomials and rationals. (Sk 3, 4)

CS3.A.4

Build graphic models of polynomial functions through identifying zeros, either by factoring or applying the Remainder Theorem. (Sk 2)

CS3.A.5

Describe and represent structures and patterns that systematically expand and simplify polynomial sums, products, and powers. (Sk 1)

CS3.A.6

Select and use tools appropriately and strategically to solve simple rational, radical and polynomial equations in one variable. (Sk 8)

CS3.A.7

Strategically select and use the appropriate tools to solve compound inequalities in one variable, including absolute value inequalities. (Sk 8)

Number System (NS)

Use complex numbers in polynomial identities and equations. Build on work with quadratic equations in Secondary Mathematics II. Emphasize the essential role of complex numbers in solving polynomial equations in satisfying the conditions for the Fundamental Theorem of Algebra. Build on the understanding of operations of real numbers to include imaginary numbers. Perform operations with complex numbers. Represent complex numbers and their operations on the complex plane. Use complex numbers in polynomial identities and equations.

CS3.NS.1

Describe and represent structures and patterns that emerge when performing algebraic operations with complex numbers on the complex plane. (Sk 1)

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CS3.NS.2

Construct, justify, and communicate clear and reasonable arguments that verify, according to the Fundamental Theorem of Algebra, the solutions to polynomial equations will always be numbers within the set of complex numbers and will have the form a + bi where and b are real numbers. Find solutions for polynomials that are limited to integer coefficients. (Sk 4)

Functions (F)

Describe and represent functional relationships and their features. Analyze functions using different representations, and contextualize functions and their features when appropriate. Develop and assess conjectures to support the work of building functions that model a relationship between two quantities. Develop an understanding of how models can be refined to model more complex or sophisticated situations. Construct and compare linear, quadratic, and exponential models and solve problems. Interpret expressions for functions in terms of the situation it models. Build and use parametric and graphic models. Extend the definition of trigonometric ratios beyond 0 to 90 degrees using the unit circle and use trigonometric functions to model periodic behavior. Prove and apply trigonometric identities.

Standards with the prefix *S3* represent standards from the *Math for All Students* content. Standards with the prefix *CS3* are specific to the Calculus Pathway.

S3.F.1

Understand and interpret the features of a function represented graphically, numerically and symbolically by *applying and removing context*. Focus on these key features: domain, range, intervals of increase and/or decreasing, location of absolute maximum and/or absolute minimum, intercepts and end behavior. *Build* graphic *models* that satisfy given key features. (Sk 2, 5)

S3.F.2

Build representations that show key features of polynomial, rational, square root, cube root, exponential, logarithmic, and trigonometric functions expressed

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symbolically. *Select and use appropriate tools to* model with and without technology. (Sk 2, 8)

S3.F.3

Compare and contrast the properties of these functions polynomial, rational, square root, cube root, exponential, logarithmic, and trigonometric functions. *Formulate and ask questions* to highlight the similarities and differences between function types. (Sk 6)

S3.F.4

Identify the effect on the graph when replacing f(x) by f(x) + k, kf(x), f(kx), f(x+k) by exploring specific values of k for polynomial, rational, square root, cube root, exponential, logarithmic, and trigonometric functions in and out of context**.** *Attend to precision* of language and notation while *constructing arguments* and carefully evaluate whether the ideas and processes observed during exploration are *reasonable*. (Sk 3, 4)

S3.F.5

Use models to explore and *describe* attributes of the *structure, patterns and relationships* of inverses for linear, exponential and quadratic functions**.** *Strategically use tools* to build a model for the inverse of a linear, exponential, or quadratic function. (Sk 1, 2, 8)

S3.F.6

Communicate clear and reasonable arguments to distinguish between situations that can be *modeled* with polynomial, rational, square root, cube root, exponential, logarithmic, and trigonometric functions. *Justify* that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. (Sk 2, 4)

