Preschool-12 Mathematics Standards Board Member Recommendations

This report contains board member recommendations as part of Step 5 of the Utah State Board of Education's approved *Standards Revision Process* (2023) - Board members are encouraged to submit recommendations pertaining to revisions for the full Board's consideration.

Board Member Cindy Davis

(Also recommended by the Standards Review Committee) The Secondary Mathematics courses outside of Secondary Mathematics I, II and III should be revised and adjusted. For example, we don't have a Data Science option and the Precalculus course has standards that don't really align with preparation for Calculus.

(Also recommended by the Standards Review Committee) Use Utah Secondary Mathematics course taking pattern data to inform decisions about the secondary mathematics courses outside of Secondary Mathematics I, II and III (2023 Utah Mathematics Course Taking Data and 2007-2021 Utah Mathematics Performance. Reduce the number of additional courses that can be used to substitute for Secondary Math III and make the additional courses in R277-700-9 more than a recommendation, so that students were taking more mathematics courses.

(Also discussed by the Standards Review Committee) What can we attribute the successes we have seen in recent years on PISA and NAEP? Those measures suggest that we are doing some good things with elementary and early secondary mathematics and probably should not make too many adjustments, rather continue the work and seek to further improve instructional practices. NAEP article about Utah 8th Grade Math PISA article about Utah math comparison

(Also recommended by the Standards Review Committee) Emphasis on the nature of instruction and the level at which a mathematics teacher is applying effective mathematics teaching practices (NCTM, 2104) is significant. In addition to content standards, we need to continue to support improvement of instructional practices.

Small revisions to the standards for Secondary Math 1, 2 and 3 should be made to increase their clarity and focus, including:

Secondary Math 1
Keep all standards as they are and consider the revisions suggested below with strikethrough.
Standard A.REI.3 Solve equations and inequalities in one variable.

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Board Member Cindy Davis Continued:

a. Solve one-variable equations and literal equations to highlight a variable of interest.
b. Solve compound inequalities in one variable, ~~including absolute value inequalities.~~
c. Solve simple exponential equations that rely only on application of the laws of exponents (limit solving exponential equations to those that can be solved without logarithms). For example, 5x = 125 or 2x = 1/16.

Standard F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, ~~positive, or negative~~; relative maximums and minimums; symmetries; ~~and end behavior~~.
(Is also in Math 2 and can have additional features added then.)

~~Standard F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.~~ (Is also in Math 2 and is a better fit there.)

Standard S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ~~Calculate the weighted average of a distribution and interpret it as a measure of center~~

Secondary Math 2
Keep all standards as they are and consider the revisions suggested below with strikethrough.
Standard F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Focus on quadratic functions and ~~consider~~ include~~ing~~ absolute value functions. Experiment with cases and illustrate an explanation of the effects on the graph using technology. ~~Include recognizing even and odd functions from their graphs and algebraic expressions for them.~~ (Even and odd functions can be a focus in Math 3.)

~~Standard G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.~~

~~Standard G.C.4 Construct a tangent line from a point outside a given circle to the circle~~

~~Standard G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane~~
~~is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin~~

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Board Member Cindy Davis Continued:

~~and containing the point (0, 2).~~ (Already in Math 1, it doesn't need to be repeated in this course.)

Secondary Math 3
Keep all standards as they are and consider the revisions suggested below with strikethrough.

~~Standard N.CN.9 Know the Fundamental Theorem of Algebra. Limit to polynomials with real coefficients.~~ (In Math 2, could be removed from there or here course.)

Standard A.SSE.4 Understand the formula for the sum of a series and use the formula to solve problems.
a. Derive the formula for the sum of an arithmetic series.
b. Derive the formula for the sum of a geometric series, and use the formula to solve problems. Extend to infinite geometric series. For example, calculate mortgage payments
(Make part of the Extended courses don't need to be in regular Math 3)

Standard A.APR.4 Use ~~Prove~~ polynomial identities ~~and use them~~ to describe numerical relationships. For example, the polynomial identity (x2 + y2)2 = (x2 – y2)2 + (2xy)2 can be used to generate Pythagorean triples.