S3.F.7

Describe structures and patterns to extend the domain of trigonometric functions beyond 0 to 90 degrees to *build and use models* of periodic phenomena with sine and cosine functions with specified amplitude, frequency, and midline. (Sk 1, 2)

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CS3.F.8

Build and use parametric and graphic *models* to represent curves, including rational functions, that are represented symbolically. *Construct and communicate clear arguments* that identify and compare properties of rational functions, including zeros, asymptotes, points of discontinuity, and end behavior. (Sk 2, 4)

CS3.F.9

Build and use representations of piecewise-defined functions, including absolute value functions. *Construct, justify, and communicate clear arguments* that compare and contrast piecewise-defined functions with all other secondary functions. (Sk 2, 4)

CS3.F.10

Build and use expressions including sigma notation to represent the sum of a finite arithmetic or geometric series. (Sk 2)

CS3.F.11

Build and use algebraic, graphic, and numeric *models* that represent series. (Sk 2)

CS3.F.12

Build a new function from a composition of functions that *models* a relationship between two quantities. (Sk 2)

CS3.F.13

Construct, justify and communicate clear arguments to show that one function is the inverse of another function using tables, graphs, and equations. *Build* an invertible function from a non-invertible function by restricting the domain. (Sk 2, 4)

CS3.F.14

Decontextualize trigonometric equations in modeling contexts and *attend to precision* while using inverse trigonometric functions to solve equations and inequalities. Use *tools strategically* to evaluate and *contextualize* solutions to determine *reasonableness.* (Sk 3, 5, 8)

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CS3.F.15

Explore representations, structures and patterns between exponential and logarithmic functions to describe the inverse relationship between them. *Strategically use* the developed understanding of representations, structures, and patterns in this inverse relationship as *tools* to solve problems involving logarithms and exponents. *Justify* the properties of logarithms, connecting them to the properties of exponents. (Sk 1, 4, 8)

Geometry/Trigonometry (G)

Apply geometric reasoning to real world modeling situations. Apply prior understanding of trigonometric ratios to all types of triangles using the Law of Sines and Cosines.

Standards with the prefix *S3* represent standards from the *Math for All Students* content. Standards with the prefix *CS3* are specific to the Calculus Pathway.

S3.G.1

Use geometric shapes, their measures, and their properties to *model* real world objects and apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost). *Use these models* to *ask targeted and probing questions* with respect to area and volume *in context*. (Sk 2, 5, 6)

S3.G.2

Apply concepts of density based on area and volume in *modeling* situations. (Sk 2)

CS3.G.3

Construct, justify and communicate clear and reasonable arguments with respect to the validity of the Law of Sines and the Law of Cosines. *Contextualize and decontextualize* situations where the Law of Sines and Law of Cosines can be used to solve problems and find unknown measures. (Sk 4, 5)

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CS3.G.4

Use special triangles as *tools* to construct the unit circle with degrees and radians. *Describe and represent patterns* in the unit circle and use them as *tools* to evaluate the trigonometric functions at multiples of π/4 and π/6. (Sk 1, 8)

CS3.G.5

Identify and strategically use the addition and subtraction formulas for sine, cosine and tangent *as tools* to solve problems. (Sk 8)

Data Science (D)

Understand independence and conditional probability and use them to interpret data and compute probabilities of compound and conditional events in a uniform probability model or using a normal distribution when appropriate. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use technology to estimate areas under the normal curve. Summarize, represent, and interpret data on a single count or measurement variable. Understand and evaluate random processes underlying statistical experiments. Draw and justify conclusions from sample surveys, experiments, and observational studies. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.

Standards with the prefix *S3* represent standards from the *Math for All Students* content. Standards with the prefix *CS3* are specific to the Calculus Pathway.

S3.D.1

Describe and represent structures and patterns of data on two categorical variables. *Attend to precision and reasonableness* when summarizing categorical data in two-way frequency models, and *construct, justify, and communicate clear and reasonable arguments* about possible associations and trends in the data. (Sk 1, 3, 4)

S3.D.2

Build and use probability models including organized lists, ven-diagrams, tree

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diagrams, and two-way tables to find or estimate probabilities of compound or conditional events. (Sk 2)

S3.D.3

Use multiple representations as tools to *describe and represent structure* to building understanding of the addition rule P(A or B) = P(A) + P(B) – P(A and B) and understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of B given A is the same as the probability of B. (Sk 1)

S3.D.4

Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. *Select and use tools appropriately and strategically* to estimate areas under the normal curve. (Sk 8)

S3.D.5

Formulate statistical investigative questions that are either summary-based or comparative**,** for surveys, observational studies, or experiments using primary and/or secondary data. Select *appropriate statistical tools* to develop a plan for data collection and analysis to address these questions. (Sk 6, 8)

S3.D.6

Select appropriate statistical tools to implement a data collection plan for primary data related to an investigative question. When using secondary data, assess its validity and relevance to answering the question. *Construct an argument justifying* the chosen data collection method (survey, observational study or experiment). Understand and apply best practices for designing sample surveys, experiments, and observational studies. (Sk 4, 8)

S3.D.7

Attend to precision and reasonableness when summarizing collected data to answer an investigative question using tables, graphs, and numerical summary statistics. *Develop and use models* from sample data to estimate population means or proportions, including constructing prediction intervals. Employ simulation models

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for random sampling to develop a margin of error, determine approximate sampling distributions, and compute p-values from those distributions. (Sk 2, 3)

S3.D.8

Use statistical evidence from the analyses to *evaluate results* and answer statistical investigative questions. *Communicate justifications* for outcomes or estimates of population characteristics, considering their plausibility compared to chance variation, in a variety of formats (verbal, written, visual). Interpret the margin of error associated with estimates and appropriately communicate the interpretation of simulated p-values. (Sk 4, 7)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of Secondary III (Calculus), students will be able to:

- understand, compare and represent functions including polynomial, rational, trigonometric, logarithmic, and inverse functions. (F)
- build on prior knowledge of key features of transformations of linear, quadratic and exponential models and extend to polynomial, rational, logarithmic and trig functions (sine and cosine). (F)
- build off prior knowledge of congruency and similarity and right triangle ratios to extend the domain of trigonometric functions (sine and cosine). (F)
- distinguish between situations that can be modeled with polynomial, rational, square root, cube root, exponential, logarithmic, and trigonometric functions. (F)

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- compute probabilities for compound and conditional events and use them to interpret data. (D)
- draw and justify conclusions from sample surveys, experiments, and observational studies. (D)
- describe, represent and build polynomials beyond quadratic, using zeros and factors. (A)
- find and use zeroes of polynomials to graph polynomial functions. (A)
- solve rational and radical equations and inequalities and compound inequalities. (A)
- perform operations with complex numbers. (NS)
- apply trigonometric identities and inverse trigonometric functions in solving equations and inequalities. (F)

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Standards: Secondary Math III (Data Science Pathway)

Introduction

In secondary math I through secondary math III, the major works of each grade will focus on learning to create, interpret, manipulate, and solve algebraic equations. Students develop an understanding of functions and learn how to compare and represent functions as defined by rates of change, multiple representations, and building functions. Students will learn to define functions and describe their features and transformations. Students will understand, apply and prove congruence of similarity as defined in terms of geometric transformations. Students will also investigate statistical questions by creating a plan to collect data for a non-biased sample from a population for primary data or ask questions about how secondary data was collected and whether it is useful. They will analyze data using graphical displays and numerical summaries of the sample data and use the evidence from that analysis to answer the statistical question of interest.

In secondary mathematics III instructional time should focus on four major works: (1) Solve algebraic equations (polynomial, logarithmic, radical, rational and trigonometric), (2) Understand, compare and represent functions (polynomial and inverse), (3) describe characteristics of functions (4) extend congruence and similarity and (4) Draw and justify conclusions from sample surveys, experiments, and observational studies.