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Board Member Matt Hymas:

I have noticed that students in grades K-5 are asked to use logical steps to formulate responses when they have not been taught the foundational pieces. How do we get back to math facts for our students? Quick recall of arithmetic without having to draw pictures, count fingers, or use manipulatives? Our upper elementary and middle school students are struggling to think logically as well because without a solid foundation of facts, each logical question causes a restart in the entire thought process. I would like to see the trivium (grammar pk-5, logic 6-9, rhetoric 10-12) used in our standards to guide educators on what they are teaching.

Board Member LeAnn Wood:

Strand: MATHEMATICAL PRACTICES - This strand doesn't seem to align with the definition of a standard -"identify the basic knowledge, skills, and competencies each student is expected to acquire or master as the student advances through the public education system;" I'm concerned that the early grades don't yet have a proper foundation to make sense of problems and persevere, reason abstractly, construct arguments and critique others, etc. This may be an appropriate strand for higher level math classes. I think we need to start with the basics and eliminate this strand for K-5.

(Also recommended by the Standards Review Committee) My biggest concern is that I have heard that throughout the nation math standards are being simplified and are not adequately preparing our students for college and career options. Whatever the writing committee does - please, please do not fall prey to this simplification. Math standards need to have appropriate age progression, they need a strong foundation of numeracy skills and computation that can be built upon itself. Memorization of basic number facts needs to be prioritized so that students can work ahead with that solid foundation. Standards need to be clearly stated as to what concept the students should be learning. We need to look at the progression from Algebra 1 to higher level math classes and make sure the standards are taught at an appropriate grade.

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Vice Chair Jennie Earl

Recommendations for math standards:
This is my expectation for Utah Math Standards as we look at improvements to our current standards for Utah students.

Currently Utah has a proficiency rate of 35-49% for grades 3-8. What will it take to create an environment where proficiency is reached by more than 80-100% of our students each year at every grade level? Clearer standards? Revised assessments? Educator training?

We can lead out with improved standards and addressed training as well as assessments later.

I envision:

- *(Also recommended by the Standards Review Committee)* Utah Standards specially designed to build student's math knowledge over time through carefully planned, intentionally sequenced strands leading to mathematical mastery at each grade level, focusing on principles, and theories that make up math discipline's content.
- *(Also recommended by the Standards Review Committee)* An environment where all students understand that they can successfully learn and apply math concepts.
- An environment where all educators understand that every student in their class can learn and apply math concepts.
- An environment where educators are adequately prepared to teach math.
- *(Also recommended by the Standards Review Committee)* Classrooms that build upon student's prior math knowledge from previous grades and effectively apply new content.
- *(Also recommended by the Standards Review Committee)* Standards that use clear, concise, and correct math language.
- *(Also recommended by the Standards Review Committee)* Adequate practice and guidance to reinforce student understanding and application of grade level math concepts.
- Students who are fluent in key foundational math concepts leading to automatic retrieval of these concepts allowing more time to be spent on complex mathematical tasks and executing multi-step mathematical procedures.
- Intentional instruction on strategies that help solve math problems.
- Concrete number sense leading to the application of math procedures with useful examples for the age of a child.

Recommendations specific to new standards:

- *(Also discussed by the Standards Review Committee)* Review the components of California's 1999 standards and use as a framework for improving Utah standards.
- Avoid weaving language from Portrait of a Graduate or other documents into math standards. Rationale: Autonomy, critical thinking, responsibility, collaboration,

innovation, etc. are the outcome of clear and concise mathematical standards applied and practiced in Utah classrooms. The end goal for elementary students should be to accurately solving straightforward problems in a straightforward way and not on "creative or innovative" ways to solve problems as an end goal. To further illustrate this our proficiency rates at each grade level have rarely exceeded 50%. This means that 50% or more are not proficient with age appropriate math content but are moving forward to the next grade without foundational mastery of basic math skills. Do we want them proficient in innovation or in their ability to solve two digit subtraction problems? We need to be careful that we are not creating an impossible objective for students at risk of not succeeding in math. Additionally, turning in assignments on time leads to responsible actions and builds responsibility and teaching deep understanding around standard algorithms leads to critical thinking.

- Calculator usage should not start until the 6th grade and shouldn't be used to replace basic math retrieval skills necessary for success in upper grade classrooms.
- Students are be prepared to take algebra by 8th grade
- Increase the number of students qualified for higher level math classes in high school (related to algebra in 8th grade)
- Allow LEAs to choose a different approach besides integrated math in secondary classrooms.
- *(Also recommended by the Standards Review Committee)* Star the most important standards needed to build foundational knowledge at each grade level.
- Articulate when specific mastery of skills should be accomplished. All students should be able to know and do math at each grade level, not just 45% of students.