The five pillars below are the foundations of the mathematics students explore in the data science pathway. The DS3 standards are an expression of these pillars:

1. Data Acquisition and Preparation:
    - Create usable datasets by collecting, filtering, cleaning, and scaling data, while prioritizing data privacy.
2. Exploratory Data Analysis:

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- Describe patterns and variations within data by analyzing visualizations and identifying trends and features. Provide contextual interpretations of visual and numerical findings. Explore relationships between variables and formulate questions to guide further analysis.

3. Computational Thinking:
    - Frame problems for computational solutions. Apply algorithmic thinking to decompose complex problems into manageable parts, enabling reuse of components across applications.
4. Data-Driven Decision Making:
    - Build and apply models relevant to specific contexts for real-world applications. Critically evaluate the ethical implications of generalizations. Justify conclusions based on rigorous data analysis.
5. Model Evaluation and Validation:
    - Assess the validity and potential bias of models. Consider the role of randomness in inference and computation. Evaluate the representativeness of data in relation to model validity. Distinguish between statistical significance and practical importance. Conduct hypothesis testing to validate findings.

Mathematical Skills

Link to Detailed Skills Description

Skill 1: Describe and represent structures, patterns, and relationships

In high school, students describe the structures of linear and exponential expressions by identifying whole expressions being made up of parts, like terms and factors. Students describe and represent underlying structures to help define dilations and similar shapes, and then use these structures to solve problems. Students will use models to explore and describe attributes of the structure, patterns and relationships of inverses for various function types.

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Skill 2: Build and use models

In high school, students will build and use models in two primary ways. Students use models to connect mathematical concepts and to model real-world phenomena. Students will create and use verbal, contextual, visual, symbolic, and physical models to connect mathematical ideas and enhance their understanding. Students will use these models to describe and represent patterns and relationships to compare function types and connect mathematical ideas such between them.

To model real-world phenomena, students will engage in *mathematical and statistical modeling* to represent, analyze, and predict real-world situations. In high school students will use geometric shapes, their measures, and their properties to *model* real world objects and apply geometric methods to solve design problems.

Skill 3: Attend to precision and reasonableness

In high school, students attend to precision and reasonableness when analyzing data to compare two or more distributions, which includes selecting appropriate graphical displays and analyzing measures of center and variability. Students will attend to precision and reasonableness when exploring associations between two quantitative variables, using tools like scatter plots and correlation to evaluate the reasonableness and strength of models. Students attend to precision of language and notation while constructing arguments and carefully evaluating the reasonableness of observed ideas and processes when exploring transformations of functions.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

In high school, students construct, justify, and communicate clear and reasonable arguments about reflection and rotational symmetries of parallelograms and regular polygons. Students construct, justify and communicate proof of the Pythagorean Theorem using triangle similarity. Students construct arguments to distinguish between situations that can be modeled by various function types. They will construct arguments to justify claims such as why exponential growth

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eventually surpasses linear, quadratic or the growth in any other polynomial function.

Skill 5: Add or remove context to make sense of mathematics

In high school, students contextualize and decontextualize functions to interpret their properties, connecting symbolic representations to meaningful situations. Students will interpret the features of a function represented graphically, numerically and symbolically by applying and removing a context. Students interpret the parts of an expression in context to make sense of the underlying structure of polynomial and rational expressions.

Skill 6: Ask questions to explore mathematical ideas

In high school, students will formulate and ask questions to highlight the difference between various function types. Students will ask targeted questions about univariate data displays to explore mathematical ideas. Students will ask questions about the underlying structure of geometric objects revealed by geometric constructions.

Skill 7: Make conjectures and evaluate the results

In high school, students develop and assess conjectures as they form hypotheses and assess their validity when evaluating results derived from statistical analyses to answer investigative questions. Students will make conjectures about the reasonableness and strength of the model they choose to describe the association between two quantitative variables. Students will develop, assess, and evaluate conjectures about properties of exponents for rational exponents.

Skill 8: Select and use tools appropriately and strategically

In high school, students will learn to select and leverage multiple types of tools while exploring mathematics. Opportunities to leverage tools include, but are not limited to, selecting appropriate algebraic tools to solve linear, exponential, or quadratic equations for a variable. Students will select and use appropriate tools to build representations for various function types with and without technology. Students will use mathematical ideas as a tool to validate the appropriateness and

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usefulness of a model.

Standards-Secondary III (Data Science)

More details about the standards, including relevant vocabulary, representations that support student learning, pre-requisite skills, and how the skills are integrated, can be found in the Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Algebra (A)

Interpret the structure and patterns of polynomial and rational expressions.

S3.A.1

Describe, represent, and interpret the structures and patterns of polynomial and rational expressions by identifying whole expressions as being made of parts, like terms and factors. Interpret these structures in a *context* and use them to produce equivalent forms that reveal different properties. (Sk 1, 5)

Functions (F)

Describe and represent functional relationships and their features. Analyze functions using different representations, and contextualize functions and their features when appropriate. Develop and assess conjectures to support the work of building functions that model a relationship between two quantities. Develop an understanding of how models can be refined to model more complex or sophisticated situations. Construct and compare linear, quadratic, and exponential models and solve problems. Interpret expressions for functions in terms of the situation it models. Extend the definition of trigonometric ratios beyond 0 to 90 degrees and use trigonometric functions to model periodic behavior.

S3.F.1

Understand and interpret the features of a function represented graphically, numerically and symbolically by *applying and removing context*. Focus on these key features: domain, range, intervals of increase and/or decreasing, location of

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absolute maximum and/or absolute minimum, intercepts and end behavior. *Build* graphic *models* that satisfy given key features. (Sk 2, 5)

S3.F.2

Build representations that show key features of polynomial, rational, square root, cube root, exponential, logarithmic, and trigonometric functions expressed symbolically. *Select and use appropriate tools* to model with and without technology. (Sk 2, 8)

S3.F.3

Compare and contrast the properties of these functions polynomial, rational, square root, cube root, exponential, logarithmic, and trigonometric functions. *Formulate and ask questions* to highlight the similarities and differences between function types. (Sk 6)

S3.F.4

Identify the effect on the graph when replacing f(x) by f(x) + k, kf(x), f(kx), f(x+k) by exploring specific values of k for polynomial, rational, square root, cube root, exponential, logarithmic, and trigonometric functions in and out of context**.** *Attend to precision* of language and notation while *constructing arguments* and carefully evaluate whether the ideas and processes observed during exploration are *reasonable*. (Sk 3, 4)

S3.F.5

Use models to explore and *describe* attributes of the *structure, patterns and relationships* of inverses for linear, exponential and quadratic functions**.** *Strategically use tools* to build a model for the inverse of a linear, exponential, or quadratic function. (Sk 1, 2, 8)

S3.F.6

Communicate clear and reasonable arguments to distinguish between situations that can be modeled with polynomial, rational, square root, cube root, exponential, logarithmic, and trigonometric functions. *Justify* that a quantity increasing

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exponentially eventually exceeds a quantity increasing linearly or quadratically. (Sk 2, 4)

S3.F.7

Describe structures and patterns to extend the domain of trigonometric functions beyond 0 to 90 degrees to *build and use models* of periodic phenomena with sine and cosine functions with specified amplitude, frequency, and midline. (Sk 1, 2)

Geometry (G)

Apply geometric reasoning to real world modeling situations.

S3.G.1

Use geometric shapes, their measures, and their properties to *model* real world objects and apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost). *Use these models* to *ask targeted and probing questions* with respect to area and volume *in context*. (Sk 2, 5, 6)

S3.G.2

Apply concepts of density based on area and volume in *modeling* situations. (Sk 2)

Data Science (D)

Understand independence and conditional probability and use them to interpret data and compute probabilities of compound and conditional events in a uniform probability model or using a normal distribution when appropriate. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use technology to estimate areas under the normal curve. Summarize, represent, and interpret data on a single count or measurement variable. Understand and evaluate random processes underlying statistical experiments. Draft and justify conclusions from sample surveys, experiments, and observational studies. Develop the concept of statistical significance informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.