Recommendations concerning "mathematical practices".

- Simplify language kindergarten example: (current language) **Make sense of problems and persevere in solving them**. "Explain the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. When a solution pathway does not make sense, look for another pathway that does. Explain connections between various solution strategies and representations." How do we measure perseverance?
- When do students have enough knowledge to "construct viable arguments and critique the reasoning of others"? And does this take precedence over a deep understanding of procedures, arithmetic and algorithms?
- **Attend to precision.** ~~Communicate precisely to others by crafting careful explanations that communicate mathematical reasoning by referring specifically to each important mathematical element, describing the relationships among them, and connecting their words clearly to representations.~~ Calculate accurately and efficiently, and use clear and concise notation to record work. Use mathematical language to describe important elements and relationships.

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Vice Chair Jennie Earl Continued

- Combine and simplify **Model with mathematics and Use appropriate tools strategically.** Consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. Choose tools that are relevant and useful to the problem at hand, such as physical objects, drawings, diagrams, models, physical tools, technologies, or mathematical tools, such as estimation or a particular strategy or algorithm. Isn't modeling a strategy used to represent mathematical problems?
- Combine and simplify **Look for and make use of structure and reasoning.** Recognize and apply the structures of mathematics, such as patterns, place value, the properties of operations, or the flexibility of numbers. See complicated things as single objects or as being composed of several objects. Notice repetitions in mathematics when solving multiple related problems. Use algorithms and procedures to observe shortcuts or generalizations. Evaluate the reasonableness of intermediate results.
- Add something around Number sense: Know and flexibly use different properties of operations, numbers, and geometric objects.
- Fluency in addition and subtraction of whole numbers by the end of grade 3, and fluency in multiplication and division by the end of grade 5.
- *(Also recommended by the Standards Review Committee)* Express confidence in students to conceptually understand and problem solve as well as understand arithmetic and basic math skills.

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Board Member Natalie Cline

- *(Also discussed by the Standards Review Committee)* My recommendation to the writing committee is that Utah's current PK-12 mathematics standards be replaced entirely with either California's 1997 math standards (which were developed in association with Stanford University's math department) or Massachusetts' 2000 math standards for the reasons explained below by Wendy Hart in her recent email to the Board. Thank you.

  Dear Utah State Board of Education Members,

  I am so excited to know you will be looking at new math standards. This is one area that parents have been, almost universally, concerned about for over a decade.

  As some of you know, I, along with two others, presented to the board in 2019 on suggested changes to our math standards, and the major problems we have seen with the current standards (https://www.youtube.com/watch?fbclid=IwAR1sWVTBXa2BJQrmioNycD1fTKpkhgEVMI7-HPFMZdARNed35aDlSK5fs58&v=NYoVUVvzJbY&app=desktop at 1hr:16).

  Where you are looking at a complete overhaul, might I suggest two options that come with a track record for success:

  California's 1997 math standards (which were developed in association with Stanford University's math department)
  or Massachussets' 2000 math standards.

  Both of these standards have at least a decade each of success. Rather than reinvent the wheel and hope that we get it right, let's adopt something that has worked, not just well, but exceptionally well, in the past. Both of these sets of standards are in the public domain, and so could be adopted without incurring costs for their development. (If you need/want a copy of either of them, let me know. I will need to search my archives to find them, but I'm sure I can get access to them shortly.) I believe that their state exams are in the public domain, as well.

  One of the most important factors is preparing students to take Algebra I in the 8th grade. Our current standards teach the bulk of Algebra I in 9th grade, putting students at a disadvantage for reaching Calculus by their Senior Year in High School. In order to take Calculus, students must enroll in the Extended track which teaches the equivalent of 4-years' worth of high-level math in only

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3 years (Algebra I, Geometry, Algebra II, and pre-calculus in grades 9-11, leaving grade 12 open for Calculus). As a math major, this would have been too much for me to handle, I believe, let along students who have the capacity for high-level math, but not a desire to make it their life's work. When California implemented these standards, which allowed for the vast majority of students to take (and succeed) in Algebra I in 8th grade, their achievement gaps between demographics and between males and females decreased significantly. It also allowed more students to be prepared for selective colleges and STEM. For a good overview of these achievements and a comparison between these 1997 standards and what we (and California) essentially have to date, please see https://www.hoover.org/research/californias-common-core-mistake.