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Standards with the prefix *S3* represent standards from the *Math for All Students* content. Standards with the prefix *DS3* are specific to the Data Science Pathway.

S3.D.1

Describe and represent structures and patterns of data on two categorical variables. *Attend to precision and reasonableness* when summarizing categorical data in two-way frequency models, and *construct, justify, and communicate clear and reasonable arguments* about possible associations and trends in the data. (Sk 1, 3, 4)

S3.D.2

Build and use probability models including organized lists, venn-diagrams, tree diagrams, and two-way tables to find or estimate probabilities of compound or conditional events. (Sk 2)

S3.D.3

Use multiple representations as tools to *describe and represent structure* to building understanding of the addition rule P(A or B) = P(A) + P(B) – P(A and B) and understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of B given A is the same as the probability of B. (Sk 1)

S3.D.4

Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. *Select and use tools appropriately and strategically* to estimate areas under the normal curve. (Sk 8)

S3.D.5

Formulate statistical investigative *questions* that are either summary-based or comparative**,** for surveys, observational studies, or experiments using primary and/or secondary data. Select *appropriate statistical tools* to develop a plan for data collection and analysis to address these questions. (Sk 6, 8)

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S3.D.6

Select appropriate statistical tools to implement a data collection plan for primary data related to an investigative question. When using secondary data, assess its validity and relevance to answering the question. *Construct an argument* justifying the chosen data collection method (survey, observational study or experiment). Understand and apply best practices for designing sample surveys, experiments, and observational studies. (Sk 4, 8)

S3.D.7

Attend to precision and reasonableness when summarizing collected data to answer investigative questions using tables, graphs, and numerical summary statistics. *Develop and use models* from sample data to estimate population means or proportions, including constructing prediction intervals. Employ simulation models for random sampling to develop a margin of error, determine approximate sampling distributions, and compute p-values from those distributions. (Sk 2, 3)

S3.D.8

Use statistical evidence from the analyses to *evaluate results* and answer statistical investigative questions. *Justify* outcomes or estimates of population characteristics, considering their plausibility compared to chance variation, in a variety of formats (verbal, written, visual). Interpret the margin of error associated with estimates and appropriately communicate the interpretation of simulated p-values. (Sk 4, 7)

DS3.D.9

Represent categorical and quantitative information in organized tables suitable for analysis. *Attend to precision* in data processing by filtering through logical statements, and transforming data with a detailed set of ordered tasks that are repeatable by others or a computer. (Sk 1, 3)

DS3.D.10

Describe univariate displays. *Ask targeted questions* about the displays. (Sk 1, 6)

DS3.D.11

Describe and represent relationships between two quantitative variables. (Sk 1)

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DS3.D.12

Build and use models to describe relationships between two continuous variables, including linear regression. *E*xplain which variable(s) should be used for prediction. *Construct reasonable arguments* for appropriate and inappropriate use of the model for practical decision making. (Sk 2, 4)

DS3.D.13

Describe the relationship between the distribution of a population, the distribution of a sample, and sampling distributions, ie., the distribution of sample averages. *Describe* sampling distributions through summary statistics, including the mean and the standard deviation. Use the central limit theorem to *represent* the distribution of sample averages as a normal distribution and *assess conjectures* about the sample averages through probability calculations from the normal distribution. (Sk 1, 7)

DS3.D.14

Use Mathematics as a tool to validate the appropriateness and usefulness of a model, including visual and numerical assessments of model fit. Use hypothesis testing and simulation to *develop and assess conjectures* about the model. (Sk 7, 8)

DS3.D.15

Evaluate results reached from data analysis. *Justify and communicate clear and reasonable* conclusions, including the appropriate scope of generalization, and *contextualize* the implications through data storytelling. (Sk 4, 5, 7)

Essential Competencies

Essential competencies have been identified at each grade level to represent foundational understandings that students need to succeed as they continue in future grade levels. This list is not exhaustive; students will engage with all grade-level standards. Educators may prioritize instructional time and, as needed, provide interventions based on the standards that support these competencies to ensure students are proficient. Through engagement with the skills and knowledge embedded within the *Utah Core Mathematics Standards*, by the end of Secondary III (Data Science), students will be able to:

- Understand, compare and represent functions including polynomial, rational, trigonometric, logarithmic, and inverse functions. (F)

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- Build on prior knowledge of key features of transformations of linear, quadratic and exponential models and extend to polynomial, rational, logarithmic and trig functions (sine and cosine). (F)
- Build off prior knowledge of congruency and similarity and right triangle ratios to extend the domain of trigonometric functions (sine and cosine). (F)
- Distinguish between situations that can be modeled with polynomial, rational, square root, cube root, exponential, logarithmic, and trigonometric functions. (F)
- Build and use probability models. (D)
- Compute probabilities for compound and conditional events and use them to interpret data. (D)
- Estimate areas under the normal curve. (D)
- Ask investigative questions and collect data related to those questions. Understand and apply best practices for designing sample surveys, experiments, and observational studies. (D)
- Summarize collected data to answer investigative questions using tables, graphs, and numerical summary statistics. Transform data. (D)
- *Describe and represent structures and patterns of* data on two categorical variables, two quantitative variables, and for univariate data. (D)
- *Build and use models* to describe relationships between two continuous variables, including linear regression. *E*xplain which variable(s) should be used for prediction. (D)
- *Develop and use models* from sample data to estimate population means or proportions. (D)
- *Describe* sampling distributions through summary statistics, including the mean and the standard deviation. Use the central limit theorem to *represent* the distribution of sample averages as a normal distribution. (D)

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- Use hypothesis testing and simulation to *develop and assess conjectures* about models. Evaluate appropriateness of models. (D)
- Evaluate results from data analysis. *Justify and communicate* conclusions, including appropriate scope of generalization, and *contextualize* the implications. (D)

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Appendix

Detailed Skills Descritptions

Mathematical Skills

The mathematical skills included in *Utah's P-12 Core Mathematics Standards* describe mathematical practices that Utah students develop throughout their educational experiences. Students become proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills. Each grade-level's standards have a description in their introductory material specifying the ways students will develop these skills as they learn math concepts. Although specific Mathematical Skills have been intentionally identified and connected to each standard, instruction and practice of other Mathematical Skills not referenced in that standard is encouraged. Below, general descriptions are provided for each skill.

Descriptions of what the skills look like in each grade band are included in the grade level standards. More details about the skills and how they are integrated throughout the standards are included in each grade level's Utah Mathematics Core Guides.

(Utah Mathematics Core Guides will be updated after these standards are approved by the Utah State Board of Education)

Skill 1: Describe and represent structures, patterns, and relationships

Students learn the language of mathematics through exploration (making sense of a mathematical task), observation (noticing patterns and relationships in mathematics), and highlighting structure (using mathematical tools to organize those relationships). Students use their understanding to make generalizations about the structure of numbers to become more flexible and efficient in applying mathematical ideas.

Students build these skills when they have opportunities to explore new topics collaboratively and independently. As they surface patterns and relationships, they apply mathematical structures to organize ideas and work toward making mathematical generalizations.

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Skill 2: Build and use models

Mathematical models use structures to represent relationships. Students build and use verbal, contextual, visual, symbolic, and physical models to represent and make sense of the relationships between quantities in real-world situations and mathematical problems.

Using models enhances student learning by providing students with multiple ways to represent and understand mathematical concepts, connect mathematical ideas to real-world situations, and develop problem-solving skills. Students explore the idea that various models can showcase mathematical thinking in valuable ways. Using multiple models helps students notice the connections between mathematical structures and promotes the development of mathematical generalizations that can be leveraged as tools to make sense of and persevere in solving novel problems.