Another factor is practice. We have eliminated the practice and memorization of math facts in the early grades. Because math is about seeing patterns, without the actual memorization of math facts, especially multiplication, students will be unable to see the patterns necessary in numbers, themselves, and in the higher math activities that these math facts prepare them for. A student who does not know, automatically that 2x3 = 6 and 2x4 = 8 will not see that 3/6 and 4/8 are the same value, anymore than one could see a/b to be the same as c/d.

Additionally, the use of calculators should be eliminated almost entirely. It will require more of the teachers and the textbooks to create problems that come out "nicely" aka have good round numbers as answers (such as the square root of 9 instead of 10), but it has the advantage of a built in check on basic math skills. If you drop a negative, you'll discover it when your answer isn't something you are expecting. This is an excellent exercise for students that comes naturally. Along with that idea of patterns, most patterns in math are not seen using decimals, but rather fractions. The use of calculators obscures those patterns by accepting decimal answers and calculations.

I would also suggest that the practice of project-based learning be rejected. The data doesn't support it, as shown here: https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=&cad=rja&uact=8&ved=2ahUKEwjy4JzFyKGDAxVVIEQIHYNkBD4QFnoECBEQAw&url=https%3A%2F%2Fwww.usf.edu%2Fatle%2Fdocuments%2Fhandout-kirschner-stop-minimally-guided-instruction.pdf.pdf&usg=AOvVaw33tS05JYH4c4c6IhERTuuV&opi=89978449

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Along those lines, Dr. Daniel Willingham, author of Why Students Don't Like School, outlines the need for direct instruction and actual practice of facts. One interesting point of research was that practice doesn't mean doing the exact same thing, but rather being able to expand on the concepts, especially in higher math. He shows a graphic that those who took one course above calculus retained a nearly perfect amount of algebra skills for 50+ years, whereas, without that level of practice, the algebra skills fell off precipitously with less courses that reinforced those skills. (See attached graphic.)

The "Practice Standards" that we hear so much about are said to be "unteachable and untestable" by Dr. W. Stephen Wilson of Johns Hopkins University.
(https://fordhaminstitute.org/national/commentary/guest-post-sbac-math-specifications-dont-add)

Another area of concern is that our geometry was mostly gutted, and the integrated approach is not done in any sort of logically consistent way. The argument that high-achieving countries use an integrated approach is insufficient if we just randomly teach concepts. It is far better to have the courses broken out, and allow students the ability to take Algebra II and Geometry at the same time, should they choose, as well as allowing students who either transfer into Utah or out of Utah to have a consistent experience. Most states do NOT use the integrated approach. Additionally, Algebra II concepts are of such a high-level, that most students are not prepared for them at the age of 14.

And finally, the concepts must be taught in a logically, coherent manner. Most students who have gone through our public education system in the last decade are unaware that math concepts build and support each other. The standards as they currently exist appear to be random and completely unrelated to the average person. Fordham Foundation rated Utah's 2007 math standards (which were jettisoned after only 3 years when we adopted the 2010 standards from the National Governors' Foundation) and said, "The standards are generally very strong and cover most of the essential content with both depth and rigor. The high school standards are particularly strong...Utah's standards are briefly stated and usually clear...the high school content is organized so that standards addressing specific topics...are grouped together in a mathematically coherent way." [emphasis mine]
(http://edexcellencemedia.net/publications/2010/201007_state_education_standards_common_standards/Utah.pdf) Math concepts must be taught in a logically, coherent way.

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Board Member Natalie Cline Continued

In conclusion, please use standards that have a proven track record, such as CA 1997 or MA 2000 math standards. Utah's 2007 standards appear to be fairly good, but they are not as strong as CA or MA previous standards. But they do appear to be much better than the ones that we currently have. I am happy to help in any way that I can on this process.

Let's not allow another generation of Utah students to be left in the lurch when it comes to mathematics. Most students have the ability to learn math when they are shown and given the necessary instruction to learn it. And as a final request, encourage the LEAs to give students textbooks from which to learn. Most high tech execs send their kids to private schools that limit computer time and exposure. Textbooks allow students a better method, especially in math, to be able to go back, study, review, and learn on their own. This is an essential part of learning math...being able to figure it out and to see those patterns that exist.

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