Skill 3: Attend to precision and reasonableness

Students exercise mathematical precision and reasonableness in their communication, calculations, and processes. They calculate flexibly, accurately, and efficiently, using clear and concise notation to record their work. They regularly evaluate whether actions, ideas, processes, and solutions make sense and are reasonable.

Precision in communication and calculation fosters a deeper understanding of mathematical concepts. Using precise language, students articulate their thoughts clearly, enhancing their ability to explain their reasoning to others and understand others' ideas. Accurate and efficient calculations and precise notation enable students to solve problems effectively and minimize errors. The continuous evaluation of reasonableness ensures that students improve their ability to identify and learn from mathematical errors.

Skill 4: Construct, justify, and communicate clear and reasonable arguments

Students construct arguments using the mathematical reasoning that underlies a

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strategy, solution, or conjecture. They justify their conjectures and clearly communicate their reasoning using concrete referents such as objects, drawings, diagrams, and actions. They justify their arguments with peers through discourse, refining their thinking to create more robust and precise arguments.

Constructing, justifying, and communicating arguments fosters a deep understanding of mathematical concepts. By defending their conjectures, students solidify their mathematical understanding and identify potential flaws in their reasoning. Clear communication enables students to articulate their thoughts effectively, enhancing their ability to explain their reasoning to others. Discourse with peers exposes students to diverse perspectives, promoting critical thinking and collaborative problem-solving.

Skill 5: Add or remove context to make sense of mathematics

Students add or remove context to help make sense of mathematical ideas. Students add context by connecting numbers and operations to images, stories, objects, and events. They remove context from a situation by representing the mathematics using numbers, symbols, and equations. Students visualize complicated concepts as being composed of simpler parts. They identify quantities in a contextual situation, use mathematical models to represent and analyze the relationships between those quantities, and draw conclusions.

When students contextualize and decontextualize mathematical ideas, they can make sense of underlying structures by connecting representations. Working in and out of context throughout a learning progression empowers students to make sense of mathematics and persevere as they explore abstract and real-world applications. This process allows students to take their knowledge of mathematics and meaningfully apply it to their lives.

Skill 6: Ask questions to explore mathematical ideas

Students examine mathematical concepts and formulate questions based on what they notice and wonder about given situations. During mathematical discourse, students ask probing questions of one another as they examine arguments and seek to understand others' ideas.

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Asking questions cultivates students' curiosity about mathematical relationships and helps them refine their understanding. As students build their capacity to ask probing questions to their peers, they engage in more meaningful discourse about mathematics. When students ask and explore rich questions, it advances the mathematical understanding of all learners.

Skill 7: Make conjectures and evaluate the results

Students make sense of mathematics when they engage in mathematical investigations. They use their understanding of structures, patterns, and relationships to speculate about the nature of a relationship and then test their conjecture to determine how to proceed. As they test their mathematical hypotheses and evaluate the outcomes of their investigation, they build mathematical generalizations.

Developing and assessing conjectures encourages active engagement and critical thinking. Conjecturing acts as an entry point to exploration for all students. When making conjectures, students have opportunities to continually revise their thinking. Students formulate conjectures, test them, and refine their understanding based on the outcomes. Students view mistakes as assets that help them better refine their mathematical understanding. This process fosters a deeper understanding of underlying mathematical principles. It promotes curiosity, students' ownership of their learning, and the development of positive mathematical identities.

Skill 8: Select and use tools appropriately and strategically

Students strategically select and use relevant and valuable tools for mathematical tasks with which they engage. Tools can be physical objects (e.g., compasses or manipulatives), technological tools (e.g. graphing calculators or virtual manipulatives) or mathematical constructs (e.g., diagrams, strategies, or algorithms).

Students consider the available mathematical problem-solving tools and choose the most relevant and useful ones. When students have the agency to select mathematical tools strategically, it empowers them to visualize, represent, and

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understand mathematical concepts effectively, fostering critical thinking, problem-solving flexibility, perseverance, and engagement.

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